Monsoon Subdivisional Rainfall Dimensionality and Predictability

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Received July 24, 1990; revised January 4, 1991

ABSTRACT

For summer monsoon rainfall purpose India is divided into 35 subdivisions. The daily rainfall series of one such subdivision (Konkan) has been analysed using the phase space approach. Fifteen years (1959–1973) of daily rainfall data have been utilised in this study. The analysis shows that the variability is due to the existing of strange attractor of dimension about 3.8. The predictability is estimated by computing the Lyapunov characteristic exponent. The computations show that the predictability is about 8 days.

I. INTRODUCTION

The atmospheric flows are complex and are governed by nonlinear partial differential equations. The interactions among the various scales of the motions generate the variability in the flows. The recent method to the study of the variability and predictability of the atmospheric flows is based on the phase space approach. The development of this approach is based on the works of Packard et al. (1980), Takens (1981), Grassberger and Procaccia (1983 a,b), Broomhead and King (1986). According to this method, the evolution of the system can be described by the trajectories in the phase space. The coordinates of the phase space are defined by the variables needed to completely describe the evolution of the system. The variability generated in the system is due to the existence of the strange attractor. The dimension of such an attractor gives the measure of variability and next higher integer gives the number of parameters necessary to control the time evolution of the flow. Further the existing of the strange attractor makes the system sensitive to initial conditions. This means that very close initial conditions may lead to completely different paths of the system after a certain amount of time into future. The limits of predictability are decided by rate of divergence of trajectories from nearby initial conditions. The measure of this divergence is given by lyapunov characteristic exponents. The inverse of maximum Lyapunov exponent is taken as the limit of predictability of the long term behaviour of the system.

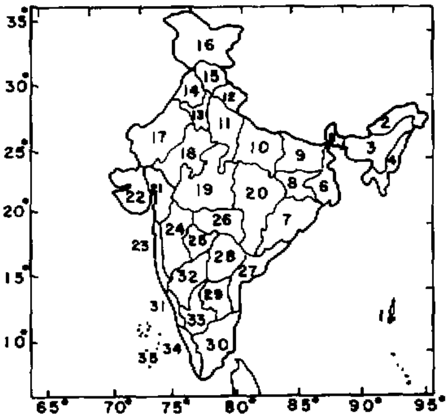
If the mathematical formulation of the system is not available, then the information about the system can be deduced from studying the time series $x(t)$ of a single variable, from that system (Packard et al., 1980; Reulle, 1981; Takens 1981). The theory behind such an approach is that a single variable observable from a dynamical system is the result of all interacting variables and thus any observable variable will contain the information of evolution of the dynamical system.

Fraedrich (1986, 1987) used this approach to the study of the time series of

(1) daily surface pressure at Berlin

(2) oxygen isotope record of planetonic species.

The variability in these parameters was accounted to the low fractal dimensionality. Nicolis and Nicolis (1984) obtained similar results for global paleoclimate; Hense (1987) for



METEOROLOGICAL SUB-DIVISIONS OF INDIA

Fig.1.

1 Andaman and Nicobar islands
2 Arunachal Pradesh
3 Assam and Meghalaya
4 Nagaland, Manipur, Mizoram & Tripura
5 Sub-Himalayan West Bengal & Sikkim
6 Gangetic West Bengal
7 Orissa
8 Bihar Plateau
9 Bihar Plains
10 East Uttar Pradesh
11 Hills of West Uttar Pradesh
12 Plains of West Uttar Pradesh
13 Haryana, Chandigarh & Delhi
14 Punjab
15 Himachal Pradesh
16 Jammu & Kashmir
17 West Rajasthan
18 East Rajasthan
19 West Madhya Pradesh
20 East Madhya Pradesh
21 Gujarat Region, Daman, Dadra & Nagar Haveli
22 Saurashtra, Kutch & Diu
23 Konkan & Goa
24 Madhya Maharashtra
25 Marathwada
26 Vidarbha
27 Coastal Andhra Pradesh
28 Telangana
29 Rayalseema
30 Tamil Nadu & Pondicherry
31 Coastal Karnataka
32 North Interior Karnataka
33 South Interior Karnataka
34 Kerala
35 Lakshadweep

southern oscillation.

The Indian summer monsoon rainfall for the months June to September is the most important factor in the agriculture economy of India. About 75% of the annual rainfall occurs during this period. The monsoon rainfall shows variability on the decadal, interannual and intraseasonal scale. Satyan (1988) used this phase space approach to study the interannual variability of the seasonal Indian monsoon rainfall. He found that the variability in the rainfall was due to the existence of strange attractor whose dimension was about 5.1. The understanding of the daily variations in the rainfall is important from the point of view of short and medium range forecasting. This new approach has not been used to study the daily rainfall variations. An attempt is made here to investigate the cause of daily variations in rainfall of one small region (subdivision) of India, using above mentioned approach. The predictability has been estimated by computing Lyapunov characteristic exponent.

II. DATA

For rainfall purpose India is divided into 35 subdivisions (Fig.1). There is large variability of rainfall from subdivision to subdivision. The selection of a subdivision is based on the criteria that it should have large number of rainy days which will make the terms in time series other than zero. Konkan (No.23 in Fig.1), lying on the west coast of India, is one such subdivision which satisfies above criteria. The mean and standard deviations of Konkan rainfall are 2756.2 mm and 481.8 mm (Shukla, 1987) and about 85% rainy days. The daily rainfall data of the Konkan subdivision are available in the Institute (obtained from India Met. Dept.). Fifteen years of data 1959−1973 have been utilised in the study.

III. COMPUTATIONS

The correlation function $C(r)$ of the attractor of the daily rainfall time series has been estimated using the method given by Grassberger and Procaccia (1983a, b). The method can be briefly explained in the following way. From the time series of daily rainfall $X(t)$, the vectors $\overline{X}(i)$ can be constructed whose space coordinates are $X(i)$, $X(i+L)$, $X(i+2L)$, ...$X(i+(n-1)L)$ where n denotes the dimension in which the given series is embedded and L is the fixed lag or time difference between two observations. The L is chosen such that

$$X(i), X(i+L), X(i+2L), \cdots\cdots, X(i+(n-1)L)$$

become linearly independent, i.e., the correlation between

$X(i)$ and $X(i+L)$, $X(i+L)$ and $X(i+2L)\cdots\cdots$ should become statistically zero. The correlation function $C(r)$ of the attractor is given by

$$C(r) = \frac{1}{N^2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} H[r - |\overline{X}(i) - \overline{X}(j)|] \ , \tag{1}$$

where H is the Heaviside function

$$H(x) = 0 \quad \text{if} \quad x \leqslant 0, \quad H(x) = 1 \quad \text{if} \quad x > 0$$

and r is the distance from $X(i)$, N is the total number of points. For small r the variation of $C(r)$ is given by

$$C(r) = r^d \ , \quad \text{where d } \textit{is the dimension.}$$

The dimensionality d of the attractor is given by the slope of $\log C(r)$ versus $\log(r)$ curve at the origin, if n is chosen sufficiently large. The dimension of the attractor is obtained as saturation value, which does not change if the system is embedded in further dimensions. The predictability depends on the speed of divergence of the trajectories from nearby initial conditions. This rate of divergence is given by Lyapunov functions. In this paper Lyapunov characteristic function (h) is estimated as has been given by Fraedrich (1987). For fixed r the correlation function for $(n+k)$ dimension is related to n dimension by the factor e^{-hLk} where k is an integer.

Thus h is given by

$$h = \frac{1}{LK} \log \frac{C(n)}{C(n+k)} \ .$$

The inverse of h gives the predictability.

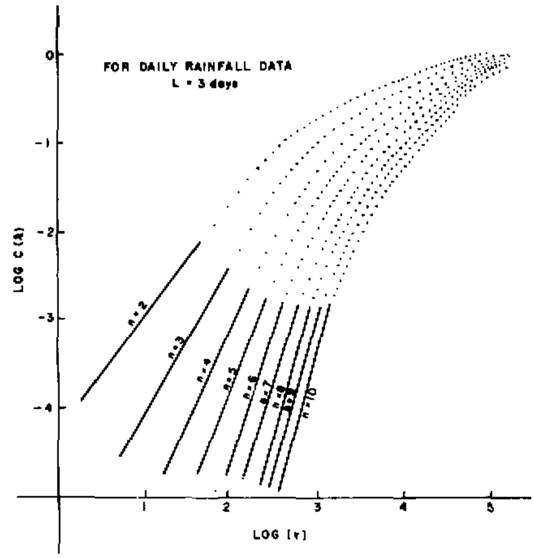


Fig.2.

IV. COMPUTATIONAL RESULTS

The above mentioned procedure has been applied to the daily Konkan subdivisional rainfall. The lag correlation was computed and it was seen that the lag correlation drops to 0.3 for $L=3$ days and becomes insignificant. Hence L was to be 3 days. The rainfall series was embedded into different dimensions from $n=2$ to $n=10$. For each embedding dimension n, different r values (log r goes from 0 to 4) were selected and the correlation function $C(r)$ was evaluated. Fig.2 shows the plots of log $C(r)$ versus log (r) for different embedding dimension n for fifteen years of composite. A random time series was attached to each years of data. It was generated from observed time series to have same length, mean and varience. Figure 3 shows the plots of the fifteen years of composites of $\log C(r)$ versus $\log(r)$ for random data. Again straight lines were fitted to estimate the slopes of the curves. Figure 4 shows the slopes d versus embedding dimension n. It is seen that for actual data the slope slowly increases whereas for random data the slope uniformly increases. The value of d for $n=10$ for actual data is 3.8. Thus the dimension of the attractor of the daily rainfall comes out to be around 3.8. Satyan (1988) has shown the dimension of annual rainfall is around 5.1. The difference in the two may be attributed to different causes of variability. The interannual variability is due to internal dynamics of the circulation and boundary forcing such as snow cover, sea surface temperature, soil moisture etc. The variability in the daily rainfall may be due to the existence of three different oscillations of periods 4–to 5–day, 14–to 19–day and 40–to 50–day (Tao and Chen, 1987). The dimension of about 3 in the daily weather element has also been observed by Fraedrich (1987). (Daily surface pressure at Berlin $d=3.2$ for winter and 3.9 for summer). For estimating the value of h different r values were taken and each value of r, $\log(Cn)$ and $\log(Cn+k)$ were computed. Table 1 summarizes the results. Here n was taken to be 9 and k equal to 1. The mean value of h comes out to be 0.12. Thus the predictability which is given by $1/h$ comes out to be 8 days.

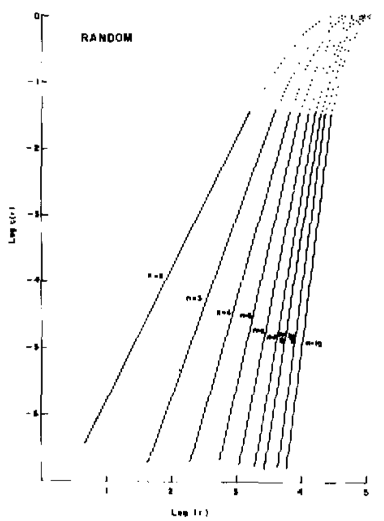


Fig.3.

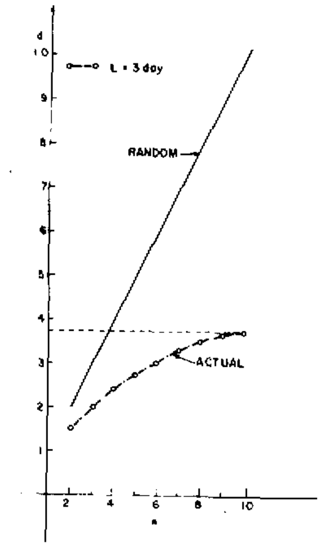


Fig.4.

V. CONCLUSIONS

Fifteen years of daily Konkan subdivisional, summer monsoon rainfall have been analysed using the procedure based on correlation function developed by Grassberger and Procaccia (1983 a,b). The analysis shows that the variability is due to existence of strange attractor. The dimension of it is about 3.8. The predictability is estimated using Lyapunov characteristic exponent. It shows the predictability around 8 days.

Table 1.

$n=9$ $k=1$			
$\log(r)$	$\log(Cn)$	$\log(Cn+k)$	$\log(Cn / Cn+k)$
3.00	−3.05	−3.35	0.30
2.95	−3.25	−3.55	0.30
3.90	−3.45	−3.75	0.30
2.85	−3.50	−3.85	0.35
2.80	−3.65	−4.00	0.35
2.75	−3.85	−4.20	0.35
2.70	−3.95	−4.35	0.40
2.65	−4.10	−4.50	0.40
2.60	−4.30	−4.70	0.40
2.55	−4.50	−4.95	0.45

The author is thankful to Dr. S. S. Singh, Mr. R. K. Verma and Dr. (Mrs.) Mary Selvam for their interest and constant encouragement. The author also wishes to thank Dr. A. D Gangal Physics Department, University of Poona for discussion and advice on points covered in this study. He is thankful to I. M. D. for providing the daily rainfall data. He is further thankful to Mr. K. D. Barne, for typing the manuscript.

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