Split–Explicit Integration of Primitive Equation Barotropic Model for the Prediction of Movement of Monsoon Depression

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ABSTRACT

The split–explicit version of a limited area primitive equation barotropic model is formulated and tested for the prediction of movement of monsoon depressions. The model is integrated upto 48 hours with split–explicit time integration scheme (Gadd, 1978a) using input of four synoptic cases. The model is also integrated explicitly. The forecast results obtained from both the versions are compared and discussed. The computational time in former version is less than half of the computational time needed in explicit version.

I. INTRODUCTION

A six level limited area primitive equation model has recently been formulated and tested for monsoon prediction (Singh et. al., 1990). The model has been applied for a number of cases of monsoon depression with satisfactory forecast results. For time–integration, the explicit leap–frog scheme with Asselin time–filter is used. For stable integration, a time–step of 3 minutes for 150 km grid–length is used and the computer time required for 48 hr integration is of the order of 90 minutes on NEC S 1000 computer. Such a large computational time is due to the use of explicit leap–frog time integration scheme in which the size of the time–step, that can be used for a given spatial resolution, is restricted by Courant–Friedrichs–Lewy criteria. During the last few years, a number of efficient time–integration schemes which allow a large time–step without the loss of accuracy of the forecast and consequently reduce the computer time significantly, have been proposed by researchers. One such scheme is split–explicit scheme designed by Gadd (1978a, 1980). To achieve more economy in computer time, we propose to use split–explicit time–integration scheme in the aforesaid six–level model. Before reformulating the model in terms of split–explicit scheme, we have initiated the work with a single level model and as such numerical experiments are carried out with a limited area primitive equation barotropic model to predict the movement of monsoon depressions using split–explicit time integratiuon scheme. The model equations are integrated by split–explicit scheme (Gadd, 1978a, 1980) in which horizontal advection terms are integrated explicitly with a time–step limited by the maximum wind speed while the gravity wave terms are integrated in succession of shorter time–steps. The model is also integrated explicitly. The forecast efficiency of the model with split–explicit integration is evaluated by comparing the forecast results with those produced by explicit version. The performance of the split–explicit integration is found reasonably well and has encouraged the author to use the scheme in the multi–level model. The paper describes the data used, the computation scheme, and the forecast results produced by split–explicit as well as explicit version of the barotropic model. Since the purpose of this experiment is to use the split–explicit scheme in the multilevel models developed in the Institute, the details of the split–explicit scheme is described in the paper and not the explicit scheme. However, it is worth–mentioning here that the split–explicit version of the model is formulated on staggered grid where as the explicit version is based on non–staggered grid. The domain of integration, grid–size initialization scheme and the lateral boundary conditions are same in both the versions. Topography has not been included in this experiment.

II. DATA

Four typical synoptic situations, all of which were dominated by monsoon–depressions, are selected for the present study. These are 00 GMT, 4 August, 1968; 12 GMT of 15 and 23 June and 6 August, 1979. The basic input to the model is wind defined at $2^\circ \times 2^\circ$latitude–longitude intersection. The wind thus defined are further interpolated on a uniform grid of 220 km on Mercator projection. The domain of integration extends from 6°N–36°N and 56°E–108°E.

III. MODEL

The model equations are the well known shallow–water equations in cartesian co–ordinate on Mercator projection describing the motion of a homogeneous incompressible fluid with a free upper–surface. The equations are

$$\frac{\partial u}{\partial t}+mu\frac{\partial u}{\partial x}+mv\frac{\partial u}{\partial y}-fv+mg\frac{\partial h}{\partial x}=0 \ ,$$

$$\frac{\partial v}{\partial t}+mu\frac{\partial v}{\partial x}+mv\frac{\partial v}{\partial y}+fu+mg\frac{\partial h}{\partial y}=0 \ ,$$

$$\frac{\partial h}{\partial t}+mu\frac{\partial h}{\partial x}+mv\frac{\partial h}{\partial y}+mh\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-hv\frac{\partial m}{\partial y}=0 \ ,$$

where u and v are the zonal and meridional components of horizontal wind vector, h is the height of the free upper–surface ($h=0$ is the mean sea level), g the acceleration due to gravity, m the map–factor (secant of latitude), f is the Coriolis parameter and t the time. The numerical experiments are carried out with free–surface at 700 hPa level and the mean–height of the free upper surface is taken to be 2000 meter.

IV. GRID AND FINITE–DIFFERENCE NOTATIONS

To integrate the model, the equations are first approximated by finite difference equations using a staggered grid shown in Fig.1. The geopotential heights are defined at the regular grid–points and the wind components are defined at the centre of the squares, the corners of which are the regular grid–points. Following notations are used in writing the finite–difference equations;

$$\bar{\theta}^x=\frac{1}{2}\left[\theta\left(x+\frac{\Delta x}{2}\right)+\theta\left(x-\frac{\Delta x}{2}\right)\right] \ ,$$

$$\bar{\theta}^y=\frac{1}{2}\left[\theta\left(y+\frac{\Delta y}{2}\right)+\theta\left(y-\frac{\Delta y}{2}\right)\right] \ ,$$

$$\theta_x=\delta_x\theta=\frac{1}{\Delta x}\left[\theta\left(x+\frac{\Delta x}{2}\right)-\theta\left(x-\frac{\Delta x}{2}\right)\right] \ ,$$

$$\theta_y=\delta_y\theta=\frac{1}{\Delta y}\left[\theta\left(y+\frac{\Delta y}{2}\right)-\theta\left(y-\frac{\Delta y}{2}\right)\right] \ ,$$

$$\bar{\theta}^{xy}=\overline{(\bar{\theta}^x)}^y=\overline{(\bar{\theta}^y)}^x \ ,$$

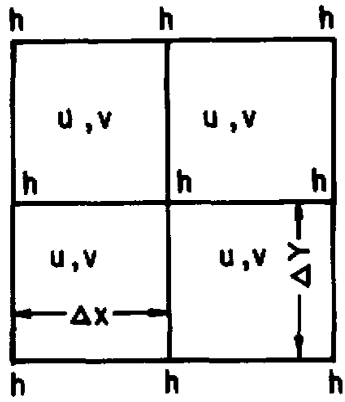

Fig.1. The distribution of variables in split–explicit version.

where θ is any variable, Δx, Δy are the grid–size in x and y directions respectively.

V. TIME INTEGRATION SCHEME

The split–explicit time integration scheme devised by Gadd (1978a) is used to numerically integrate the model equations. In this scheme the complete integration of the governing equations for each time–step, is performed in two stages, viz., the adjustment stage and the advection stage and as such the terms of the governing equations are splitted into two groups. The adjustment stage includes the Coriolis terms and the pressure gradient terms of the momentum equations and also the entire continuity equation. The advection stage consists of the horizontal advection terms of the momentum equations. Thus the equations to be integrated in adjustment stage are.

$$\frac{\partial u}{\partial t} = fv - mg\frac{\partial h}{\partial x} ,$$

$$\frac{\partial v}{\partial t} = -fu - mg\frac{\partial h}{\partial y} ,$$

$$\frac{\partial h}{\partial t} = -\left[mu\frac{\partial h}{\partial x} + mv\frac{\partial h}{\partial y} + mh\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] + hv\frac{\partial m}{\partial y} ,$$

and the equations for advection stage are

$$\frac{\partial u}{\partial t} = -mu\frac{\partial u}{\partial x} - mv\frac{\partial u}{\partial y} ,$$

$$\frac{\partial v}{\partial t} = -mu\frac{\partial v}{\partial x} - mv\frac{\partial v}{\partial y} .$$

The advantage of this splitting is that it enables the advection terms and the gravity–wave term to be dealt with separately and hence more efficiently. Since gravity–wave terms are removed from the second group of terms, only the adjustment stage has to be integrated with a small time–stap while a large time–stap can be used for advection stage. A typical integration cycle of one time–step consists of marching forward with terms of the adjustment stage for several small time–steps followed by the integration of the horizontal advective term in advection stage with a large time–step. The large time–step for advection stage must be an integral multiple of the small time–step used for the integration of adjustment stage and the initial values of the predicted variables to be used in the advection stage are those obtained from the final step of several successive integration of adjustment stage.

The adjustment stage is integrated using forward–backward scheme (Mesinger and Arakawa, 1976; Gadd, 1978a) in which a forward difference in time is used for height (h) equation and the new value of h is then used in u and v equations. The finite–difference equation for the forward part of the scheme is

$$h(t+\delta t)=h(t)-\delta t\Big[m\overline{u(t)}^{xy}\,\overline{H_x(t)}^{y}+m\overline{v(t)}^{xy}\,\overline{H_y(t)}^{x}$$
$$+mh(t)\Big\{\overline{u_x(t)}^{y}+\overline{v_y(t)}^{x}\Big\}\Big]+\delta t h(t)\overline{v(t)}^{xy}\,\overline{m_y}^{y}\ ,$$

where $H=\overline{h}^{xy}$. δt is the time–step for adjustment stage.

The equations for the backward part of the integration scheme are

$$u(t+\delta t)=u(t)+\delta t\Big[\frac{1}{2}f\Big\{v(t)+v(t+\delta t)\Big\}-\overline{m}^{xy}\,\overline{h_x(t+\delta t)}^{y}\Big]\ ,$$
$$v(t+\delta t)=v(t)-\delta t\Big[\frac{1}{2}f\Big\{u(t)+u(t+\delta t)\Big\}+\overline{m}^{xy}\,\overline{h_y(t+\delta t)}^{x}\Big]\ .$$

A note–worthy feature of the scheme is that the R. H. S. of u and v equations contains the value of h at the new time–step $t+\delta t$. Thus the scheme is technically implicit but explicit in practice because, the continuity equation is integrated first and so when u and v equations are integrated, $h(t+\delta t)$ is known by that time. Another important feature is that in u and v equations the Coriolis terms are integrated by trapezoidal implicit rule (Mesinger, 1977) in order to eliminate the source of instability noted by Gadd (1980). The scheme still remains, however, explicit since the u and v equations may be rearranged to get

$$u(t+\delta t)=W1*u(t)+W2*v(t)-W3*\overline{h_x(t+\delta t)}^{y}-W4*\overline{h_y(t+\delta t)}^{x}\ ,$$
$$v(t+\delta t)=W1*v(t)-W2*u(t)-W3*\overline{h_y(t+\delta t)}^{x}+W4*\overline{h_x(t+\delta t)}^{y}\ ,$$

where

$$W1=(1-\frac{1}{4}f^2\delta t^2)/WW\ ,$$
$$W2=f\delta t/WW\ ,$$
$$W3=g\delta t/WW\ ,$$
$$W4=\frac{1}{2}gf\delta t^2/WW \text{ and } WW=1+\frac{1}{4}f^2\delta t^2\ .$$

The integration of advection stage is performed by two–step Lax–Wendroff scheme (Gadd, 1978b) which reduces the phase and amplitude errors. In the first step, $u^{n+1/2}$, $v^{n+1/2}$ are obtained at the regular grid points in forward sense. In second step, u^{n+1}, v^{n+1} are predicted at the centre of the boxes by centred difference scheme. The finite difference equations for the first step are

$$u^{n+\frac{1}{2}}=\overline{u^n}^{xy}-\frac{m}{2}\Delta t\Big[\overline{u}^{xy}\,\overline{u_x}^{y}+\overline{v}^{xy}\,\overline{u_y}^{x}\Big]^{n}\ ,$$
$$v^{n+\frac{1}{2}}=\overline{v^n}^{xy}-\frac{1}{2}\Delta tm\Big[\overline{u}^{xy}\,\overline{v_x}^{y}+\overline{v}^{xy}\,\overline{v_y}^{x}\Big]^{n}\ .$$

In the second step fourth order approximation is used for the derivatives. The equations are

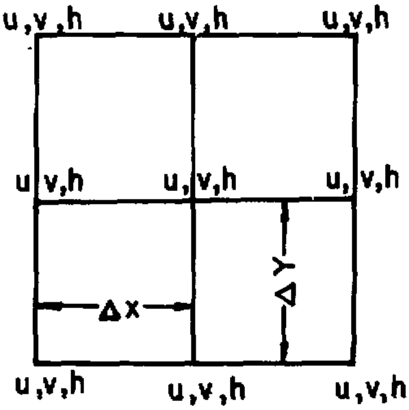

Fig.2. The distribution of variables in explicit version.

$$u^{n+1} = u^n - \left[(1+a)\left\{ \mu\overline{(\delta_x u)}^y + \nu\overline{(\delta_y u)}^x \right\} - a\left\{ \mu\overline{(\delta_{3x} u)}^y + \nu\overline{(\delta_{3y} u)}^x \right\} \right]^{n+\frac{1}{2}} ,$$

$$v^{n+1} = v^n - \left[(1+a)\left\{ \mu\overline{(\delta_x v)}^y + \nu\overline{(\delta_y v)}^x \right\} - a\left\{ \mu\overline{(\delta_{3x} v)}^y + \nu\overline{(\delta_{3y} v)}^x \right\} \right]^{n+\frac{1}{2}} ,$$

where

$$a = \frac{3}{4}(1-\lambda^2) \ ,\lambda^2 = \mu^2 + \nu^2 \ ,$$

$$\mu = mu\Delta t / \Delta x, \ \ \nu = mv\Delta t / \Delta y \ ,$$

$$\delta_{3x}\theta = \frac{1}{3\Delta x}\left[\theta(x + \frac{3}{2}\Delta x) - \theta(x - \frac{3}{2}\Delta x) \right] \ ,$$

$$\delta_{3y}\theta = \frac{1}{3\Delta y}\left[\theta(y + \frac{3}{2}\Delta y) - \theta(y - \frac{3}{2}\Delta y) \right] \ ,$$

and Δt is the time–step for advection stage. Here in the advection stage u^n and v^n should be those values which are obtained at the end of adjustment stage. n is the index for time–level.

It should be pointed out that for the explicit integration of the model, the corresponding finite difference equations are formulated following the scheme suggested by Shuman (1962). The grid is shown in Fig.2.

VI. INITIALIZATION

To achieve the initial balance between the mass and wind field, first, we solve the Poisson equation $\nabla^2\psi = \zeta$, for the stream–function ψ, ζ, the relative vorticity, is obtained from the basic input of u and v. The non–divergent wind components u and v are then evaluated from the relations $u = -m \cdot \partial\psi / \partial y$, $v = m\partial\psi / \partial x$, and finally the geopotential heights are computed by solving the non–linear balance equation

$$\nabla^2 h = \frac{1}{g}\left\{ f\zeta - \beta u_\psi + 2J\left[u_\psi , v_\psi \right] \right\} \ ,$$

where $\beta = m\frac{\partial f}{\partial y}$. ∇^2 and J are the Laplacian and the Jacobian operators respectively.

VII. BOUNDARY CONDITIONS

During time–integration, a simple boundary condition is used. All the three variables at the boundary points are kept constant throughout the period of integration.

VIII. TIME–STEP USED

The adjustment terms are integrated with a time–step of 8 minutes and five successive adjustment steps are taken for each advection step. Thus, a complete cycle of integration consists of integration of the adjustment stage five times in succession with a time–step of 8 minutes followed by one integration of advection terms with a large time–step of 40 minutes. The details of the stability analysis of the scheme is given by Gadd (1978a).

The explicit version of the model is, however, integrated with a time–step of 4 minutes only.

IX. RESULTS AND DISCUSSIONS

The performance of the split–explicit version of the model (upto 48 hours) is evaluated by comparing the forecast results with the observed as well as with those obtained from the explicit version. Although, experiments are carried out for all the four synoptic cases mentioned in Section II, here we have restricted the presentation of flow pattern to the case of 4 August, 1968 only. Fig.3 shows the initial flow–pattern at 700 hPa level of 00 GMT, 4 August, 1968. Figs.4 presents the 24 hr forecast obtained from both the versions together with the corresponding verification chart (700 hPa, 00 GMT, 5 August, 1968). The 48 hr forecast results and the corresponding observed (00 GMT, 6 August, 1968) flow–pattern are shown in Fig.5. It can be seen from Figs.4 and 5 that the cyclonic circulation associated with the depression is well predicted in 24 hr and 48 hr forecasts by both the versions. In both the cases, the location of centre of 24 hr forecast–circulation is about 2° south of the actual location. However, the 48 hr forecast position of the centre almost coincides with the observed location. Table 1 shows the 24 and 48 hr forecast position, the corresponding observed positions and the position vector errors of the centre of cyclonic circulation. The observed broad

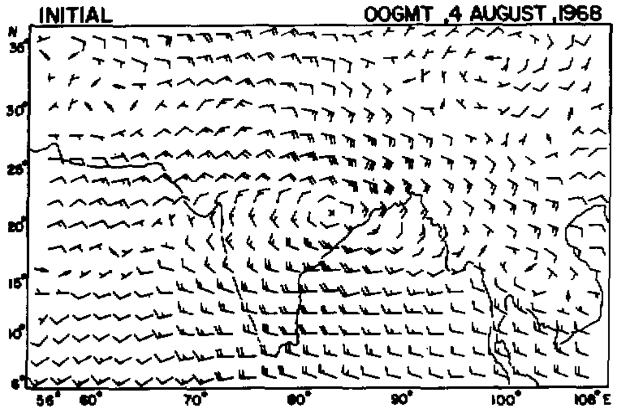

Fig.3. Initial flow pattern at 700 hPa level of 00 GMT, 4 August, 1968.

belt of westerlies over the Arabian Sea and the peninsular India are fairly well simulated by both the versions of the model in both 24 and 48 hr forecast. On average the forecast results suggest that the split–explicit model is stable upto 48 hours. Two predictions are almost identical. The flow field is well simulated by both the versions of the model and no significant deterioration of the map–feature in 48 hr period is noticed in either of the versions. In both the versions the predicted movement of the depression is comparable with the actual.

Table 1. 24 and 48 Hour Forecast and Corresponding Observed Positions and Position Vector Errors of Centre of Cyclonic Circulation Associated with the Depression

Input	24 hour			48 hour		
	Observed position on 5.8.68	Forecast position	Vector error (km)	Observed position on 6.8.68	Forecast position	Vector error (km)
			(Split–explicit version)			
4.8.68	23°N, 79°E	21°N, 79°E	220	23°N, 76°E	23°N, 75°E	110
			(Explicit version)			
4.8.68	23°N, 79°E	21°N, 79°E	220	23°N, 76°E	23°N, 75°E	110

Table 2. RMS Errors, RMS Vector Wind Errors and Correlation Coefficients of 24 and 48 Hours Forecasts. Reference level: 700 hPa

Input	24 hour					48 hour				
	RMS error (ms^{-1})		RMS Vector wind error (ms^{-1})	Correlation coefficient		RMS error (ms^{-1})		RMS Vector wind error (ms^{-1})	Correlation coefficient	
	u	v		u	v	u	v		u	v
				(split–explicit version)						
4.8.68	4.2	4.4	6.1	0.8	0.4	4.2	3.6	5.5	0.8	0.2
15.6.79	4.1	3.7	5.3	0.8	0.6	5.8	3.6	6.8	0.6	0.6
23.6.79	3.8	3.7	5.3	0.9	0.7	5.1	4.2	6.6	0.9	0.5
6.8.79	4.9	4.7	6.5	0.9	0.5	5.0	4.8	6.9	0.9	0.3
Average	4.3	4.1	5.8	0.9	0.6	5.0	4.1	6.5	0.8	0.4
				(Explicit version)						
4.8.68	4.3	4.4	6.1	0.8	0.4	4.8	3.8	6.2	0.7	0.3
15.6.79	4.5	4.3	6.2	0.7	0.4	6.5	5.0	8.2	0.5	0.4
23.6.79	3.9	3.7	5.4	0.8	0.6	5.1	4.5	6.8	0.8	0.5
6.8.79	5.5	4.4	7.1	0.9	0.5	6.2	5.3	8.2	0.8	0.3
Average	4.8	4.3	6.4	0.8	0.5	5.7	4.7	7.4	0.7	0.4

The efficiency of the split–explicit model in predicting the meteorological variables (u and v) is evaluated by computing the verification–statistics, viz., the root–mean–square error

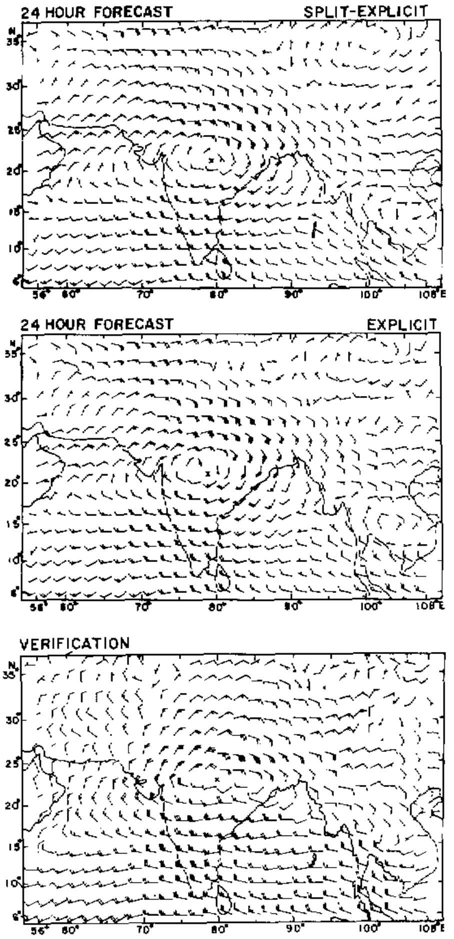

Fig.4. 24 hr forecast flow patterns produced by both the versions and the corresponding verification chart (00 GMT, 5 August, 1968).

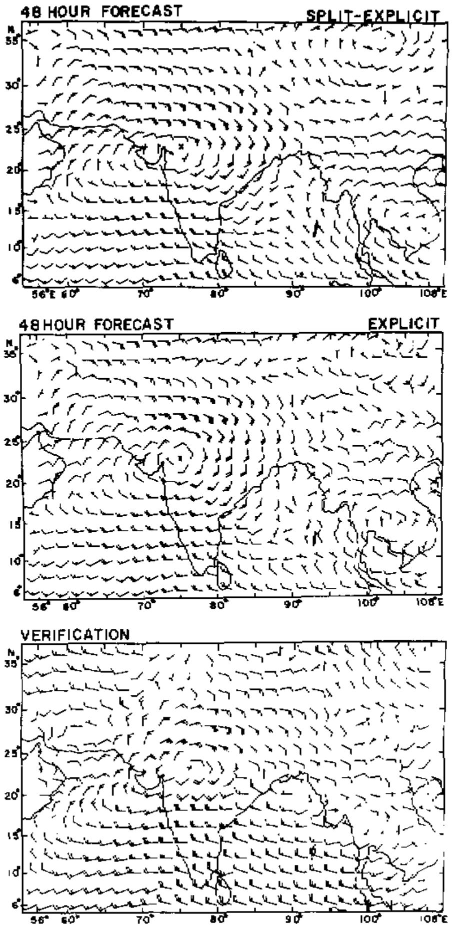

Fig.5. 48 hr forecast flow patterns predicted from both the versions and the corresponding verification chart (00 GMT, 6 August, 1968).

(r.m.s. error), r.m.s. vector wind error and the correlation coefficient between the forecast and the corresponding observed field. Table 2 presents the verification statistics of u and v fields for 24 hr and 48 hr forecasts obtained from both split–explicit and explicit version. It may be seen from Table 2 that the split–explicit version perfroms reasonably well in all the four cases. On average, r.m.s. errors, r.m.s. vector errors and the correlation coefficients of u and v in 24 and 48 hr forecast from split–explicit version are found to be comparable with those obtained from explicit version. In general, r.m.s. errors of the zonal wind (u) in both the versions are higher than those of meridional wind (v).

Table 3. 12 Hourly Domain Averaged Values of Potential Vorticity $(\zeta+f)/h$ Geopotential Height (z) and Total Energy Parameter (kz+.5 gz^2) for the Cases of 4 August 1968. Split–explicit Integration

Input	Forecast time in hour	Potential Vorticity ($\times 10^{-7}m^{-1}s^{-1}$)	Geopotential height ($\times 10^4$m)	Total energy parameter ($\times 10^8 m^3 s^{-2}$)
4.8.68	0	0.2587	0.2000	0.1960
	12	0.2589	0.1999	0.1963
	24	0.2590	0.2000	0.1965
	36	0.2592	0.2001	0.1967
	48	0.2594	0.2002	0.1970

Since a unmber of approximations (lateral boundary condition, finite difference approximation etc.) are involved in formulating a limited area model, it is necessary to evaluate the forecast efficiency of the model by examining the conserving properties of shallow–water equation during its forward integration. The domain invariant quantities examined in this experiment are mean potential vorticity, mean total energy and mean geopotential height. These invariant quantities are computed 12 hourly during entire 48 hr integration period. In all the cases, all the three quantities remain almost constant and are very nearly conserved by both the versions of the model during the entire 48 hr period of integration. However, we have restricted the presentation of these quantities to the case of 4 August, 1968 only. The computed invariant quantities in split–explicit experiment are presented in Table 3 and those in explicit integration are shown in Table 4. It can be seen from Tables 3 and 4 that the quantities are almost invariant.

The forecast efficiency of a model depends on stability, accuracy and economy of the schemes employed for the time integration and for the approximation of space derivatives. The split–explicit model is found stable upto 48 hr. The economy is the significant achievement in split–version as compared to the explicit–version. Table 5 shows that the computational time (upto 48 hr) required in split–explicit version is less than half of the computational time in explicit version. The little inaccurate movement of the depression by both the versions of the model may be attributed to the fact that a barotropic model lacks many physical processes important for atmospheric motions. The only predominant terms are the advective terms which normally counter–act the tendency of westward movement of the systems. A barotropic model, thus, cannot be expected to produce best possible forecast. Nevertheless, performance of the split–explicit version is reasonably well and encouraging.

Table 4. Same as Table 3, but for Explicit Integration

Input	Forecast time in hour	Potential Vorticity ($\times 10^{-7}m^{-1}s^{-1}$)	Geopotential height ($\times 10^4$m)	Total energy parameter ($\times 10^8 m^3 s^{-2}$)
4.8.68	0	0.2587	0.2000	0.1960
	12	0.2592	0.2001	0.1969
	24	0.2589	0.2002	0.1971
	36	0.2596	0.2003	0.1971
	48	0.2598	0.2004	0.1973

Table 5. CPU Time (on ND–560 Computer)

Scheme	CPU time in second
Split–explicit	45
Explicit	110

X. CONCLUSIONS

The split–explicit version of limited area primitive equation barotropic model has been formulated and tested over Indian region for the prediction of movement of monsoon depressions. The forecast results are comparable with those obtained from the explicit–version. The significant achievement in split–explicit version is the saving of considerable amount of computational time. A successful application of split–explicit integration has encouraged the author to apply this scheme to the multilevel models developed in the Institute.

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