The Structure and Propagation of Stationary Planetary Wave Packet in the Barotropic Atmosphere

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ABSTRACT

Monthly or seasonally mean anomalies of large-scale atmospheric circulation are better represented by wave packets or their combination. Both qualitative and quantitative analyses of equations of wave packet dynamics, which are obtained by the use of WKB approximation, are very helpful for the understanding of structure, formation and propagation of stationary and quasi-stationary planetary wave packet patterns in the atmosphere. Indeed, these equations of wave packet dynamics can be directly solved by the method of characteristic lines, and the results can be simply and clearly interpreted by physical laws. In this paper, a quasi-geostrophic barotropic model is taken for simplicity, and the wave packets superimposed on several ideal profiles of the basic current and excited by some ideal forcings are investigated in order to make comparison of the accuracy of calculation with the analytical solution. It is revealed that (a) the rays of stationary planetary wave packet do not coincide with but go away from the great circle with significant difference if the shear of the basic zonal flow is not too small; (b) being superimposed on a westerly jet flow with positive shear ($\partial U_{\lambda} / \partial y > 0$), the stationary wave packets excited by low-latitudinal forcing are first intensified during their northeastward propagation in the Northern Hemisphere, then reach their maximum of amplitude at some critical latitude, and after that weaken again; (c) the connected line of extremes (the positive and negative centres) of wave packet does not coincide with but crosses the ray by an angle, the larger the scale of external forcing, the larger the angle; and (d) the whole pattern of a trapped stationary wave packet is complicated by the interference between the incident and reflected waves.

I. INTRODUCTION

In the monthly mean circulation of the middle and upper troposphere the quasi-stationary disturbances such as troughs, ridges and even closed vortices, superimposed on the basic zonal flow can be considered as compositions of various planetary waves. However, these disturbances-planetary waves take shape of wave packets, because their amplitude and orientation all vary with longitude and latitude. Therefore, to study the structure, formation and the propagation characteristics of these stationary or quasi-stationary wave packets will undoubtedly very helpful for the understanding of monthly mean atmospheric circulations (Zeng et al., 1986; Huang, 1986, 1990).

In this paper we will directly apply the equations for wave packet dynamics to the investigation because the representing disturbances by wave packets is geometrically proper, simple and convenient, and the physical meanings of equations for wave packet dynamics are also very clear.

II. THE MODEL AND THE DYNAMICS EQUATIONS FOR WAVE PACKET

Taking barotropic quasi-geostrophic model in Marcator projection, the non–dimensional potential vorticity equation linearzed with respect to a (non–dimensional) zonal basic flow $\bar{u}$ is written as follows (see, Zeng et al., 1986):

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial t}\right)\left[\frac{\partial^2\psi'}{\partial x^2}+\frac{\partial^2\psi'}{\partial y^2}-\rho^2\psi'\right]+\bar{\beta}\,\frac{\partial\psi'}{\partial x}=Q', \tag{1}$$

where ψ' and Q' are the perturbations of the stream function and the source intensity (such as heating and dissipation) respectively; $\bar{u}=U_\lambda / U^* \sin\theta$ the (nondimensional) angular velocity of the (nondimensional) velocity of the zonal basic flow $\bar{U}_\lambda, \bar{\beta}=R_0^{-1}\partial\bar{q}/\partial y$, $\bar{q}=1+R_0\left(\beta y-\frac{\partial\bar{u}}{\partial y}-\rho^2\bar{\psi}\right)$ the (nondimensional) potential vorticity of the basic flow whose stream function is $\bar{\psi}$, and ρ^2 is a function of y but simplified as a constant here ($\rho^2=1$); $x=a\lambda / L^*$ and $y=(a / L^*)\ln[(1+\cos\theta)/\sin\theta]$ are the coordinates along λ (longitude) and $\frac{\pi}{2}-\theta$ (θ the colatitude) respectively; U^*, L^* and $T^*=L^* / U^*$ are the characteristic (dimensional) velocity, scale and time respectively; a is the radius of the Earth; and $R_0=U^* / f_0 L^*$ is the Rossby–Kibel' Number (see Zeng et al., 1986, in detail).

The wave packet is represented as follows:

$$\psi'(x,y,t)=\sum_j \varepsilon^j \Psi_j(X,Y,T)exp[i\varepsilon^{-1}\alpha(x,y,t)] , \tag{2}$$

where ε is small parameter, and $(X,Y)=\varepsilon(x,y)$ and $T=\varepsilon t$ are the stretched spatial and temporal coordinates respectively.

Substituting (2) into (1), applying WKB method, and denoting

$$\sigma=-\frac{\partial\alpha}{\partial t}, \quad m=\frac{\partial\alpha}{\partial x}, \quad n=\frac{\partial\alpha}{\partial y}, \tag{3}$$

we obtain the dispersion relationship

$$(\sigma-m\bar{u})(m^2+n^2+1)+\bar{\beta}m=0, \tag{4}$$

and an equation determining the evolution and structure of the zero–order approximation of amplitude Ψ_0 as follows:

$$\left(\frac{\partial}{\partial T}+\vec{C}_g\cdot\nabla\right)A=-A\nabla\cdot\vec{C}_g+HA(\delta-2\gamma\bar{\beta}A), \tag{5}$$

where $\vec{C}_g=\vec{i}C_{gX}+\vec{j}C_{gY}$ is the group velocity whose two components are given by

$$\begin{cases} C_{gX}=\frac{\partial\sigma}{\partial m}=\bar{u}-\bar{\beta}[(m^2+n^2+1)-2m^2]/(m^2+n^2+1)^2, \\ C_{gY}=\frac{\partial\sigma}{\partial n}=2mn\bar{\beta}/(m^2+n^2+1)^2; \end{cases} \tag{6}$$

A is the wave action

$$A=|\Psi_0|^2(m^2+n^2+1)^2 / 2\bar{\beta}, \tag{7}$$

and the last term on the right hand side of (5) is the source intensity, which consists of dissipation and the release of latent heat of condensation by ascending motion and is here taken as proportional to potential vorticity of the wave packet in a quadratic form with some parameters H,δ and $\gamma.H=0$ means that there is a free wave packet.

In the stationary case, $\sigma=0$, we have

$$(m^2+n^2+1)=\bar{\beta} / \bar{u}, \tag{8}$$

$$C_{gX} = C_{gx} = 2m^2\bar{u} / (m^2 + n^2 + 1), \tag{9}$$

$$C_{gY} = C_{gy} = 2mn\bar{u} / (m^2 + n^2 + 1). \tag{10}$$

For a given m which is independent on x, y in our case, the local wave number n along y is determined by equation (8); the ray of the stationary wave is determined by (9), (10) and the following differential equation:

$$\frac{dx}{dy} = \frac{C_{gx}}{C_{gy}} = \frac{m}{n}; \tag{11}$$

and the variation of wave action A along the ray can by obtained by solving equation (5) by the method of characteristic lines (the rays). As A is known, Ψ_0 can be determined by (7), and the structure of stationary wave packet is eventually determined with zero order approximation by substituting Ψ_0 and $\alpha = mx + n(y)y$ into (2).

In our calculation we take the Rossby radius of deformation $L_0 = 3 \times 10^3 km$, $L^* = L_0$, and $U^* = 10m \cdot s^{-1}$. Before having our concrete examples of wave packets we first give Tables 1–3 which represent the relations between y and θ, x and λ, and the actual and nondimensional zonal wave numbers M and $m = L^* M / a$ respectively.

Table 1

y	0	1	2	3	∞
θ	90°	64°	42.6°	27.4°	0°
φ(°N)	0°	26°	47.4°	62.6°	90°

Table 2

x	0	6.7	13.4
λ(°E)	0°	180°	360°

Table 3

M (actual zonal wave number)	2	3
m (nondimensional wave number)	0.94	1.41

Besides, the location of maximum $\bar{u}$ (equivalent to $\dot{\lambda}$) does not coincide with the zonal velocity $\overline{U_\lambda}$ if the basic flow is jetlike one, and the first is northerner than the second. For example, let

$$\bar{u} = U_0 + U_1 y + U_2 y^2 / 2, \tag{12}$$

and take $U_0 = 0.2$, $U_1 = 1$, $U_2 = -0.5$, $\bar{u}_{max}$ is located at $y = 2(\varphi = 47.4^o \text{ N})$, but $\overline{U}_{\lambda max}$ at $y = 1.4(\varphi = 35.3^o \text{ N})$.

III. PENETRATING STATIONARY WAVE PACKET

Suppose that there is an external forcing source located in a localized region in equatorial zone. After emitted from the source region the wave packet is governed by (5).

It is clear from (8) that travelling from Equator to the Northern Hemisphere the wave packet can penetrate all latitudes if $\bar{u} > 0$, $\bar{\beta} > 0$ and

$$S \equiv (\bar{\beta} / \bar{u}) - (m^2 + 1) > 0 \tag{13}$$

everywhere in $0 \leqslant y < \infty$.

Now, taking a basic flow given by (12) with $U_0 = 0.2$, $U_1 = 1$ and $U_2 = -0.5$, we have

$$(\bar{\beta} / \bar{u}) = [(1 - U_2) / \bar{u}] + 1. \tag{14}$$

From (13) we know that waves with $m^2 \leqslant 1.25$are all penetrating. Taking $m = 1$(the actual zonal wavenumber M is slightly larger than 2), the n, C_{gx} and C_{gy} as well as $\bar{u}$are given in Fig.1, the rays [computed by solving (11)] and constant phase lines

$$mx + ny = \alpha_k \, (constant) \tag{15}$$

are in Fig.2; and A and Ψ_0 as functions of y along one characteristic line (ray) with $A(0) = 1$ are in Fig.3, where $\delta = 2\gamma = 1$ and different $H = 0, 0.05, 0.1, 0.2$ and 0.3 are taken for demonstrating the role of internal heating.

The wave packets ψ' generated in the Equator with initial conditions (external sources)

$$A(x,0) = e^{-0.64x^2} \, , \tag{16}$$

$$A(x,0) = 0.5\left\{e^{-0.64x^2} + e^{-(x+2)^2} + e^{-(x-2)^2} + 1 \Big/ \left[\left(\frac{x}{2}\right)^2 + 1\right]\right\}, \tag{17}$$

$$A(x,0) = 1 \, , \tag{18}$$

are given in Figs.4,5 and 6 respectively. (16) is a locally isolated wave packet with scale $|x| = (0.8)^{-1}$, while (17) is rather wide with scale $|x| = 3$, and (18) is uniform along the Equator.

It can be seen from these figures that due to (a) every constant phase line crosses latitudinal cycles, and that these lines (the troughs and ridges) are not perpendicular to the rays, and (b) $\bar{u}$ increases but C_{gy} decreases within $0 \leqslant y \leqslant y_c$ (y_c is the location of $\bar{u}_{max}$), we have:

(1) The intensity or amplitude of the wave packet increases with y, and the intensification is more pronounced if the internal heating is larger and exceeds dissipation ($\delta > 0$).

(2) The connected line of extremes of ψ' (the centres of wave packet) does not coincide with but crosses the ray by an angle. The larger the scale of external forcing, the larger the angle. *It is worth pointing out that one must avoid a confusion between the wave ray and the* connected line of the centres of the disturbance.

(3) It is possible to have more than one centres of the disturbance along a constant phase line if the scale of external source is very large, and the feature is enhanced by the internal source (see Fig.5(b), two centres are visible although too weak due to the too weak secondary center of the external forcing).

IV. TRAPPED STATIONARY WAVE PACKET

Suppose again the external source is located in the Equator. For a given m, the wave packet restricts its propagation only in $0 \leqslant y \leqslant y_c$, if

$$\begin{cases} S(y_c) = \left|[(\bar{\beta} / \bar{u})[(m^2 + 1)]\right|_{y=y_c} = 0, & \\ S(y) > 0 & (y < y_c), \\ S(y) < 0 & (y > y_c). \end{cases} \tag{19}$$

In this case every ray emitted from $y = 0$ reaches $y = y_c$. The incident wave possesses $n > 0$, but the reflected wave $n < 0$.

Taking basic flow given by (12) with $U_0 = 0.2$, $U_1 = 1$ and $U_2 = -0.5$, the wave with $m^2 > 1.25$ are all trapped, and y_c depends on m. Now, for simplicity of calculation we take a linear profile of $\bar{u}$,

$$\bar{u} = 0.2 + 0.5y, \tag{20}$$

in which all waves are of trapped type.

Taking $\bar{u}$ given by (20) and $m = 0.9$ (the correspondent M is slightly smaller than 2), we have $y_c = 2.09$. $\bar{u}$, $|n|$, C_{gx} and C_{gy} are given in Fig.7, wave rays in Fig.8, and the wave packet ψ' in Fig.9, where $A(x,0) = 2\exp[-0.64x^2]$. One can see remarkable difference between the penetrating and trapped waves by comparing Fig.9 with Figs.4–6. However, the characteristics (1) and (2) mentioned in Section 3 are still valid for trapped waves, especially, the amplitude of trapped waves becomes very large in the vicinity of y_c, and therefore the energy of the wave is concentrated there. The essential difference is that in the case of trapped waves, there exists interference of incident and reflected waves, which leads to rather complicated wavy pattern.

It should be pointed out that according to (6) we have $C_{gy} \to 0$, hence $A \to \infty$ and $\Psi_0 \to \infty$ as $y \to y_c$. It is necessary to take other factors which are omitted in (5) into account in order to calculate the amplitude and structure of the wave in the vicinity of y_c. However, by using (5) we can still determine the reflected wave pattern in the region far away from y_c. In fact, taking a channel enveloped by two adjacent rays including both incident and reflected parts, integrating (5) over the channel with the cross section at $y = y_i$ (incident section) and $y = y_c$ (reflection section), we can calculate $A(y_c)$ of reflected wave if $A(y_i)$ of incident wave is known. This is namely the method we have adopted in our calculation of Fig.9. Nevertheless, the improvement of the method is desirable.

V. STATIONARY WAVE PACKET SUPERIMPOSED ON ASYMMETRIC JET STREAM

The jet streams taken in the previous sections are too ideal. In the actual atmosphere, the basic flow always possesses an asymmetric jet, i.e., its angular velocity in the northern side of the jet stream is larger than that in the southern side. In order to investigate the influence of the jet asymmetry on the orientation and the propagation of stationary wave packet, we now take $\bar{u}$ similar to the reality as follows

$$\bar{u} = 0.8e^{-5(y-2)^2} + \frac{0.4}{\pi}\left[\frac{\pi}{2} + \mathrm{arctg}(y-2)\right], \tag{21}$$

which is shown in Fig.10, where the n, C_{gx} and C_{gy} corresponding to $m = 0.9$ are also given, and the rays are given in Fig.11. This is the case of penetrating wave packet. It can be seen from these figures, that the constant phase lines are more tilted and similar to the situation of actual disturbances in the monthly mean map in winter.

Of course, it would be better to calculate all the characteristics of wave packets by taking the actual monthly mean zonal circulation as the basic flow. This will be the topic of our further papers.

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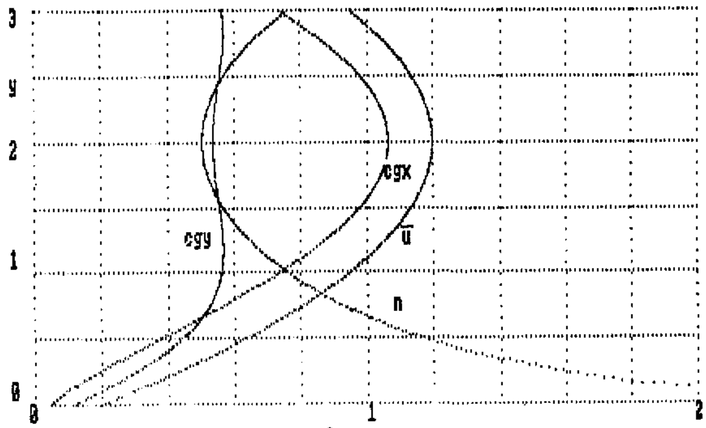

Fig.1. The basic flow $\bar{u}(y) = 0.2 + y - 0.25y^2$ and some characteristics of a penetrating stationary wave packet such as $n(y)$, $C_{gx}(y)$ and $C_{gy}(y)$. The zonal wave number $m = 1$.

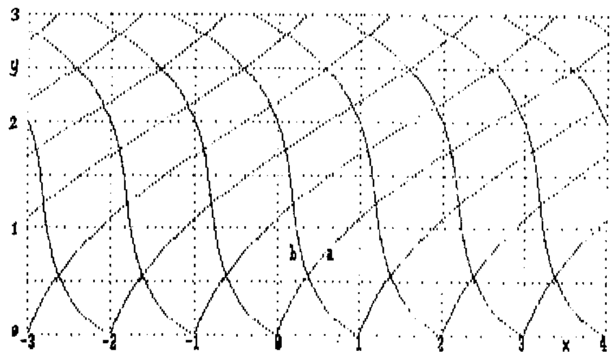

Fig.2. The rays (a) and constant phase lines (b) of a penetrating stationary wave packet with $m = 1$. $\bar{u}$ is given in Fig.1.

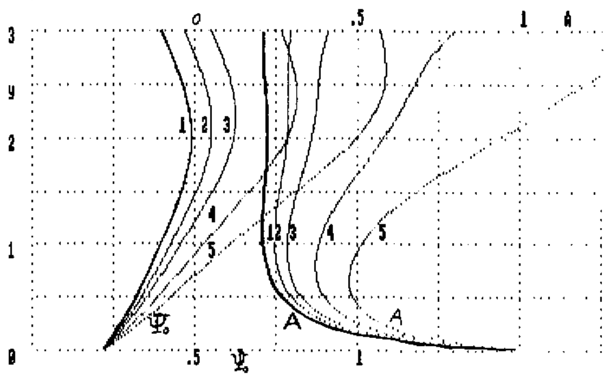

Fig.3. The variation of A and Ψ_0 with y. The basic flow and the Characteristics of wave packet are given in Fig.1. $A(0) = 1$, $\delta = 2r = 0.1$; $\varepsilon H = 0$ [label (1)], 0.5(2), 1(3), 2(4) and 3(5).

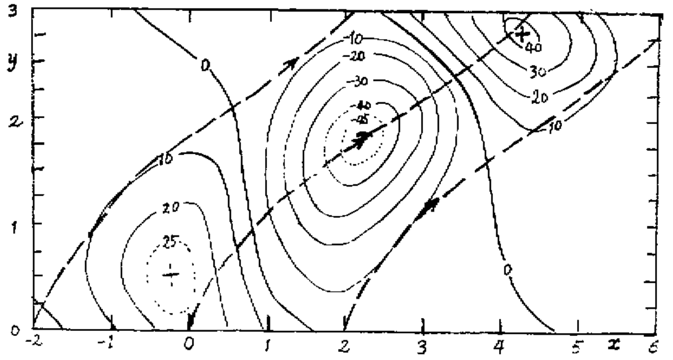

Fig.4. A penetrating stationary wave packet ψ'. The basic flow is given in Fig.1. $A(x,0) = \exp(-0.64x^2)$. $m = 1$, $\varepsilon H = 0$.

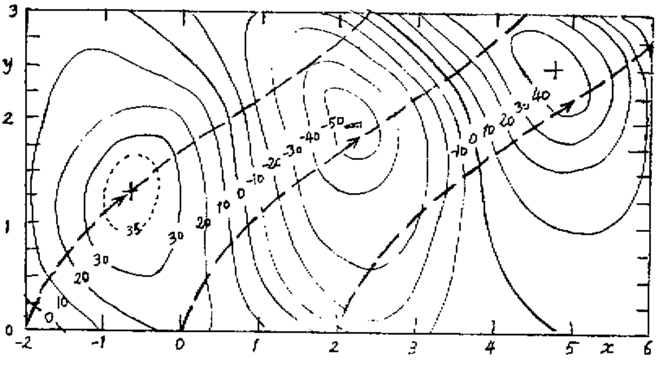

Fig.5. The same as Fig.4, but $A(x,0) = 0.5\{\exp(-0.64x^2) + \exp[-(x+2)^2] + \exp[-(x-2)^2] + 1/(0.25x^2+1)\}$.

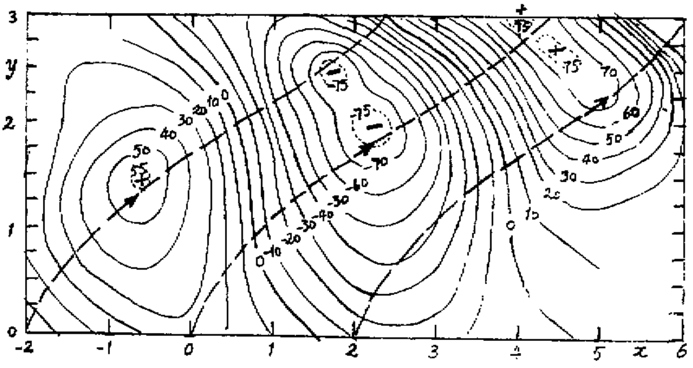

Fig.5b. The same as Fig.5a., but $\varepsilon H = 2$.

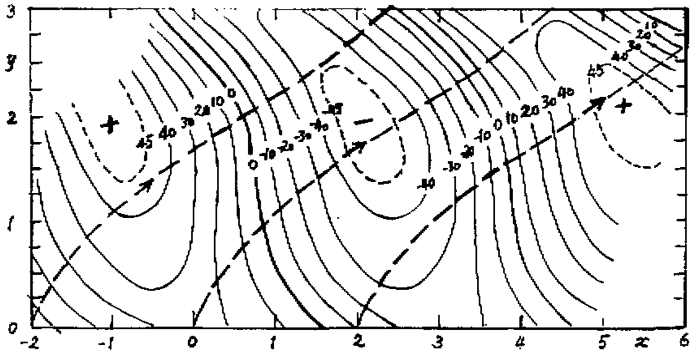

Fig.6. The same as Fig.4, but $A(x,0)=1$.

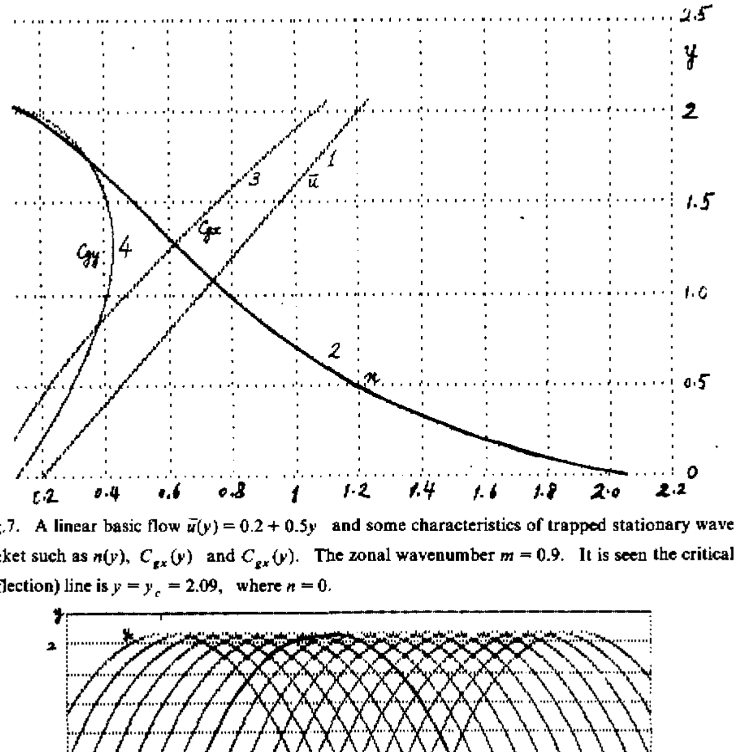

Fig.7. A linear basic flow $\bar{u}(y)=0.2+0.5y$ and some characteristics of trapped stationary wave packet such as $n(y)$, $C_{gx}(y)$ and $C_{gx}(y)$. The zonal wavenumber $m=0.9$. It is seen the critical (reflection) line is $y=y_c=2.09$, where $n=0$.

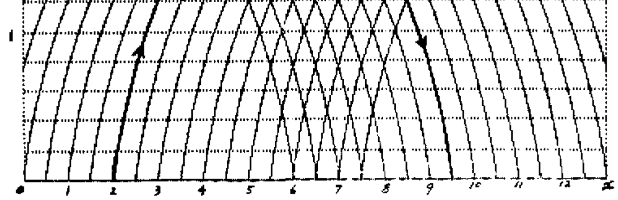

Fig.8. The rays of a trapped stationary wave packet with $m=0.9$ and superimposed on $\bar{u}=0.2+0.5y$.

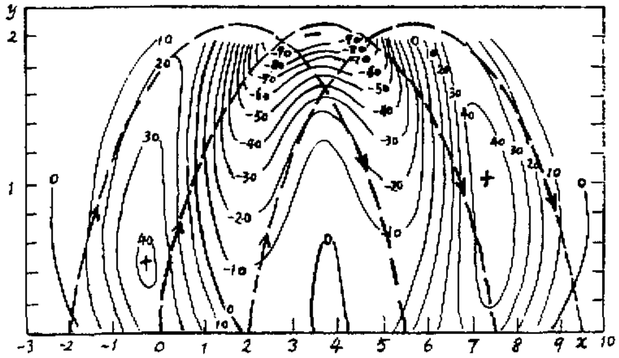

Fig.9. A trapped stationary wave packet ψ'. The basic flow and characteristics of the wave packet are given in Fig.7, and $H = 0$.

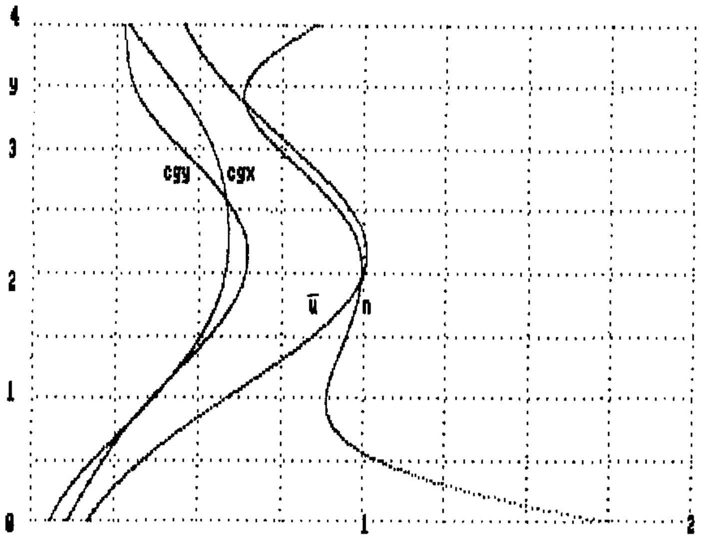

Fig.10. The basic flow $\bar{u}(y) = 0.8\exp\{-0.5(y-2)^2\} + \frac{0.4}{\pi}\{\frac{\pi}{2} + \mathrm{arctg}(y-2)\}$, and $n(y)$, $C_{gx}(y)$ and $C_{gy}(y)$ of a penetrating stationary wave packet with $m = 0.9$.

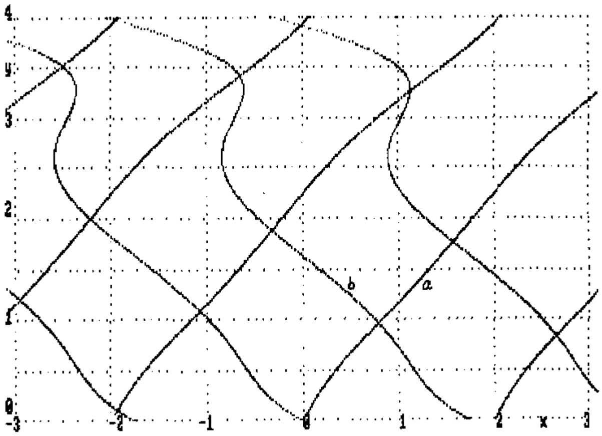

Fig.11. The rays (a) and constant phase lines (b) of a penetrating stationary wave packet with $m = 0.9$. The basic flow is given in Fig.10.