

## Nonlinear Planetary Wave Instability and Blocking

*Yong. L. McHall*

Dept. of Meteorology, University of Edinburgh, EH9 3JZ, U.K.

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### ABSTRACT

The instability of geostrophic wave circulations related to the nonlinear processes involved in the zonal mean heat balance equations is studied. It is found that the planetary waves may be destabilized by thermal forcing in specific baroclinic layers, called the breaking layers. The critical conditions of the instability will be given. In the troposphere, these conditions may be provided in blocking regions and the development of planetary perturbations is characterized distinctly by the onset, maintenance and decay of observed blocks. The whole blocking episode cannot be described as either the barotropic or baroclinic process only. The limitations on the study of wave-wave interaction using spectral models or spectrum analyses will be discussed also.

### 1. INTRODUCTION

The blocking events in the atmosphere attracted many studies in the last 40 years. The modern developed comprehensive numerical models may forecast rather successfully the onset and development of blocks when the realistic initial conditions are incorporated. However, the dynamic interpretation of blocking mechanism remains a great challenge in atmospheric sciences. The general review and comments on this subject may be referred to the study of Wallace and Blackmon (1983). The most recent studies on blocking events are concentrated greatly on the theories of wave interactions (Hoskins et al., 1983; Illari, 1984; Metz, 1987), bimodality distributions of probability density (Sutera, 1986; Hansen and Sutera, 1990) and the solitons or modons (Butchart et al., 1989; Haines, 1990).

In a study of McHall (1991c), the distributions of observed blocking actions and their seasonal variations have been explained by evaluating the zonal momentum balance in forced planetary stationary waves. This study did not reveal the dynamic mechanism of blocking events. It implies that consideration of zonal momentum balance is not enough for us to understand blocking dynamics. The mechanism and whole process of blocking events will be investigated in the present study in terms of the heat balance and instability of the planetary wave circulations.

In general, wave instabilities may be classified into two categories. One is the stochastic instability of transient waves. The heat or momentum fluxes and their convergences produced by the transients need not satisfy the local balance of heat or momentum. This category of instability is most responsible for the band-pass variabilities in the atmosphere, and can be studied by linearized perturbation equations only without considering the long-term balances related to nonlinear processes involved in the primitive equations. Most of the instabilities studied theoretically before fall into this category.

The other category of instability is an important cause of the time mean circulation breakdown in a planetary scale. It may produce the low frequency variabilities in the atmosphere. Since the time averaged circulations rely on the physical balances associated with nonlinear processes in the primitive equations, this instability cannot be illustrated by using

the perturbation equations only. Till now, it is studied mostly by numerical experiments and has not been dealt with fully without using numerical models.

Usually, the nonlinearities were considered by solving the simplified nonlinear equations such as the KdV equation (Benney, 1966). In the recent studies of McHall (1991a, b, c, referred to as M1, M2, M3, respectively) the nonlinear processes were treated differently as wave-wave interactions related to the eddy fluxes of heat and momentum and their convergences in planetary wave circulations. The results were used to explain the observed geographical distributions of planetary stationary waves and blocking events including their seasonal variations in both hemispheres. With the same method, we may also deal with the nonlinear processes in blocking events. It will be shown that the dynamic mechanism and main properties of blocking may be explained by the nonlinear development of the planetary perturbations. Discussions on some other theories of blocking will be given also in the text.

## II. BREAKING LAYERS

We have noted that the instability associated with the stochastic anomalies in synoptic or smaller scales may be studied without consideration of the time averaged balances. But the low frequency variabilities cannot be explained by ignoring the physical balances associated with the nonlinear processes in the atmosphere, because the nonlinear terms related to wave-wave interactions may make important contribution to the balances in the long life cycles of them. In M3 we discussed the zonal momentum balance in planetary stationary waves to explain blocking distributions in the atmosphere. In this study, the typical dynamic processes and characteristics of blocking are discussed further by considering the time averaged heat balance in geostrophic wave circulations.

When the zonal mean vertical velocity is included, the balance equation of symmetric heating derived in M2 gave

$$\frac{\delta k m}{2fR} \left[ \frac{\sigma_z p}{f\sigma_y} \frac{(k^2 - k_T^2)}{(k^2 + k_T^2)} \left( \beta + \frac{f^2}{a^2 \beta} \right) + \varepsilon \right] \Phi_i^2 - \frac{p}{R} (\sigma_y \bar{v} - \sigma_z \bar{\omega}) + \frac{\bar{H}}{C_p} = 0, \quad (1)$$

where,  $\Phi_i$  denotes the initial amplitude before the waves are split as discussed later on, and

$$k_T^2 = \frac{\delta^2 f^2 \sigma_z m^2}{(1 + \delta)^3 \sigma_y^2}, \quad \varepsilon = \frac{C_v}{C_p}. \quad (2)$$

It can be derived also for transient geostrophic waves and is rewritten as

$$\left[ \frac{\sigma_z p}{f\sigma_y} \left( \beta + \frac{f^2}{a^2 \beta} \right) + \varepsilon \right] \delta m \Phi_i^2 k^3 + 2fQk^2 - \left[ \frac{\sigma_z p}{f\sigma_y} \left( \beta + \frac{f^2}{a^2 \beta} \right) - \varepsilon \right] \delta m \Phi_i^2 k_T^2 k + 2fQk_T^2 = 0, \quad (3)$$

where,

$$Q = R\bar{T}_d - (\sigma_y \bar{v} - \sigma_z \bar{\omega})p,$$

and  $\bar{T}_d$  measures the zonally averaged diabatic change rate of temperature. For a provided meridional wavenumber, Eq. (3) may be viewed as a polynomial about  $k$ . In general, there exists at least one real solution which satisfies the heat balance equation. The transient waves with a different zonal wavenumber will cause zonally averaged heating or cooling in geostrophic wave circulations.

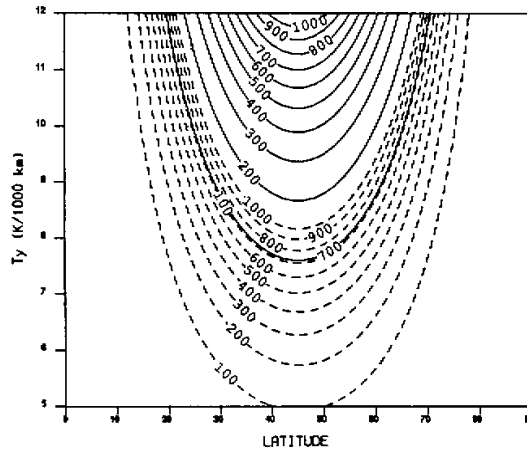


Fig.1. The breaking layers in the troposphere denoted by the numbers on the lines (unit: hPa). Solid and dashed lines are drawn for  $\Gamma = 0.65, 0.75 \text{ K} / 100 \text{ m}$  respectively.

If the solution of  $k$  is a complex, the amplitude of geostrophic waves will vary along latitude. Also, since  $k$  is involved in the phase speed of the waves, the angular frequency may become a complex too, and the waves will amplify exponentially with time. For  $Q \neq 0$ , one of the condition for the possibility of no real solutions of  $k$  reads

$$\frac{\sigma_z}{f\sigma_y} \left( \beta + \frac{f^2}{a^2\beta} \right) p + \frac{C_v}{C_p} = 0. \quad (4)$$

It is noted that this is a necessary but not the sufficient condition. Another necessary condition of the wave instability will be discussed in Section 4. In a statically stable atmosphere, this condition holds only when  $\sigma_y < 0$  in the Northern Hemisphere or  $\sigma_y > 0$  in the Southern Hemisphere. The layers where the nonlinear breakdown occurs are, then, given by

$$\tilde{p} = -\frac{aC_v\sigma_y}{2C_p\sigma_z} \sin 2\varphi. \quad (5)$$

It is referred to as the breaking layers of geostrophic wave circulations. When parameters  $\sigma_z$  or  $\sigma_y$  varies with height, a breaking layer may extend to some height, and is not only an isobaric surface.

In isobaric surfaces, (5) may be rewritten as

$$\tilde{p} = \sqrt{-\frac{aC_v R}{2C_p\sigma_z} \frac{\partial \bar{T}}{\partial y}} \sin 2\varphi. \quad (6)$$

In the stratified troposphere with constant temperature lapse rate  $\Gamma$ , we may apply

$$\sigma_z = \frac{R^2 \bar{T}}{g p^2} \Delta \Gamma, \quad \bar{T} = \bar{T}_s \left( \frac{p}{p_s} \right)^{R\Gamma/g},$$

in which,  $\bar{T}_s$  measures surface temperature (We will use  $\bar{T}_s = 280\text{K}$  in this study), and  $\Delta \Gamma = g/C_p - \Gamma$ . Thus (6) is replaced by

$$\tilde{p} = p_s \left( -\frac{agC_v \sin 2\varphi}{2C_p RT_s \Delta\Gamma} \frac{\partial \bar{T}}{\partial y} \right)^{\kappa/R\Gamma}$$

The variations in breaking layer with baroclinity and static stability in the troposphere are displayed in Fig. 1. The breaking layers reach the lowest height at midlatitude when the other conditions are the same, and may be much higher at lower and higher latitudes. The breaking height decreases with increasing baroclinity and static instability. For example, a breaking layer occurs on 500 hPa at middle latitudes when temperature gradient is over 10 K / 1000 km as  $\Gamma = 0.65$  K / 100 m, but is reduced to be about 7 K / 1000 km for  $\Gamma = 0.75$  K / 100 m. This gradient in the middle troposphere may be observed frequently in blocking regions at the beginning time. As the baroclinity in the Southern Hemisphere is generally weaker than that in the Northern Hemisphere, blocking event there takes place most frequently just in the vicinity of mid-latitude (Coughlan, 1983).

It is emphasized that the breaking layers appear merely in the atmosphere with equatorward component of mean meridional temperature gradient. This may happen at most latitudes in the troposphere all the year, but only at high latitudes in the lower and middle stratosphere of winter hemisphere. So the condition of the nonlinear instability is asymmetric with respect to the mean temperature field. This important property may be employed to investigate some anomalous circulation events which occur particularly in the lower stratosphere during winter time, such as the stratospheric sudden warmings and thermal pools. These will be discussed in other places.

### III. SUPERPOSITION WAVES IN BREAKING LAYERS

As (6) is not a sufficient condition of the wave instability, the geostrophic waves in breaking layers are not always unstable. Thus, we discuss firstly the stable waves in breaking layers required by heat balance.

In a breaking layer, the time averaged heat balance relationship (3) becomes

$$fQk^2 + \delta k_T^2 m \epsilon \Phi_i^2 k + fQk_T^2 = 0.$$

When  $\nabla \geq 0$ , where

$$\nabla = k_T^2 (\delta^2 \epsilon^2 k_T^2 m^2 \Phi_i^4 - 4f^2 Q^2). \quad (7)$$

The real solutions of zonal wavenumbers are given by

$$k_1 = -\frac{\delta \epsilon k_T^2 m \Phi_i^2 - \sqrt{\nabla}}{2fQ}, \quad k_2 = -\frac{\delta \epsilon k_T^2 m \Phi_i^2 + \sqrt{\nabla}}{2fQ}. \quad (8)$$

They are positive as  $Q < 0$ , and the magnitudes increase with growth of baroclinity or reduction of geopotential amplitude.

For a provided meridional wavenumber, the wave components possessing the discussed zonal wavenumbers propagate in different directions. These waves, when they occur in a wide area, may superimpose on each other to form the combined perturbations. For example, the superposition of geopotential perturbations gives (referring to M1)

$$\varphi' = i \left( \Phi_1 e^{i(\nu_1 t - k_1 + my + lp)} + \Phi_2 e^{i(\nu_2 t - k_2 + my + lp)} \right).$$

If wave energy is conserved as the wave split into two components, there will be

$$\Phi_1^2 + \Phi_2^2 = \Phi_i^2 .$$

This relationship which is similar to the Pythagorean proposition for right triangles is required also by heat and momentum balances in breaking layers. It turns out that the maximum amplitudes of the superposition waves in breaking layers may have the maximum amplitude greater than that of the original wave.

For the problems discussed in the following, differences in the amplitudes may be ignored. Thus,

$$\varphi' = -2\Phi \cos\left(\frac{\Delta v}{2}t - \frac{\Delta k}{2}x\right) \sin(\bar{v}t - \bar{k}x + my + lp) , \quad (9)$$

where

$$\Delta v = v_2 - v_1 , \quad \Delta k = k_2 - k_1 ; \quad \bar{v} = \frac{v_2 + v_1}{2} , \quad \bar{k} = \frac{k_2 + k_1}{2} ;$$

$v_1$  and  $v_2$  are similar to the  $v$  derived in M1 for  $k_1$  and  $k_2$  , respectively. Applying (8) and  $k_1 k_2 = k_T^2$  yields

$$\bar{v} = \bar{u}\bar{k} - \frac{1}{2k} \left( \beta + \frac{f^2}{a^2 \beta} \right) ,$$

$$\bar{k} = -\delta \frac{C_v k_T^2 \Phi_i^2}{2f C_p Q} m , \quad (10)$$

$$\Delta v = \bar{u} \Delta k , \quad (11)$$

$$\Delta k = -\frac{1}{fQ} \sqrt{k_T^2 (\delta^2 \varepsilon^2 k_T^2 m^2 \Phi_i^4 - 4f^2 Q^2)} . \quad (12)$$

Here,  $\bar{v}$  denotes the angular frequency of the superimposed perturbation, of which zonal wavelength is

$$L_m = \frac{2\pi}{|\bar{k}|} . \quad (13)$$

While,  $\Delta v / 2$  indicates the frequency of wave envelope of the superposition wave with the wavelength

$$L_e = \frac{4\pi}{|\Delta k|} .$$

It is noted that the superposition waves are different from the classical wave group composed of the waves propagating in one direction.

Eq.(9) shows that this wave envelope propagates at the velocity called the dispersive velocity:

$$c_e = \frac{\Delta v}{\Delta k} . \quad (14)$$

Using (11) yields  $c_e = \bar{u}$  . So the dispersive velocity is always along the zonal direction. Its magnitude is identical to the velocity of mean zonal flow in this circulation model. While, the zonal phase speed of the superposition waves is evaluated by

$$c_m = \bar{u} - \frac{1}{2k^2} \left( \beta + \frac{f^2}{a^2 \beta} \right), \quad (15)$$

and follows, from (10), that

$$c_m = \bar{u} - \frac{2C_p^2 f^2 Q^2}{\delta^2 C_v^2 k_T^4 m^2 \Phi_i^4} \left( \beta + \frac{f^2}{a^2 \beta} \right). \quad (16)$$

It is slowed down by strong baroclinity, diabatic heating and wave amplification, and is in the direction different from the dispersive velocity in general.

#### IV. UNSET OF BLOCKING

##### 1. *Thermally Forced Baroclinic Instability*

We have noted that the nonlinear instability of normal geostrophic waves in breaking layers may occur only if  $\nabla < 0$ , which gives another necessary condition and may be represented approximately by

$$\bar{T}_d^2 > \frac{\delta^4 g^4 C_v^2 \sigma_z m^4 H_{rs}^4}{4(1+\delta)^3 C_p^2 R^2 \sigma_y^2}, \quad (17)$$

in which,  $H_{rs} = \sqrt{H_1^2 + H_2^2}$  indicates the root-square geopotential perturbation height. The instability will occur if diabatic heating exceeds a certain limit, so it is the thermally forced instability as supposed by White and Clark (1975). They wrote:

"Because the formation of mid-ocean blocking ridges occurs most often in autumn and winter months when the mid-latitude westerlies and sensible heat exchange are at their maximum values, the hypothesis is formed that blocking activity is due to baroclinic instability processes which are modified by sensible heat transfer between ocean and atmosphere. ....

The seasonal variability of blocking ridge activity emphasizes the importance of sensible heat exchange between ocean and atmosphere to the baroclinic instability of stationary waves. In the absence of sensible heat exchange, stationary waves cannot be made unstable since their wavelengths are too long. However, with sensible heat exchange the long waves are made unstable, and stationary waves can grow. We observe them as mid-ocean blocking ridges. The reason blocking ridge activity is much reduced in spring and summer over that in autumn and winter is because from May to September the net sensible heat exchange over the mid-latitude North Pacific is almost negligible."

It is clarified that the planetary wave instability which they talked about was considered to be produced by anomalous diabatic heating and induced baroclinity, but not the land-sea thermal contrast only. The effect of diabatic heating on long wave instability was suggested also by the numerical study of Haltiner (1967). He showed clearly that when diabatic heating was included, the long waves of wavelength comparable with those of blocking waves became unstable. That the critical unstable wavelength is enlarged by diabatic heating was manifested also by Wiin-Nielsen et al. (1967). But in their linear theory, the growth rate of ultralong waves was very small.

The effect of anomalous diabatic heating on blocking events has been examined by the comprehensive numerical models as well. In their 9-level Southern Hemisphere model, Simpson and Downey (1975) found that the warm midlatitude sea surface temperature anomaly could cause the systematic shift of instability toward lower wavenumbers and induce a blocking event at about 55° S, 10° south of the temperature anomaly.

As hypothesized by White and Clark, this instability is a kind of baroclinic instability, since it occurs only in the baroclinic breaking layers and (17) may be replaced by

$$\sigma_y^2 > \frac{\delta^4 g^4 C_v^2 \sigma_z m^4 H_{rs}^4}{4(1+\delta)^3 C_p^2 R^2 \bar{T}_d^2}.$$

For the instability happens only in the atmosphere with poleward decrease of mean temperature, it gives, on isobaric surfaces,

$$\frac{\partial \bar{T}}{\partial y} \begin{cases} < -B_b & \text{(Northern Hemisphere)} \\ > B_b & \text{(Southern Hemisphere)} \end{cases}$$

where

$$B_b = \frac{\delta^2 g^2 C_v m^2 \sqrt{\sigma_z} p H_{rs}^2}{2\sqrt{(1+\delta)^3} C_p R^2 |\bar{T}_d|}.$$

is the breaking baroclinity for nonlinear breakdown of geostrophic waves. In breaking layers, we may substitute (6) into it producing

$$B_b = \frac{\delta^4 a g^4 C_v^3 m^4 H_{rs}^4}{8(1+\delta)^3 C_p^3 R^3 \bar{T}_d^2} |\sin 2\varphi|. \quad (18)$$

The breaking baroclinity depends on wave amplitude and may be reduced by diabatic heating. If baroclinity is not intense enough, wave development will be limited until the baroclinity in a breaking layer increases further and exceeds the new threshold.

When the planetary waves become unstable, the amplitudes vary exponentially with time and along latitude as well. If the conditions of forming breaking layer and instability are satisfied also in downstream or upstream, the geostrophic waves there will amplify too. This downstream or upstream effect may in turn produce the double and triple blocks.

## 2. Blocking Positions

The fact that blocking events occur most frequently in some regions implies that onset of blocking depends on zonal asymmetry of forcing. It was widely recognized that this forcing is produced by synoptic transient eddies. The studies of Hoskins (1983) and Hoskins et al. (1983) showed that the effect of high frequency eddies is to force the mean flow via vorticity fluxes, while they act as a damping mechanism via the heat flux. Green (1977) found from his synoptic study that the high frequency transients might act through vorticity flux to sustain the existing low-frequency anomalies. Whereas, some other authors (Savijarvi, 1977; Holopainen et al., 1982; Holopainen, 1983) argued that the dumping produced by the heat flux tends to be predominant if compared with the vorticity flux. As concluded by Wallace and Blackmon (1983), apart from the high frequency transients, other processes might be more influential in determining when and where the anomalies develop, and in anchoring them in certain favoured geographical positions. The forcing of low frequency variability is not an entirely random process.

Being an alternative argument, it is forwarded that the asymmetric forcing may be produced by the planetary waves themselves. As discussed in M3, blocking is favoured in the area where zonal momentum balance would be destroyed by the asymmetric zonal acceleration produced in the forced stationary waves, which provide a direct internal forcing in the observed blocking areas. The Fig.2 in M3 showed that the orographically forced normal sta-

tionary waves may produce zonal acceleration and deceleration behind and in front of the stationary ridges, respectively, in order to balance the opposite momentum generations by the orographical forcing in the Northern Hemisphere. Consequently, there will be velocity convergence near the ridges and divergence near the troughs at high levels. According to vorticity equation, the ridges will be strengthened but the troughs may be weakened to form the typical "Ω" pattern of blocking height after the normal wave pattern becomes unstable. While in the Southern Hemisphere (referring to Fig.5 in M3), the same mechanism reduces the stationary wavelength in one area and enhances the amplitude. This process may form the specific dipole blocks as observed there.

Since the eddy fluxes of heat and momentum are represented by the products of perturbations, they include the components with half, but not equal to, the wavelength and period of the perturbations (M1 and M2). While, the fluxes of the same wavelength as the perturbations are produced by waves and mean flow interaction. The half scale fluxes may be seen evidently in the diagnostics of Atlantic and Pacific blocks provided by Hansen and Chen (1982). These fluxes produced by the interactions of planetary perturbations correspond to no real weather systems and so cannot be regarded as an example of the interaction between cyclonic scale waves and planetary waves as suggested by the authors. It can be expected that they may increase in blocking waves with greatly enhanced amplitudes. This was proved also by the study of Hansen and Sutera (1984).

### 3. Growth Rate

It was reported by Dole (1986) that the development rates of Pacific blocking were often rapid so that the blocks could be established fully in less than a week. Thus, one would argue that the blocking events may not be associated with baroclinic instability, because the growth rate of planetary waves evaluated by many authors (e. g., Gall, 1976; Simmons and Hoskins, 1977) is very small. However, the baroclinic instability discussed here depends entirely on nonlinear process, and involves the different mechanism from those in the previous studies. From (9), the growth rate of wave amplitude is given by

$$G = \frac{1}{2} \text{Im}(\Delta v) .$$

Using (11) and (12) yields

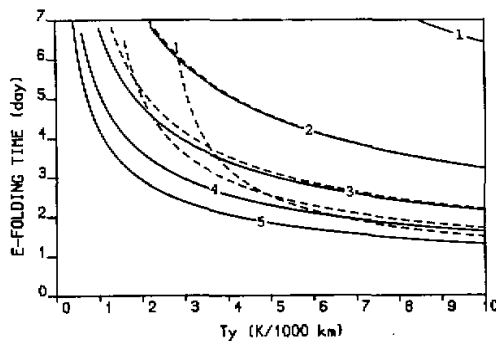


Fig.2. The growth rate of blocking waves on 500 hPa. The solid and dashed lines are drawn for initial root-square height being 100 and 250 gpm respectively. The values indicate  $N_y$ .

$$G = -\frac{\bar{u}}{2fQ} \sqrt{k_r^2 (4f^2 Q^2 - \delta^2 e^3 g^4 k_r^2 m^2 H_{rs}^4)} .$$

Here, inserting (6) into (2),

$$k_r^2 = -\frac{4\delta^2 C_v \Omega^2 N_y^2}{(1+\delta)^3 a C_p R T_y} \sin^3 \varphi \cos \varphi ,$$

Where,  $N_y = am$  signifies nondimensional meridional wavenumber. The growth rate depends on wavenumber, so blocking waves grow inhomogeneously in the spectra as confirmed by Hansen and Chen (1982) and Hansen and Sutera (1984). Also, it is shown clearly that the growth is self-limited by wave amplification. So the blocking ridges do not usually form the strong closed system like the surface anticyclones.

To estimate approximately, the mean zonal flow is represented by thermal wind

$$\bar{u} = \bar{u}_s - \frac{R}{f} \frac{\partial \bar{T}}{\partial y} \ln \frac{p_s}{p}$$

and we set  $Q \approx R\bar{T}$ . For  $\bar{T} = 1\text{K/day}$  at 500 hPa (Newell et al. 1970),  $\bar{u}_s = 0\text{ m/s}$  and  $\delta = 0.21$ , the variation of growth rate with baroclinity is displayed in Fig.2. At the initial stage when perturbation amplitude is small, the synoptic scale components in a blocking spectrum grow quicker than planetary components. After waves have been amplified to some extent and baroclinity has been reduced, the planetary components will develop more rapidly as shown by the dashed lines.

## V. MAINTENANCE OF BLOCKING EVENTS

### 1. Stationary Baroclinity

Retgression of the superposition waves may take place in breaking layers if baroclinity therein is strong enough or, approximately from (16),

$$\sigma_y^2 \geq \frac{\delta^3 g^2 C_v |m^3| \sigma_z H_{rs}^2}{(1+\delta)^3 C_p R |\bar{T}_d|} \sqrt{a\Omega \bar{u} \sin^2 \varphi \cos \varphi} . \quad (19)$$

In breaking layers we may substituting (6) into it, producing

$$\frac{\partial \bar{T}}{\partial y} \begin{cases} \leq -B_s & (\text{Northern Hemisphere}) \\ \geq B_s & (\text{Southern Hemisphere}) \end{cases}$$

where

$$B_s = \frac{\delta^3 a g^2 C_v^2 |m^3| H_{rs}^2}{(1+\delta)^3 C_p^2 R^2 |\bar{T}_d|} \sqrt{a\Omega \bar{u} \cos^3 \varphi \sin^2 \varphi} \quad (20)$$

is the stationary wave baroclinity in breaking layers. Unlike the usual geostrophic waves discussed in M1, the condition of retrogression depends more directly on baroclinity, and may be independent of the variation in wavelength. This may be found also from observations. The blocks may regress temporally when there is baroclinic advection brought by synoptic troughs. In this episode, the wavelength of block may not increase significantly.

### 2. Blocking Amplitude

When the usual geostrophic waves outside the breaking layers are stable, they may

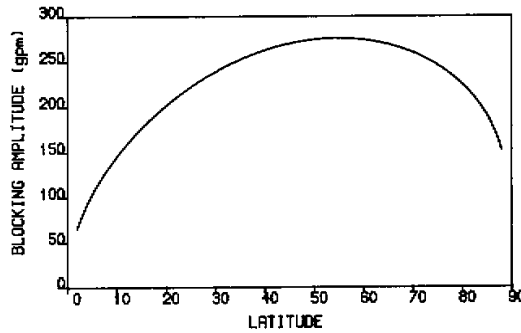


Fig.3. The root-square blocking height on 500 hPa.

retrograde continuously until the mean circulation fields are changed. But in an unstable breaking layer, the stationary baroclinity increases rapidly as waves grow. So wave regression may be halted by wave amplification. When wave amplification and retrogression cease at the threshold of the breaking and stationary baroclinity, amplified blocking waves may be maintained stationary.

The amplitude of blocking geopotential perturbation at the threshold may be resumed approximated from  $B_b = B_s$ , giving

$$H_{rs}^2 = 4 \left| \frac{a C_p R \bar{T}_d}{\delta g^2 C_e N_j} \sqrt{a \Omega \bar{u} \cos \varphi} \sin \varphi \right|. \quad (21)$$

This blocking amplitude is peaked at latitude about  $55^\circ$ , which is coincident with the most frequent latitudes of blocking events around  $56^\circ$  in the Northern Hemisphere (Coughlan, 1983). It is found also that in the same spectrum of a mature block, planetary components have greater amplitudes than synoptic components.

For  $\bar{u} = 15 \text{ m/s}$ ,  $N_j = 4$  and the parameters used in Fig.2, the meridional variation in blocking amplitude on 500 hPa is depicted in Fig.3. It shows that the root-square height of blocking waves is about 280 gpm at the peaked latitude. In the Southern Hemisphere, as the meridional scale of blocking is usually smaller than that in the Northern Hemisphere so that the blocks look more like closed highs, the blocking waves have smaller amplitudes.

### 3. Block Baroclinity

We have noted that blocking events are initiated in the baroclinic breaking layers. The baroclinic condition is essential for formation of blocking as suggested by observations. Dole (1986) pointed out after analysis of North Pacific blocking events that the vertical structures of blocking display considerable tilts with height as they amplify and, in several aspects, resemble those of amplifying baroclinic waves, indicating that their developments may be significantly influenced by baroclinic processes. He concluded then that these observations raise the distinct possibility that barotropic or equivalent barotropic models will be inadequate for modeling important aspects of the initial development.

Moreover, the kinetic energy for blocking development is converted eventually from available potential energy which relies on the baroclinity. This may be proved by the studies of Gall et al. (1979a) and Young and Villere (1985). When available potential energy is

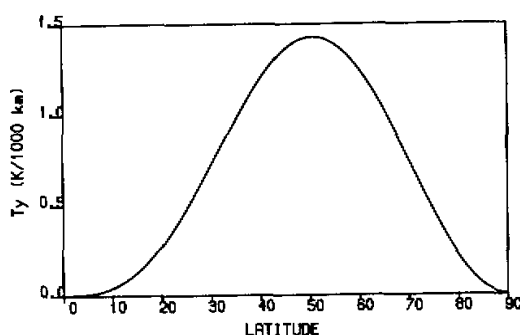


Fig.4. The block baroclinity for maintenance of blocks.

exhausted after amplification of unstable perturbations, developed disturbances may gain a barotropic structure and cannot survive for a considerable time like the occluded cyclones. For maintenance of blocking systems, the required baroclinity may be estimated by substituting (21) into the breaking baroclinity or stationary baroclinity, giving

$$T_y = -\frac{\delta^2 \Omega C_v N_y^2 \bar{u}}{(1 + \delta)^3 C_p R} \sin \varphi \sin^2 2\varphi.$$

It tells clearly that the spectral components of synoptic scale are more baroclinic than the planetary components. This is consistent with the observation analysed by Mo et al. (1987). For the parameters used previously, this baroclinity called as the block baroclinity is demonstrated graphically in Fig.4.

In general the block baroclinity is very low, so the mature blocks manifest a nearly barotropic structure. When mean zonal velocity at lower levels is not zero, they cannot be purely barotropic. This agrees the numerical experiments of Frederiksen (1987). He found that baroclinic conversion is still important although barotropic processes are coming into play in the dipole onset-of-blocking models. The feeble block baroclinity may be retained presumably by the baroclinic flows brought from synoptic disturbances. Some examples of the important effect of synoptic baroclinic flows on maintenance of blocks were given by Green (1977) and Colucci (1985, 1987). Unlike the extratropical cyclones, the quasi-barotropic blocking systems may therefore sustain for a relatively long period in a weakly baroclinic area. They exhibit frequently the weakening and reintensification process as well as the overall zonal translation during their lifetime (Shutts, 1986), produced by the variations in baroclinity. These particular features of blocking systems cannot be revealed by the barotropic theories, such as the barotropic solitary waves.

#### 4. Decay of Blocking

After baroclinity has been weakened in blocking regions, the breaking layer can no longer exist at the original height. So the blocking waves will revert to be the usual geostrophic waves outside breaking layers. This may be seen in the following.

The stationary wavelength of the superimposed waves in breaking layers is obtained

by setting  $c_m = 0$  with (15), showing

$$L_{ms} = 2\pi \sqrt{\frac{a\bar{u}\cos\phi}{\Omega}} \quad (22)$$

It gives  $L_{ms} \approx 5450$  km at latitude  $55^\circ$  for  $\bar{u} = 15$  m/s.

Furthermore, inserting the blocking amplitude (21) into (19) yields another expression of the stationary baroclinity:

$$\sigma_y^2 = 4\delta^2 a\Omega\sigma_z m^2 \bar{u}\cos\phi \sin^2\phi = 2\delta^2 \frac{f^2 \sigma_z m^2 \bar{u}}{\beta + \frac{f^2}{a^2}\beta}$$

It is twice the lowest baroclinity for the usual geostrophic waves to be able to retrograde outside the breaking layers (M1), when the wave-length is longer than the stationary wavelength. Applying it for the stationary wavelength (M1) yields

$$L_s = L_{ms}$$

So the zonal wavelength of the blocking waves is identical to the stationary wavelength of the usual geostrophic waves. It implies that blocks may remain stationary when they transmute from the anomaly pattern to the normal pattern with greatly enhanced amplitudes and the same wavelength. The difference between the stationary baroclinities of these two wave patterns may allow the blocked waves to remain stationary even longer until the baroclinity decreases as half intense as the block baroclinity. Afterwards, the blocked waves will start to progress and then collapse. This process of blocking destruction is well familiar to weather forecasters. In particular, when baroclinic flows approach from the west, they may produce successive generation of new warm high cells on the west side, characterized by westward movement as demonstrated by Palmen and Newton (1959). While the component high cells on the east side of blocking complex may drift slowly eastward.

The discussed whole process of blocking may be seen from the diagnosis of a blocking event given by Mo et al (1987). The daily spectral energetics of the control experiment in his general circulation model showed that the baroclinic amplification of planetary waves played an fundamental important role in the evolution of the blocking events, especially at the earlier stage. The analysis did not give any information about wave-wave interaction in different scales. The maintenance of the block was related to the activities of shorter baroclinic spectral components. When the shorter baroclinity components became less important than ultralong components in the energy cycle, the block started to move and decay.

## VI. ON WAVE INTERACTIONS

The instability theory developed in this study may account effectively for the statistic behaviours of blocking events. In phenomenology, the discussed onset of blocking appears as the interference between planetary waves. This is supported strongly by the statistic survey of Austin (1980). However, the theory does not draw the detailed picture related to the transient behaviours of individual blocking systems, since the discussions are based on time averaged heat balance. It is possible that blocking events may be triggered and maintained by stochastic disturbances in smaller scales. The observational evidences have been provided by many authors. Clucci (1985) reported two cases of the blocking events related to explosive cyclogenesis. Noar (1983) gave an example of a blocking high in the Southern Hemisphere commenced by a moving cold front. He postulated that the processes of replacement/dis-

placement occur through the action of weak, short wave disturbances which are very difficult to be represented accurately in numerical prediction models. However, it is not difficult to understand that the sub-scale transients may make the contribution only if the conditions of blocking, such as the appearance of breaking layer and breaking baroclinity required by thermally forced instability, can be completed by them.

In mathematics, any continuous function different from a harmonic wave may be expanded into Fourier series, so that it may be considered approximately as the composition of the harmonic components. The blocking patterns may also be represented by means of the mathematical spectrum, and many studies on the wave interactions were made by diagnosing the mathematically expanded spectral fields. One might believe that each of the wave components has its own behaviour and propagates independently like the wave components in a white light. With the spectrum analysis, Hansen and Chen (1982) found that the variation in energetic spectrum of Atlantic block was more inhomogeneous than that of Pacific block. They concluded then that the development of the Atlantic block was forced by nonlinear interaction of intense baroclinic synoptic waves with barotropic ultralong waves, while the Pacific blocking resulted from the baroclinic amplification of planetary waves.

However, it is important to point out that there may be the essential difference between atmospheric perturbations and lights. The latter can be dispersed into spectrum by a prism physically other than mathematically. When a circulation pattern is expanded mathematically into a spectrum, a component of it need not be a physical wave in the atmosphere. It may be the artificial separation from the others corresponding no independent wave mechanism in physics, and so illustrate only one aspect of the whole behaviour different from the simple superposition of physical waves. Thus, the inhomogeneous variations in blocking spectrum do not necessarily suggest the interaction between the weather systems of different scales. The intermediate scales diagnostic fields of blocking events shown by Hansen and Chen (1982) and Hansen and Sutera (1984) may also be produced by planetary waves themselves as discussed earlier, and so may not correspond to any synoptic weather system.

In fact, a blocking system is essentially different from an ultralong harmonics in the spectral numerical models. It may possess a complete spectrum of itself even without including any sub-scale weather system. Each of the spectral components cannot be separated physically from the others and may not vary synchronously with the others. Therefore, there is the substantial limitation on the study of wave interaction by using spectral numerical models. For example, in the spectral model of Gall et al. (1979b), wave 3 began to grow ten days later than wave 7. However, the growth rate of kinetic energy of the wave 7 was as slow as the wave 3 and did not show any remarkable difference except the time lag. This implied that the waves 3 and 7 might be the artificial components of the same integrated circulation pattern other than the different weather systems. Similarly, a synoptic system may include planetary components as well in the expanded mathematical spectrum. The spectral components are usually all time-dependent. So the development of a regional baroclinic cyclone over the eastern coast of North America may be shown in spectral representation as a change in all wavenumbers, noted by Holopainen (1983). He, thus, reminded that the spectral representation is to some extent an artificial way of describing the changes in the atmosphere.

If one believes that the inhomogeneous behaviours of the spectral components do always represent the physical interactions, he will be disappointed in finding the physical representations of them in each case. Because expressions of these components can be given by so many

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different mathematical methods of orthogonal expansion with arbitrarily selected wavelengths, and every weather system like blocking cannot in principle be identified with a single component in the mathematical spectrum. The overall development of blocking is then different from the growth of a planetary component involved in its own spectrum. Also, it will be necessary to distinguish the interactions within the same weather system from those between different systems, since they represent different physical processes. This, however, cannot be accomplished by the spectrum analysis itself. Till now, we have not found the clear observational picture of the role played by synoptic systems in blocking episodes. The study of Savijarvi (1978) did not give the systematic differences in vorticity and temperature balances between the blocking and nonblocking cases. Some other studies (e. g., Colucci et al., 1981; Hansen and Chen, 1982) showed that there are also the blocks forced by the interactions between planetary waves.

## VII. SUMMARIES AND REMARKS

The time averaged heat balance in geostrophic wave circulations cannot always be established in the baroclinic regions called the breaking layers, which occur only in the atmosphere with poleward decrease of zonal mean temperature. The breaking layers may take place at the observed blocking levels when baroclinity is strong, and the normal geostrophic waves there may become unstable. This instability is associated with the nonlinearities involved in the primitive equations, and so cannot be investigated with the linearized perturbation equations only. The induced wave amplification is characterized by observed blocking process in the troposphere.

It is discussed in M3 that the zonal momentum balance in planetary stationary waves is most responsible for the statistic properties of blocking events. While, the present study suggests that the temporal behaviours of blocking depend mostly on heat balance. The observational evidences may be found from the studies on the relation of blocking to the anomaly heat exchanges between the surface and atmosphere (e. g., Namias, 1964; Ratcliffe and Murray, 1970; White and Clark, 1975; Davies, 1978; Noar, 1983). It is possible in the real atmosphere that the temperature gradient required for the instability is connected with surface temperature contrast. In the two-level quasigeostrophic models of Everson and Davies (1970), blocking occurred only when zonally differential heating was included. They found that the model blocks took place in the area at broad scale minimum of eddy kinetic energy, when the minimum was associated with large scale longitudinal land-sea temperature differential of about 2 K. The time scale of the simulated blocking depended approximately linearly on the longitudinal differential heating function. These numerical outcomes were closely analogous to their observational analysis of the synoptic winter charts for 1956–1957. Smith (1973) also reported using the data of 1954–1956 that the onset of blocking activity generally coincided with minima of eddy kinetic energy coupled necessarily with a significant land-sea temperature contrast.

The numerical integration with a two-level general circulation model of Gordon and Davies (1977) found that the distinct surface temperature contrast which may lead to blocking could be occasionally built up through the changes of variable fields in the model atmosphere with consequences on the computed varying albedo distribution without adding external heating. During the simulated 3 years, characteristic blocks were produced noticeably more frequently at higher reductions of solar heating.

As the blocking events are related more intimately to the forced stationary waves other than topography (M3), they may occur principally also in the numerical simulations when the forced perturbations but not the asymmetric topography are incorporated (Mo et al., 1987). The asymmetric fluxes produced by the planetary perturbations act as a net forcing mechanism to the large scale circulations (Reinhold and Pierrehumbert, 1982). The role played by this internal forcing in blocking events cannot nevertheless be excluded by the numerical experiments with the simplified models of Egger (1978) and Kalnay-Rivas and Merkin (1981), though they thought that the blocking could not occur without external forcing. Because the used simple models might not produce the internal forcing.

Moreover, the definitions of blocking given by variant authors were made by comparing with the normal circulations in the earth's atmosphere surrounded by its peculiar environments. As the physical dynamics in a numerical model cannot be completely the same as in the real atmosphere, the blocking in the model atmospheres should be redefined in comparison with their own normal circulation patterns as did by Egger (1978). Thus, it would be significant to find the normal circulation fields in a model atmosphere before we use the model to study blocking events. The large amplitude waves produced by the experiments of Kalnay-Rivas and Merkin (1981) with a simple model and idealized orographic forcing were the time averaged circulation. So they were actually the normal stationary waves of the model atmosphere forced by exaggerated external forcing. The phase distribution and wavelength over the topography was similar to the linearly forced orographic waves, of which the dynamic mechanism has been elucidated in M1.

Consideration of the topographic effect on blocking events need not require that the blocking actions in a model atmosphere without topography occur also at observed time and place. Although no blocks appeared in the observed position when the topography was eliminated in their experiments with a forecasting model, Tibaldi and Buzzi (1983) noted that not all the Northern Hemisphere flows became zonal and a sort of blocking ridge appeared at a different place in the first few days of integration. The similar situation may be found more obviously in the numerical experiments for another blocking process (Ji and Tibaldi, 1983).

The present study shows that development of blocking is connected with thermally forced baroclinic instability in breaking layers. Thus, the observed blocking actions are initiated in baroclinic areas and associated frequently with cyclonic activities. The theoretically derived growth rate depends on wavenumbers and possesses the observed order of blocking development. The components of shorter wavelengths in a blocking spectrum grow more rapid than the longer components at the earlier stage. After the waves are amplified and baroclinity is reduced, the longer components may grow quicker and eventually become larger than the short components. Since the critical baroclinity of the instability and growth rate depends highly on the amplitude of perturbations, the wave development is self-limited.

After blocking systems grow to a certain extent, their development and movement may be halted at the threshold of block and stationary baroclinity. The blocking amplitude is maximized near latitude  $55^\circ$ , by which the blocking events are most frequent in the Northern Hemisphere. The mature blocks exhibit a quasi-barotropic structure, because we have proved on theory that the maintenance requires only a feeble baroclinity. It means that the whole process of blocking events cannot be described solely by either a baroclinic or barotropic model. The barotropic modons or solitons (Redekopp, 1977; McWilliams, 1980; Butchart et al, 1989) might capture some mechanic features of blocks in the mature stage.

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Although the wave interaction discussed in the present study is produced mainly by the planetary waves themselves, the influence of sub-scale weather systems is not ruled out. This interaction can be investigated also in many ways other than the spectrum analysis. We have found that the blocking strength is maintained by the weak baroclinic flows, which may be provided by the back cascade from synoptic transients. This baroclinity is also necessary to prevent the blocks from progressing downstream. The later effect can be achieved by the eddy advection of potential vorticity including temperature advection, as shown by the calculations of Illari (1984) with space-centred finite difference method.

The effect of blocking events on zonal momentum balance has been revealed in M3. This study shows that the blocking events may also have important contribution to heat and kinetic balance in the atmosphere. From M2, the zonal mean transport of heat is proportional to the square amplitudes of perturbations. When heat transport is inefficient in the normal regime with relatively small amplitudes, temperature contrast may increase at high levels in the rotational atmosphere to form the breaking layers. As a result, the normal regime is broken down and the developed blocking waves may then increase heat transport. While, the kinetic energy converted from the available potential energy in blocking areas may compensate the kinetic energy dissipation in the atmosphere.

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