An Impact of Hydrostatic Extraction Scheme on BMRC's Global Spectral Model

Sheng Hua (盛 华)
Institute of Atmospheric Physics, Academia Sinica, Beijing
William Bourke and *Terry Hart*
Australian Bureau of Meteorology Research Centre
Received December 3, 1990; revised February 1, 1992

ABSTRACT

There are two important features in geophysical fluid dynamics. One is that the atmospheric and oceanic equations of motion include the Coriolis force; another is that they describe a stratified fluid. The hydrostatic extraction scheme, or standard stratification approximation, posed by Zeng (1979), reflects the second aspect of geophysical fluid dynamics. There exist two major advantages in this scheme; accurate computation of the pressure gradient force can be obtained over steep mountain slopes, and the accumulation error in vertical finite differencing can be reduced, especially near the tropopause.

Chen et al (1987) introduced the hydrostatic extraction scheme into a global spectral model, which attained preliminary success at low resolution. Zhang and Sheng et al (1990) developed and improved the hydrostatic extraction scheme in a global spectral model, in which C_0, the parameter that represents the stratification of the reference atmosphere, changes not only with height, but also with latitude. The scheme has been incorporated BMRC's global spectral model (IAPB). Four 5–day forecasts have been performed to test the IAPB with the hydrostatic extraction scheme. Objective verifications demonstrate a positive effect of the hydrostatic extration scheme on BMRC's model, particularly at upper levels, over the tropics and the Antartic region.

I. INTRODUCTION

The sigma coordinate proposed by Phillips (1957) has been widely used in models as a way of conveniently incorporating variations in the height of the earth's surface. However, the terrain–following vertical coordinates still have some problems. For example, the pressure gradient force in σ coordinate comprises two terms which tend to be large with opposite signs over steep mountain slopes. It is such difficult to calculate accurately the pressure gradient on tilted sigma–surface over mountains.

The hydrostatic extraction or standard stratification approximation, posed by Zeng (1979) has been introduced into a spectral model by Chen et al (1987). However, in version A, the parameter C_0 depicting the stratification of the reference atmosphere is constant, and computational instability may occur for high vertical resolution in their scheme. Zhang and Sheng et al (1990) developed and improved the hydrostatic extraction scheme (version B and version C). In version C, C_0, the parameter that represents the stratification of reference atmosphere, changes not only with height, but also with latitude. This version of hydrostatic extraction makes the reference atmosphere even more realistic than versions A and B. The deviations of temperature from a reference temperature which is a function both of height and latitude become smaller everywhere. With this hydrostatic reference atmospher, the pressure gradient force terms in the σ–coordinate formulation can be rewritten to represent more exactly over steep mountain slopes; in addition, the vertical derivation of temperature can be partially deduced in term of reference atmosphere and be calculated more accurately. The accumulation error in usual vertical advection of temperature is reduced, especially near the tropopause.

In BMRC's global spectral model, the horizontal mean of the initial data is chosen as the reference temperature profile in BMRC's global spectral model, the horizontal mean of the forecast temperature for each level must be calculated for every time step. The difference between the horizontal mean of the initial data and that of the forecast for every time step has been treated in an explicit fashion for time integration. This method eliminates potential for a week unstability in semi–implicit time integration.

In Section 2, the definition of reference atmosphere is given. In Section 3, governing equations are deduced. We describe the differences of dynamical framework between the IAP model and BMRC model and the modification to the BMRC model using version C of the hydrostatic extraction scheme. The modified BMRC model is referred to as IAPB. The BMRC semi–implicit time integration is modified in IAPB system, as described in Section 4. In Section 5, we give results using four real cases, one for each season of the year. We have compared objective verification scores of 5–day prediction by both the IAPB and BMRC models at resolutions R21L9 and R31L9 respectively, integrated without physical processes, i.e., an adiabatic model. Particular attention has been paid to the beneficial impact of hydrostatic extraction scheme on the BMRC model at resolutions R21L9 and R31L9 respectively. Section 6 consists of final conclusions and remarks.

II. REFERENCE ATMOSPHERE

As is well known, the vertical temperature profile of the real atmosphere not only changes with height, but also with latitude. The tropopause is about 100 hPa in the equatorial region, but about 400 hPa at high latitudes. In the hydrostatic extraction method, the stratification parameter $C_0(\mu,p)$ describing the reference atmospheric state is introduced into the model. It is defined as

$$\frac{R^2\overline{T}}{g}\left(\frac{g}{C_p}+\frac{d\overline{T}}{dz}\right)=C_0^2(\mu,p)\ , \tag{2.1}$$

where $\overline{T}$ is the temperature of the reference atmosphere, depending upon both height and latitude, and μ is $\sin\theta$ (0 denotes latitude). In this paper, $C_0(\mu,p)$ has been calculated from climatology for each month. C_0 can equally be specified from initial data.

Assuming that the reference atmosphere satisfies the hydrostatic equation

$$\frac{\partial\overline{\varphi}}{\partial\ln\sigma}=-R\overline{T}\ , \tag{2.2}$$

the temperature and geopotential height of the reference atmosphere at any pressure surface can be derived as follows:

$$\overline{T}(\mu,p)=\frac{C_p C_0^2(\mu,p)}{R^2}+\left(\overline{T}_0(\mu)-\frac{C_p C_0^2(\mu,p)}{R^2}\right)\left(\frac{p}{p_0}\right)^{R/C_p}\ , \tag{2.3}$$

$$\overline{\varphi}(\mu,p)=\frac{C_p}{R}\left(R\overline{T}_0(\mu)-\frac{C_p C_0^2(\mu,p)}{R}\right)\left[1-\left(\frac{p}{P_0(\mu)}\right)^{R/C_p}\right]$$

$$-\frac{C_p C_0^2(\mu,p)}{R}\ln\left(\frac{P}{P_0(\mu)}\right) , \tag{2.4}$$

where $\overline{T}_0$ and P_0 are the temperature and pressure at sea level. Both depend upon latitude only. The temperature and geopotential height of the model atmosphere are then separated into those of the reference atmosphere and deviation from it, as follows:

$$T(\lambda,\mu,p,t)=\overline{T}(\mu,p)+\acute{T}(\lambda,\mu,p,t) , \tag{2.5a}$$

$$\varphi(\lambda,\mu,p,t)=\overline{\varphi}(\mu,p)+\acute{\varphi}(\lambda,\mu,p,t) , \tag{2.5b}$$

where λ is longitude. Apparently, the temperature deviation with this reference atmosphere of version C is expected to be substantially smaller than in version A or B.

III. PROGNOSTIC EQUATIONS

The temperature T and geopotential height φ of the model atmosphere are separated into those of a reference state and deviation as shown in Eq.(2.5). It can easily be verified that

$$\frac{\partial\overline{\varphi}}{\partial\lambda}+R\overline{T}\frac{\partial\ln P_*}{\partial\lambda}=0 , \tag{3.1}$$

$$\frac{\partial\overline{\varphi}}{\partial\mu}+R\overline{T}\frac{\partial\ln P_*}{\partial\mu}\neq 0 . \tag{3.2}$$

Horizontal momentum equations can then be written as follows:

$$\frac{\partial U}{\partial t}=-\frac{U}{a(1-\mu^2)}\frac{\partial U}{\partial\lambda}-\frac{V}{a}\frac{\partial U}{\partial\lambda}-\dot{\sigma}\frac{\partial U}{\partial\sigma}+fV-\frac{1}{a}\left(\frac{\partial\varphi'}{\partial\lambda}+RT'\frac{\partial\ln P_*}{\partial\lambda}\right) ,$$

$$\frac{\partial V}{\partial t}=-\frac{U}{a(1-\mu^2)}\frac{\partial U}{\partial\lambda}-\frac{V}{a}\frac{\partial V}{\partial\lambda}-\dot{\sigma}\frac{\partial V}{\partial\sigma}+fU-\frac{U^2+V^2}{a(1-\mu^2)}\mu$$
$$-\frac{(1-\mu^2)}{a}\left(\frac{\partial\varphi'}{\partial\lambda}+RT'\frac{\partial\ln P_*}{\partial\lambda}\right)-\frac{(1-\mu^2)}{a}\left(\frac{\partial\overline{\varphi}}{\partial\mu}+R\overline{T}\frac{\partial\ln P_*}{\partial\mu}\right) . \tag{3.3}$$

They are transformed into the vorticity and divergence equations:

$$\frac{\partial\zeta}{\partial t}=-\frac{1}{a(1-\mu^2)}\left(\frac{\partial A}{\partial\lambda}+(1-\mu^2)\frac{\partial B}{\partial\mu}\right)-2\Omega\left(D\mu+\frac{V}{a}\right) , \tag{3.4}$$

$$\frac{\partial D}{\partial t}=-\frac{1}{a(1-\mu^2)}\left(\frac{\partial B}{\partial\lambda}-(1-\mu^2)\frac{\partial A}{\partial\mu}\right)+2\Omega\left(\zeta\mu+\frac{U}{a}\right)-\nabla^2\left(\frac{U^2+V^2}{2(1-\mu^2)}+\varphi'\right) , \tag{3.5}$$

where the quantities A and B are given by

$$A=\zeta U+\dot{\sigma}\frac{\partial V}{\partial\sigma}+\frac{RT'}{a}\frac{\partial\ln P_*}{\partial\mu}-\frac{(1-\mu^2)}{a}\left(\frac{\partial\overline{\varphi}}{\partial\mu}+R\overline{T}\frac{\partial\ln P_*}{\partial\mu}\right) ,$$

$$B=\zeta V-\dot{\sigma}\frac{\partial U}{\partial\sigma}-\frac{RT'}{a}\frac{\partial\ln P_*}{\partial\lambda} ,$$

ζ is the vertical component of the relative vorticity, other symbols are as same as usual.

The modification equations differ from BMRC's in the horizontal pressure gradient terms. In the modification equations, some of the unexact representation of the cancellation between the $\nabla\varphi'$ and $RT'\nabla\ln P_*$ terms which occurs in the standard formulation is eliminated over steep topography. In addition, the new term $\frac{(1-\mu^2)}{a}\left(\frac{\partial\overline{\varphi}}{\partial\mu}+R\overline{T}\frac{\partial\overline{\ln P_*}}{\partial\mu}\right)$ is included in the nonlinear term A in Eqs.3.4 and 3.5.

The thermodynamic equation is as follows:

$$\frac{dT}{dt}-\frac{RT}{C_p}\frac{\omega}{P}=0\ .$$

Substituting (2.5a) into the above equation, we have:

$$\frac{dT'}{dt}-\frac{RT'}{C_p}\frac{\omega}{P}+\frac{d\overline{T}}{dt}-\frac{R\overline{T}}{C_p}\frac{\omega}{P}=0\ .$$

Using the definition of C_0 in (2.1), the thermodynamic equation becomes:

$$\frac{\partial T'}{\partial t}=-\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}(UT')-\frac{1}{a}\frac{\partial}{\partial U}(VT')+DT'-\dot{\sigma}\frac{\partial T'}{\partial\sigma}$$
$$+\left(\frac{C_0^2}{R}+\frac{R}{C_p}T'\right)\frac{\omega}{P}-\frac{V}{a}\left(\frac{\partial\overline{T}}{\partial\mu}-\frac{\partial\overline{T}}{\partial\ln\sigma}\frac{\partial\overline{\ln P_*}}{\partial\mu}\right).\qquad(3.6)$$

The prognostic variable in the thermodynamic equation becomes the temperature deviation from the reference temperature profile, rather than the temperature itself as in the BMRC model. There is an additional term in Eq.(3.6), i.e.,

$$\frac{V}{a}\left(\frac{\partial\overline{T}}{\partial\mu}-\frac{\partial\overline{T}}{\partial\ln\sigma}\frac{\partial\overline{\ln P_*}}{\partial\mu}\right).$$

The moisture mixing prognostic ratio equations is usual:

$$\frac{\partial q}{\partial t}=-\frac{1}{a(1-\mu^2)}\left[\frac{\partial}{\partial\lambda}(Uq)+(1-\mu^2)\frac{\partial}{\partial\mu}(Vq)\right]+qD-\dot{\sigma}\frac{\partial q}{\partial\sigma}\ .\qquad(3.7)$$

We derive the prognostic equation of surface pressure by vertical integrations of the continuity equation, yielding:

$$\frac{\partial(\ln P_*)'}{\partial t}=\overline{D}+\overline{\vec{V}}\cdot\nabla\ln P_*\ ,\qquad(3.8)$$

in which the prognostic variable$(\ln P_*)'$ is the deviation of from $(\overline{\ln P_*})$ which is defined by the reference atmosphere and surface topography.

Since the reference atmosphere is assumed to be hydrostatic (see Eq.(2.2)), the hydrostatic equation becomes:

$$\frac{\partial\varphi'}{\partial\ln\sigma}=-RT'\ .\qquad(3.9)$$

A diagnostic equation for the vertical velocity σ can then be expressed as:

$$\dot{\sigma}(\sigma)=\{(1-\sigma)\overline{D}-\overline{D^\sigma}\}+\{(1-\sigma)\overline{\vec{V}}-\overline{\vec{V}^\sigma})\}\cdot\nabla\ln P_*\ ,\qquad(3.10)$$

where $(\overline{\ \ })^\sigma\int_{\sigma-1}^{\sigma}(\ \ )d\sigma$ and $\overline{(\ \ )}$ denotes the evaluation of this integral with the upper limit $\sigma=0$.

The energy transformation term is written in the following form:

$$\frac{\omega}{P}=\vec{V}\cdot\nabla\ln P_*-\frac{1}{\sigma}\int_0^\sigma(D+\vec{V}\cdot\nabla\ln P_*)d\sigma$$
$$=\frac{1}{\sigma}\{(\overline{D}-\overline{D^\sigma})+[(\overline{V}-\overline{V^\sigma})+\sigma V]\cdot\nabla\ln P_*\}\ . \quad (3.11)$$

Eqs.(3.4)–(3.9) form a complete set of governing equations for an adiabatic spectral model. The main differences between the above equations and the BMRC model's equations are: (1) The calculation of horizontal pressure gradient employs the hydrostatic extraction method, (2) The reference temperature changes with both height and latitude; and (3) Some addtional terms appear in the vorticity, divergence and thermodynamic equations. These new terms may be calculated on σ surface or on pressure surfaces as required. It should be emphasized that the prognostic variables in thermodynamic equation and tendency equation are now deviations from their reference atmosphere values.

IV. SEMI–IMPLICIT TIME INTEGRATION

The prognostic equations (3.4)–(3.9), after further maniqulation, and omitting spectral subscripts and diffusion terms, may be written as:

$$\dot{\zeta}=\gamma \quad (4.1)$$
$$\dot{q}=S \quad (4.2)$$
$$\dot{D}=x+l(l+1)(R'\underline{\underline{B}}\overline{T'^{t}}+RT_0\overline{\ln P'^{t}_*}) \quad (4.3)$$
$$\dot{T}'=Y+\underline{\underline{G}}\overline{D^t} \quad (4.4)$$
$$(\ln\dot{P}_*)'=Z+\pi\overline{D^t} \quad (4.5)$$

The time–integration of the vorticity and moisture tendency equations is performed using centered explicit time differencing, as these equations are not associated with undue time step restrictions.

The other three equations are coupled, and their trmporal finite–differencing may be written in matrix form for nine levels following the normal BMRC formulation:

$$\begin{bmatrix}\dot{D}_1\\ \vdots\\ \dot{D}_9\\ \dot{T}'_1\\ \vdots\\ \dot{T}'_9\\ (\ln \dot{P}_*)\end{bmatrix}=\begin{bmatrix}X_1\\ \vdots\\ X_9\\ Y_1\\ \vdots\\ Y_9\\ Z\end{bmatrix}+\begin{bmatrix}0 & \frac{R}{a^2}l(l+1)\underline{\underline{B}} & \underline{C}\\ \underline{\underline{G}} & 0 & 0\\ \Delta\sigma_1\cdots\Delta\sigma_9 & 0 & 0\end{bmatrix}\begin{bmatrix}\overline{D}^t_1\\ \vdots\\ \overline{D}^t_9\\ \overline{T}'^t_1\\ \vdots\\ \overline{T}'^t_9\\ \overline{\ln P'^t_*}\end{bmatrix} \quad (4.6)$$

$\underline{\underline{G}}$ is an upper triangular matrix, $\underline{\underline{G}}=\frac{C_0^2(\sigma)}{R\sigma}$, whose elements are differen from those of the BMRC model. In addition, $\underline{C}$ in the 19th column is a vector with 9 identical components, namely, $l(l+1)\frac{R}{a^2}T_0$. For simplicity, the above equation can be written as follows:

$$\underline{W} = \underline{\underline{A}}\,\underline{W} + \underline{N} \ . \tag{4.7}$$

The vector $\underline{N}$ with 19 components, represents the nonlinear terms. The vector $\underline{W}$ represents the prognostic variables $(D, T', \ln P'_*)$ on each level. $\underline{\underline{A}}$ is a 19 by 19 coefficient matrix for nine levels.

The equation can be solved by the usual methods:

$$\mathrm{PROJ1} = [\underline{\underline{I}} - \Delta t\underline{\underline{A}}]^{-1}[\underline{\underline{I}} + \Delta t\underline{\underline{A}}] \ , \tag{4.8}$$

$$\mathrm{PROJ2} = 2\Delta t[\underline{\underline{I}} - \Delta t\underline{\underline{A}}]^{-1} \ , \tag{4.9}$$

$$\underline{W}^{+1} = \mathrm{PROJ1} * \underline{W}^{-1} + \mathrm{PROJ2} * \underline{N} \ . \tag{4.10}$$

Here, it is not necessary for A to be calculated for every time step. The semi–implicit method of time integration in the BMRC model is finally written as follows:

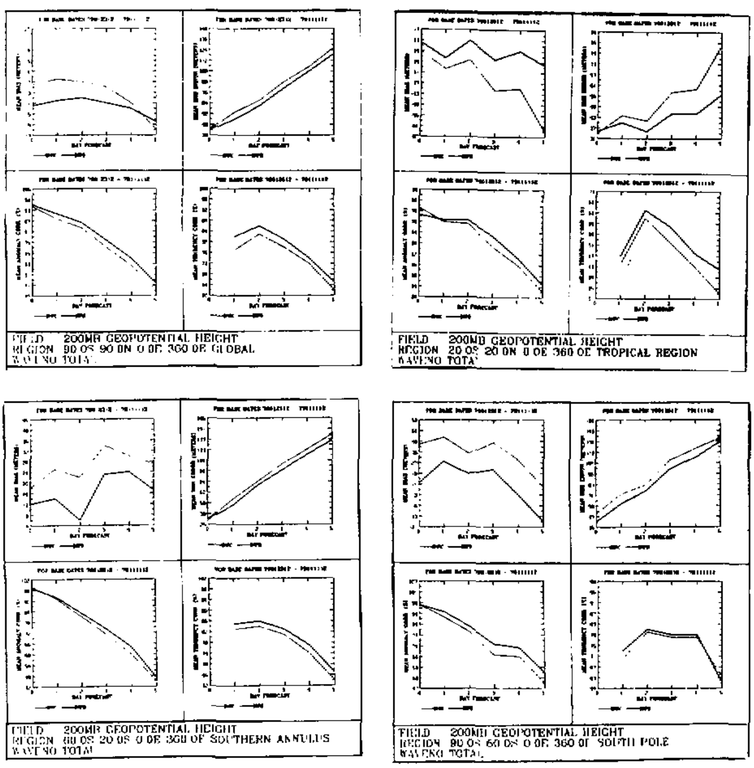

Fig.1. Verification scores of mean four cases prediction for 200 hPa. BMRC denotes BMRC's global spectral model (R21L9). IAPB denotes modified BMRC model (R21L9).

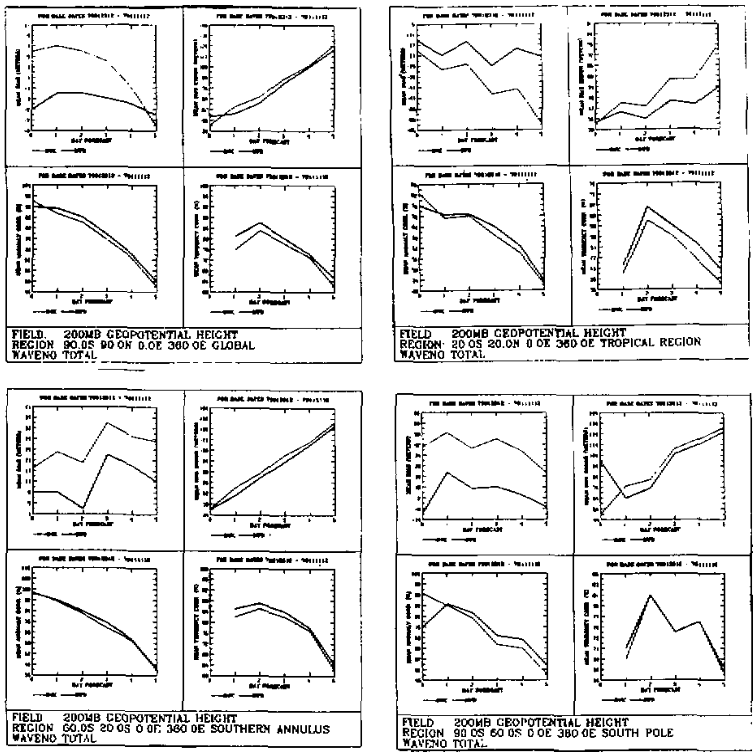

Fig.2. Verfifcation scores of mean four cases predicitions for 200 hPa. BMRC denotes BMRC model (R31L9). IAPB denotes modified BMRC model (R31L9).

$$\underline{W} = \underline{\underline{A}}(t=0)\underline{W} + \underline{\underline{A}}'\underline{W} + \underline{N} \ , \tag{4.11}$$

$$\underline{\underline{A}} = \underline{\underline{A}}(\overline{T}_0(\sigma,t)) - \underline{\underline{A}}(\overline{T}_0(\sigma,0)) . \tag{4.12}$$

$\underline{\underline{A}}'$ receives explicit time treatment and is readily specified each time step.

V. COMPARISONS OF COMPUTATIONAL RESULTS

1. *Intercomparison of the Numerical Prediction of BMRC Model and IAPB Model at R21L9 Resolution.*

For simplicity, we use IAPB to denote the modified BMRC global spectral model which employs the hydrostatic extraction scheme. Its prognostic equations appear in Sections. BMRC represents the BMRC global spectral model. In this subsection, we will compare the results of 5–day prediction with both the BMRC and IAPB models at R21L9 resolution, for the same four cases as before.

Over the globe, (as seen in Fig.1) the mean anomaly correlation of the 200 hPa height field (correlations between observed and forecast deviation from climatology) of the IAPB model is 5 percent higher than that of the BMRC throughout the forecast range. The tendency correlation (correlations between observation an forecast) of the IAPB model is 5 percent better from day 1 to day 5 of the forecast. Over the tropics, the BMRC model has a pronounced cold bias. Over the Antarctic, however, the BMRC model has a warm bias. In both regions, the mean biases of the IAPB model are superior. At 100 hPa (not shown), the beneficial impact of hydrostatic extraction scheme on the BMRC model is also remarkable throughout the forecast range.

2. *Intercomparison of the Numerical Prediction of BMRC and IAPB Model at R31L9 Resolution.*

5–day prediction was performed with both the BMRC and IAPB model R31L9 resolution for the same four cases as before,. We still focus our attention on objective verification over the Southern Hemisphere at high levels. Objective verifications were carried out separately for the global, tropical (20°N–20°S), Southern Hemisphere midlatitude (20°S–60°S) and the Antarctic (60°S–90°S).

Fig.2 presents objective verification scores for the 200 hPa height field averaged over the set of 4 R31L9 cases for different regions. Over the entire globe, mean anomaly correlation of the IAPB model is 3 percent higher than that of the BMRC model during the entire 5–day forecast period. The mean bias of the IAPB model varies from–2 meters to–6 meters. It is quite steady. Comparing other objective verification scores, namely root mean square error and tendency correlation, the beneficial impact of the hydrostatic extraction scheme on BMRC model is apparent. In order to examine the sensitivity of the change in different regions, we will discuss the improvement for the tropics, the Southern Hemisphere mid–latitude and Antarctic respectively.

It is noteworthy that the biggest impact of the hydrostatic extraction scheme on the BMRC model occurs in the tropics (Fig.2b). The mean anomaly correlation of the IAPB model is 2–5 percent higher than that of the BMRC model. The other objective verification scores, namely mean bias, root mean square error and tendency correlation, have substantial improvement over the BMRC model's standard scheme in the Antarctic region at all forecast range. The distinctive feature of the BMRC model is the cold bias in the tropics, along with the warm bias over Antarctic. The performance of the IAPB demonstrated an improvement in these deficiencies.

At 100 hPa (not shown), similar results were obtained the hydrostatic extraction scheme gives the biggest benficial impact throughout the forcast range over the tropics and Antarctica.

3. *Systematic Characteristics of the Model Predictions*

Fig.3 shows the geoptential field at 200 hPa of the ECMWF analysis, and day 5 predictions with the IAPG (T42L9) which is IAP model, and BMRC (R31L9) models and IAPB (R31L9) respectively for the case ofJune 14, 1979. In Fig.3a, there exists a deep Australian trough. Comparing this trough of analysis with these of forecast in detail, the forecasts of the Asurtralian trough with the IAPB and IAPG model are better than the BMRC model's forecast. For the big trough over the Atlantic Ocean the models give a similar performance. Particularly, the equatorward displacement of the 12360 meter contour line in Fig.3c (lower left) shows that the prediction by BMRC model displays a marked cold bias. This consists with the objective verification. A large improvement over BMRC can be seen in Fig.3d. Meanwhile, it can be seen that the prognosis by the IAPB model captures essentially the same improvement as the IAPG model.

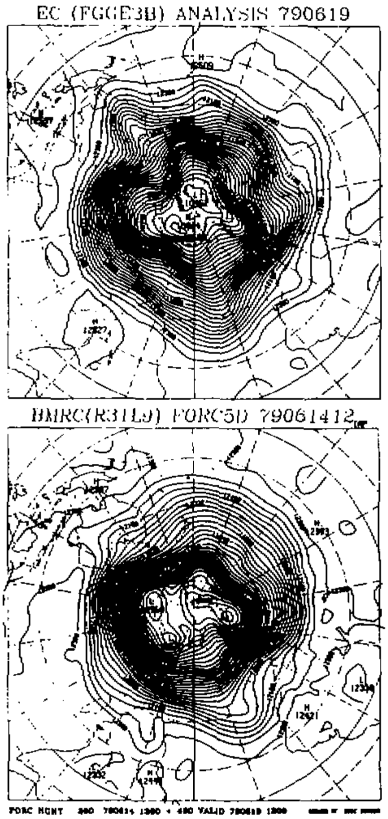

Fig.3. 200 hPa height field. (a) Analysis for June 19, 1979; (b) day 5 forecast with IAPG (T42L9) integration from June 14, 1979; (c) day 5 forecast with BMRC (R31L9) integration from June 14, 1979; (d) day 5 forecast with IAPB (R31L9) integration from June 14, 1979.

VI. CONCLUSIONS AND REMARKS

At both R21L9 and R31L9 resolution, the beneficial impact of the hydrostatic extraction on the BMRC model is apparent, especially in the Southern Hemisphere. The biggest beneficial impact occurs at higher levels, such as 200 hPa and 100 hPa, and over the tropics and Antarctica. IAPB overcomes a systematic excessive cold bias over the tropics and an excessive warm bias over Antarctica to some extent. It strengthens the subtropical high.

There may exist two advantages in the hydrostatic extraction scheme. One is that the calculation of the pressure gradient is modified to eliminate the unexact numerical representation of the cancellation or balance between the $\nabla\varphi$ and $RT\nabla\ln P$, terms which occur in the standard formulation, especially over steep mountain slopes. Using the hydrostatic extraction scheme, the pressure gradient term int he horizontal momentum equations can be calculated accurately. Another advantage is that $\partial\overline{T}/\partial P$ can be calculated accurately as well. The computational error caused by vertical finite differencing is reduced, especially for lower vertical resolution near the tropopause, because C_0, which represents the stratification $\partial\overline{T}/\partial p$

of the reference atmosphere is introduced in the thermodynamical equation. Furthermore, the ability to post process from σ to P coordinate is considered to be more accurate as it only involves interpolation of T' and reconstruction of the geopotential in pressure coordinate by adding the perturbation geopotential to the reference atmosphere. Further assessment of the incorporating of hydrostatic reference atmosphere is being considered within the model incorporating full physics.

This study has been carried out during Dr. Sheng's visit at BMRC partly under the sponsorship of the exchange agreement between the Chinese Academy of Sciences and the Australian Acdemy of Sciences, with partial support by the BMRC. Dr. Sheng wishes to express thanks to both the Academy and IAP, and to thank Dr. M. Manton for giving him an opportunity to visit BMRC. Thanks are due to Mr. Tony Bevan for the assistance in using the FACOM M200 computer.

REFERENCES

Baedde, A. P. M. et. al. (1980), Adiabatic formulation and organization of ECMWF's spectral model, ECMWF technical Report No.15.

Bourke, W. (1974), A multi–level spectral model, formulation and hemispheric integrations, *Mon. Wea. Rev.*, **100**: 683–689.

Bourke, W. et. al. (1977), Global modelling of atmospheric flow by spectral methods, Methsos in Comp. Phys., 17, General circulation methods of the atmosphere, J. Chang, Ed. Acdemic Press., 247–324.

Chen Jiabin, Ji Liren and Wu Wanli (1987), Design and test of an improved scheme for global spectral model with reduction truncation error, *Advances in Atmospheric Sciences* **4**: 156–168.

Simmons, A.J. et al. (1978), Stability of the semi–implicit method of the time integration, *Mon. Wea. Rev.*, **104**: 405–412.

Simmons A. J. and Chen Jiabin (July 1990), The calculation of geopotential and the pressure gradient in the ECMWF atmospheric models influence on the simulation of the polar atmosphere and on temperature analyses, Technical Report No.66.

Zeng Qingcun (1979), *Physical and mathematical fundament for numerical weather forecasting* (in Chinese), Science Press 22–25, 543 pp.

Zhang Daomin, Sheng Hua and Ji Liren (1990), Development and test of hydrostatic extraction scheme in spectral model, *Advances in Atmospheric Sciences* **7**: 142–153.