Effect of Counter–Gradient in the Computation of Turbulent Fluxes of Heat and Moisture in a Regional Model

S. S. Vaidya, V. N. Lykossov① and S. S. Singh
Indian Institute of Tropical Meteorology, Pune–411008, India
Received January 18, 1992; revised May 24, 1992

ABSTRACT

The counter–gradient terms in the computations of turbulent fluxes of heat and moisture have been included in the PBL parameterization of a regional model for monsoon prediction. Results show that inclusion of counter–gradient terms has a marginal impact in the prediction of large scale monsoon circulation and rainfall rates.

I. INTRODUCTION

The eddy heat flux $<W'\theta'>$ in the planetary boundary layer of the atmosphere is generally parameterized by

$$<W'\theta'> = -K_h \frac{\partial<\theta>}{\partial z} \ , \tag{1}$$

where W' and θ' are the departure of vertical velocity and the potential temperature from the area mean of the respective quantities. K_h is an eddy coefficient for heat and $<\theta>$ is the mean temperature over a square grid. the angled bracket ($<\quad>$) represents an area average of the order of magnitude 10 km by 10 km but they could represent a much larger averaging area. In case of neutral or stable atmosphere ($\frac{\partial<\theta>}{\partial z} \geqslant 0$), the heat flux would be zero or downward. The laboratory experiments and the actual observations in the lower troposphere have shown that the upward heat flux does exist even in case of neutral or stable atmosphere ($\frac{\partial<\theta>}{\partial z} \geqslant 0$).

The counter–gradient heat flux was first explained by Priestley and Swinbank (1947) and later by Deardorff (1966). Deardorff (1966) suggested that the main effect could be crudely parameterized by

$$<W'\theta'> = -K_h \left(\frac{\partial<\theta>}{\partial z} - \gamma_\theta\right), \tag{2}$$

where γ_θ is the counter–gradient for the potential temperature gradient having the suggested value,

$$\gamma_\theta = 0.7\times10^{-3} \quad \mathrm{Km}^{-1} \tag{3}$$

in clear air.

①Department of Numerical Mathematics, Soviet Academy of Science 29, Ryleev Street, Moscow.

The γ_θ value given by Eq.(3) is arbitrary and hence the dependence of γ_θ on large scale meteorological parameters can not be taken into account.

Deardorff (1972) suggested the functional relationship between γ_θ and large scale parameters. He showed that eddy heat flux could be expressed as

$$<W'\theta'> = -K_h\left(\frac{\partial<\theta>}{\partial z} - \frac{g}{\theta_0}\frac{<\theta'^2>}{<W'^2>}\right), \tag{4}$$

where

$$K_h = \frac{l<W'^2>}{CE^{\frac{1}{2}}} \tag{5}$$

and

$$\frac{g}{\theta_0}\frac{<\theta'^2>}{<W'^2>} = \gamma_\theta\ , \tag{6}$$

where l is mixing length and is assumed 30 meters, $<W'^2>$ is vertical velocity variance, C is a constant and E is the turbulent energy, θ_0 is average value of potential temperature in a layer and g is acceleration due to gravity. Eq.(6) is the desired expression for counter–gradient. The γ_θ is evaluated by expressing $<\theta'^2>/<W'^2>$ in terms of large scale parameters and turbulent fluxes are evaluated. The method of computation of counter–gradient is given in Section IV.

Although, observational evidence of upward heat and moisture fluxes in the planetary boundary layer in case of neutral or stable atmosphere are not known over Indian monsoon region, in the present study an attempt has been made to compute counter–gradient terms of heat and moisture fluxes following Lykossov (1990). The counter–gradient terms so computed are included in the PBL parameterization of the regional model to investigate the impact of counter–gradient terms on monsoon prediction.

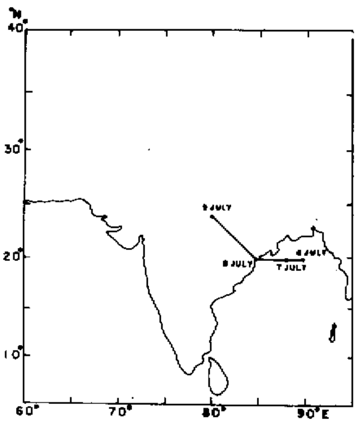

Fig.1. Track of the depression from 12 GMT 6 July to 12 GMT 9 July 1979 at 850 hPa level.

II. DATA

The FGGE IIIb data of 12 GMT, for three depression days, were used as input to the model. These depression days are 6, 7 and 8 July 1979. The feeble low pressure area over northeast Bay on 4th and 5th July became well marked low pressure area and it was centered at 20°N, 90°E on 6th July. It intensified into depression on 7th July with centre at 20°N, 88°E. It further moved westward and on 8th July it crossed the Orissa coast. During next 24 hours the depression apparently weakened into a well marked low and on 10th it has merged with seasonal low pressure area. Fig.1 shows the track of the depression from 6 to 9 July 1979.

Since, the model includes turbulent fluxes of heat and moisture over the sea surface only, the impact of counter-gradient terms is examined when the centre of depression lies over the sea during the period of integration i.e., for the next 2 days from the input date. The impact is also investigated when the depression centre lies for a part of period over the sea and remaining part over the land. Furthermore, study is also done for that phase of depression when the centre remained entirely over the land. The depression days of 6, 7 and 8 July 1979 respectively are the dates which satisfy three phases of the depression tracks needed for the present experiment.

III. MODEL

A six-level primitive equation model in a (x,y,σ,t) coordinate system has been used for the present study (Singh et al., 1990). In vertical, model levels correspond to the pressure values of 950, 850, 700, 500, 300 and 150 hPa. The wind components, mixing ratio, potential temperature and geopotential height are defined at these levels. The vertical σ-velocity $(\dot{\sigma})$ is defined at intermediate levels. In horizontal also, variables are staggered, Arakawa B-type of staggering is used i.e., wind components are defined at centre of the grid and the other variables at the corners of the grid lattice. The horizontal grid of 200 km on Mercator projection is used. The horizontal domain extends from 10°S to 40°N and 45°E to 120°E. Mass, energy, potential temperature and variance of potential temperature conserving finite difference scheme (Arakawa and Mintz, 1974) for space derivatives is used. For time integration leap-frog scheme is used, with time step of 4 minutes. In vertical $\dot{\sigma}=0$ at $\sigma=1$ and $\sigma=0$ and in horizontal tendency modification scheme following Perkey and Kreitzberg (1976) is used for lateral boundary conditions. The model physics includes the large scale precipitation, Kuo (1974) type of cumulus parameterization; dry convective adjustment, the horizontal diffusion, the sensible heat supply and evaporation over the sea. The PBL scheme with counter-gradient terms and the smoothed orography are incorporated in the model. A simple scheme of adjusting the divergence in a vertical column following Sasaki et al. (1979) based on variational adjustment of horizontal wind components has been applied to suppress the external gravity waves in the initial data.

IV. METHOD OF COMPUTATION OF COUNTER-GRADIENT

1. *Counter-Gradient for Heat Flux*

For the computation of counter-gradient for heat flux, $\gamma_\theta \left(= \frac{g}{\theta_0} \frac{<\theta'^2>}{<W'^2>} \right)$, Lykossov (1990) used the balance equations of second moments and closure assumptions for third moments. The stationary approximations are also used and its vertical turbulent transforms are neglected. Following Lykossov (1990) γ_θ in terms of large scale parameters is given by

$$\gamma_\theta = \frac{\frac{g}{\theta_0}\left(\frac{\partial<\theta>}{\partial z}\right)^2}{\frac{g}{\theta_0}\left(\frac{\partial<\theta>}{\partial z}\right)+\mu_0\omega^2}\,, \tag{7}$$

where μ_0 is a constant and ω is the turbulent frequency given by, $\omega=\frac{K}{l^2}$, where K is eddy diffusivity. In the present study K is parameterized in terms of wind shear and thermal stability (see Section V). Once K is known ω can be computed. When atmosphere is neutral $(\frac{\partial<\theta>}{\partial z}=0)$ the γ_θ in the above expression becomes zero instead of attaining a neutral case counter-gradient value. To overcome this situation Lykossov (1990) modified the above expression for γ_θ in the following form

$$\gamma_\theta = \frac{\frac{g}{\theta_0}\left(\frac{\partial<\theta>}{\partial z}\right)^2+\gamma_{\theta N}\mu_0\omega^2}{\frac{g}{\theta_0}\left(\frac{\partial<\theta>}{\partial z}\right)+\mu_0\omega^2}\,, \tag{8}$$

where $\gamma_{\theta N}$ is neutral case counter-gradient for heat flux and its value is equal to 0.7×10^{-3} Km^{-1}.

2. *Counter-Gradient for Moisture Flux* (γ_q)

Lykossov (1990) also suggested a method for computation of counter-gradient for moisture flux. The expression for γ_q corresponding to equation (8) is given by

$$\gamma_q = \frac{\frac{g}{\theta_0}\frac{\partial<q>}{\partial z}\left(2\frac{\partial<\theta>}{\partial z}-\gamma_\theta\right)+2\mu_0\omega^2\gamma_{qN}}{\frac{g}{\theta_0}\frac{\partial<\theta>}{\partial z}+2\mu_0\omega^2}\,, \tag{9}$$

where γ_{qN} is the neutral case counter-gradient value for moisture and given by

$$\gamma_{qN}=\frac{8747q\gamma_{\theta N}}{T^2}\,,$$

here q is mixing ratio and T is temperature.

V. PBL PARAMETERIZATION

The turbulent fluxes of heat, momentum and moisture are included in the lowest two model levels. The fluxes are assumed zero at the third level. The vertical exchange of surface fluxes is computed using bulk aerodynamic formulation. The transfer coefficients for momentum, heat and water vapour are computed following Kondo (1975). The distributions of the surface fluxes in vertical are evaluated by the K-theory (or mixing length theory).

1. *Surface Fluxes*

Surface eddy stress of momentum and the turbulent fluxes of heat and moisture are computed as

$$\vec{\tau}_s = \rho_s C_D |\vec{V}_s| \vec{V}_s \quad ,$$
$$H_s = \rho_s C_p C_H |\vec{V}_s| (T_g - T_s) \quad , \tag{10}$$
$$E_s = \rho_s C_E |\vec{V}_s| (q_g - q_s) \quad ,$$

where ρ_s is the density at the surface, C_D, C_H and C_E are transfer coefficients, T_g is the monthly normal mean sea surface temperature and q_g is saturated mixing ratio for T_g. The subscript 's' is used for quantity related to lowest layer values. C_p is the specific heat at constant pressure and $\vec{V}_s$ is the horizontal velocity at 10 meter height. The fluxes for heat and moisture are computed over sea only.

The vertical exchange of momentum τ, heat H and moisture E are computed as follows:

$$\vec{\tau} = -\frac{\rho^2 g}{\pi} K_v \frac{\partial < \vec{V} >}{\partial \sigma} \quad ,$$
$$H = \frac{C_p \rho^2 g}{\pi} K_\theta \frac{\partial < \theta >}{\partial \sigma} \quad , \tag{11}$$
$$E = \frac{\rho^2 g}{\pi} K_q \frac{\partial < q >}{\partial \sigma} \quad ,$$

where K_v, K_θ, K_q are eddy viscosities for momentum, heat and moisture and are functions of vertical wind shear, thermal stability and mixing length.

$$K_v = K_\theta = K_q = l^2 \psi$$

and

$$\psi = \sqrt{\frac{\rho g}{\pi} \{ \frac{\rho g}{\pi} (\frac{\partial < \vec{V} >}{\partial \sigma})^2 + \frac{g}{\theta_0} (\frac{\partial < \theta >}{\partial \sigma}) \}}$$

where ρ is density and g is acceleration due to gravity and π is defined as $P_s - P_{TOP}$, P_s = surface pressure, P_{TOP} = pressure at the top of the model atmosphere.

It is seen from Eq.(11) that the heat flux is downward or zero (i.e., $H \leqslant 0$) when $\frac{\partial < \theta >}{\partial \sigma}$ is negative or zero (atmosphere is stable or neutral). As discussed in Section I, the upward heat flux also exists in case of stable and neutral atmosphere. This effect is taken into account by incorporating the counter-gradient terms for heat and moisture as follows;

$$H = \frac{C_p \rho^2 g}{\pi} K_\theta (\frac{\partial < \theta >}{\partial \sigma} + \gamma_\theta \frac{\pi}{\rho g}) \quad ,$$
$$E = \frac{\rho^2 g}{\pi} \quad K_q (\frac{\partial < q >}{\partial \sigma} + \gamma_q \frac{\pi}{\rho g}) \quad ,$$

and the expression for ψ is given by

$$\psi = \sqrt{\frac{\rho g}{\pi} \left\{ \frac{\rho g}{\pi} \left(\frac{\partial < \vec{V} >}{\partial \sigma} \right)^2 + \frac{g}{\theta_0} \left(\frac{\partial < \theta >}{\partial \sigma} + \gamma_{\theta N} \frac{\pi}{\rho g} \right) \right\}} \quad ,$$

the γ_θ and γ_q are given in Eqs.(8) and (9) respectively.

VI. MODEL RUNS

Three model runs are made. In the first run, the constant values of counter-gradient are

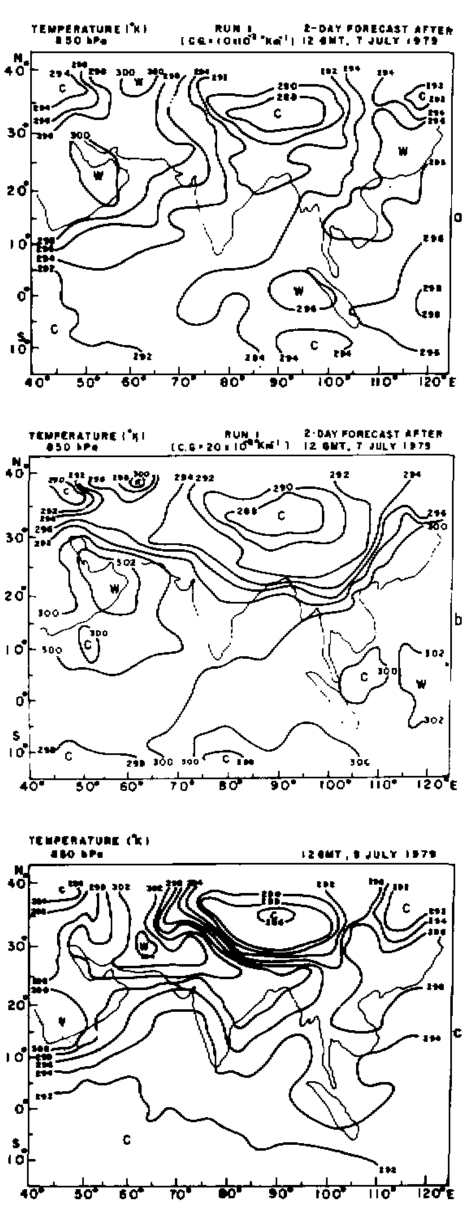

Fig.2. Temperature at 850 hPa a) 2-day forecast from Run 1 (C.G. = 10×10^{-3} Km^{-1}), b) 2-day forecast from Run 1 (C.G. = 20×10^{-3} Km^{-1}) and c) 12 GMT 9 July 1979 analysis.

used for the computations of turbulent fluxes of heat. The values are 10×10^{-3} Km^{-1} and 20×10^{-3} Km^{-1}. This run hereafter will be referred to as Run 1. In the second run, the counter-gradient terms for heat and moisture fluxes are computed using relations (8) and (9). This run will be called Run 2. The third run does not include counter-gradient terms. This will be referred to as Control Run.

In each run model is integrated upto 2 days. The model produced wind, temperature and rainfall in Run 1 and Run 2 are presented and compared with those of Control Run.

1. *Results of Run 1*

For this run, the input of 12 GMT 7 July 1979, with two values of γ_θ namely; 10×10^{-3} Km^{-1} and 20×10^{-3} $Km^{-1}$are used.

The forecast wind fields (not presented) in Run 1 and Control Run are found similar. Fig.2 shows observed and 2-day forecast temperature field at 850 hPa of Run 1. RMS error of temperature is presented in Table 1. It is seen from the figure that the high temperature area associated with the heat low over Pakistan and adjoining northwest India is absent in Run 1. The low temperature area over northeast India is predicted well in Run 1. Over Arabian sea predicted temperatures are higher by 2 K, over Bay of Bengal, they are comparable or lower by 2 K and over Indian ocean predicted temperatures are higher by 2–4 K in case of $\gamma_\theta=10\times10^{-3}$ Km^{-1}. In case of $\gamma_\theta=20\times10^{-3}$ Km^{-1}, the predicted temperature over sea are higher by 6–8 K. Thus, it is found that at lower levels the temperature fields are deteriorated, however at upper levels (not shown) the temperature fields are comparable with those found in the Control Run. Similar inference could be drawn from Table 1 also.

The accumulated rainfall during 12–36 hrs (not presented) in Run 1 is examined. The predicted maximum rainfall rates are 22 mm / d at 18°N, 84°E for $\gamma_\theta = 10 \times 10^{-3}$ Km^{-1}, 4 mm / d at 18°N, 80°E for $\gamma_\theta = 20 \times 10^{-3}$ Km^{-1} whereas observed rainfall rate is 50 mm / d at 20°N, 76°E. In order to understand why rainfall has decreased substantially in case of $\gamma_\theta = 20 \times 10^{-3}$ Km^{-1}, the three conditions necessary for invoking the convection were monitored. It was found that conditional instability always existed in the vicinity of the depression. The moisture convergence was found to increase with increase of γ_θ. However, the third condition that mean relative humidity in the cloud layer rarely exceeded 0.81 when γ_θ was set equal to 20×10^{-3}Km^{-1}, resulting in very poor prediction of rainfall. From the above discussion it can be inferred that constant values of counter–gradient instead of improving the forecast have deteriorated the predicted temperature and rainfall and as such not found suitable for use in PBL parameterization.

Table 1. RMSE for Temperature (K) in Run 1 and Control Run for 7 July 1979

Runs	1–day			2–day		
	Run 1		Control	Run 1		Control
Levels (hPa)	$\gamma_\theta = 10 \times 10^{-3}$ Km^{-1}	$\gamma_\theta = 20 \times 10^{-3}$ Km^{-1}	Run	$\gamma_\theta = 10 \times 10^{-3}$ Km^{-1}	$\gamma_\theta = 20 \times 10^{-3}$ Km^{-1}	Run
850	2.1	5.8	1.7	2.5	6.2	2.9
500	1.6	2.0	1.7	1.9	2.1	2.1
200	1.8	2.0	1.7	1.8	2.0	1.6

2. *Results with Run 2*

For this run the model is integrated using input of 12 GMT 6, 7 and 8 July 1979. Figures. With 7 July as input, are presented and discussed. Results with 6 and 8 July are also discussed wherever necessary.

Table 2. RMSE for *u* and *v* Fields for 2–day Forecast

	U ms^{-1}			*V* ms^{-1}		
Input dates / Levels hPa	850	500	200	850	500	200
6 July 1979	5.9	4.0	5.4	3.7	3.4	5.0
7 July 1979	5.9	4.0	5.0	3.3	3.6	4.8
8 July 1979	5.7	3.3	4.3	3.0	3.0	4.6
Mean	5.6	3.7	4.8	3.3	3.3	4.8

Table 3. *RESE for T(K) Field in 2–day Forecast for Run 2 and Control Run*

	Run 2			Control Run		
Input dates / Levels hPa	850	500	200	850	500	200
6 July 1979	2.1	2.2	1.9	2.4	2.2	1.8
7 July 1979	2.6	2.1	1.7	2.9	2.1	1.6
8 July 1979	2.4	2.0	1.6	2.8	2.0	1.5
Mean	2.4	2.1	1.8	2.7	2.1	1.7

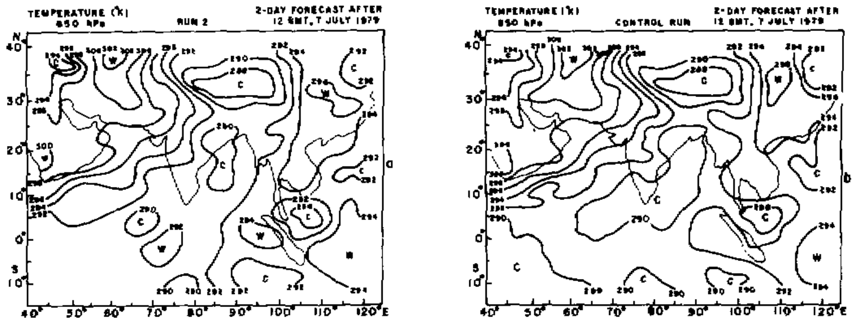

Fig.3. Temperature at 850 hPa a) 2-day forecast from Run 2 and b) 2-day forecast from Control Run.

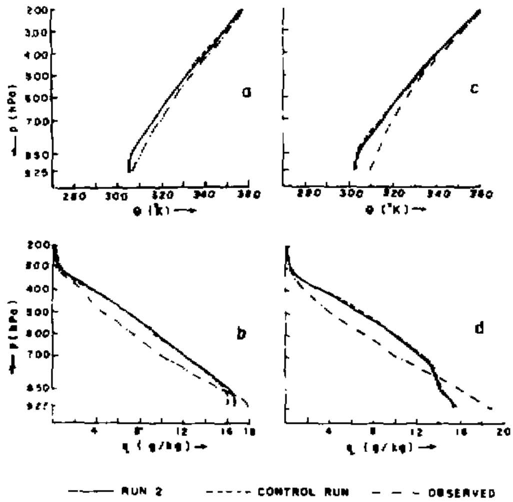

Fig.4. Vertical distribution of potential temperature (θ) and mixing ratio (q) a) 1-day forecast θ, b) 1-day forecast q, c) 2-day forecast θ and d) 2-day forecast q.

The predicted wind field (not presented) in this experiment are found similar to the Control Run, the circulation associated with the depression, cross-equatorial flow and circulation along the equator are predicted satisfactorily. Fig.3 presents, 2-day predicted temperature for Run 2, and Control Run at 850 hPa level. It is seen from the figure that the high temperature area and associated thermal trough are predicted well. The predicted temperature is lower by 1-3K than the observed. Over sea region temperature with Run 2 is slightly better predicted e.g., over west Indian ocean. Fig.4 presents 1-day and 2-day predicted vertical distribution of mean potential temperature (θ) and mixing ratio (q) for nine points in the vicinity of depression. It is found that the vertical distribution for θ and q in Run 2 and Control Run is similar.

1-day accumulated rainfall during 12-36 hrs of integration with Run 2, Control Run and observed are presented in Fig.5. Note that predicted maximum rainfall with Run 2 is located at 18°N, 82°E with rainfall rate 16 mm / d. In Control Run predicted maximum rainfall

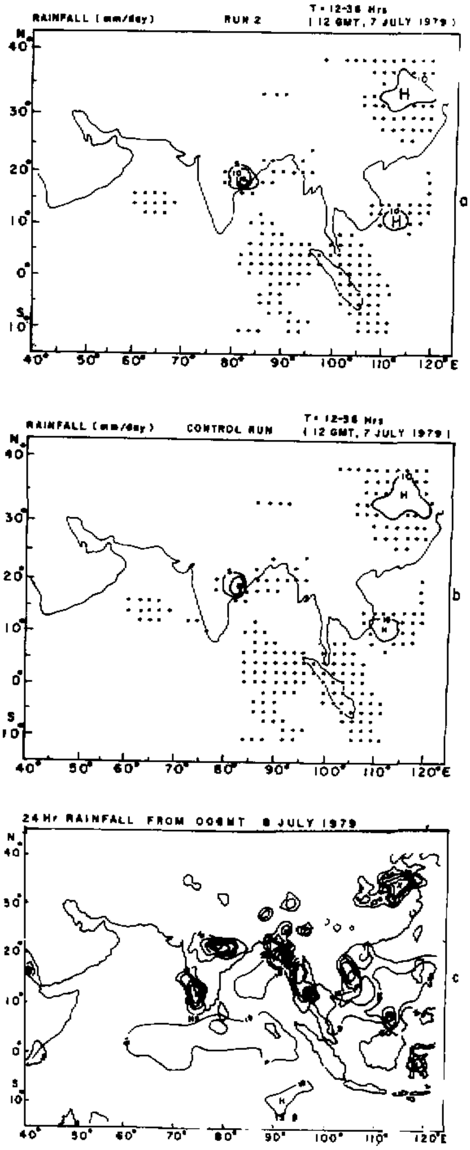

Fig.5. Observed versus predicted rainfall rates. Input 12 GMT 7 July 1979 a) Predicted rainfall (mm); Run 2, b) Predicted rainfall (mm): Control Run and c) Observed rainfall (mm) for 24 hours period ending 00 GMT 9 July 1979 (Source: Krishnamurti et al., 1983).

rate is 13 mm / d at same location, where as observed rainfall is 50 mm / d at 20°N, 76°E. Inclusion of counter–gradient with functional form has marginally improved the predicted rainfall rates. The impact of counter–gradient on the model forecast is further examined through computing RMS errors of wind and temperature fields using data of 6,7 and 8 July 1979 which covers three phases of monsoon depression. RMSE of Run 2 and Control Run for 2–day forecast for u and v fields are found identical and the same is presented in Table 2. Table 3 presents 2–day RMSE for temperature field. Table 3 shows that RMSE for temperature is slightly less in Run 2 than the Control Run. The present experiment has shown that the impact of counter–gradient is marginal even when the depression is centered over the sea.

VII. CONCLUDING REMARKS

A regional model has been used to investigate the impact of counter–gradient terms in the computation of turbulent fluxes of heat and moisture in the planetary boundary layer. Two constant values of counter–gradient for heat flux, and the counter–gradient terms for heat and moisture fluxes computed using functional relation have been used. The salient features of the results that emerged from the study are summarised as follows:

i) Application of counter–gradient terms does not affect wind field.

ii) Constant values of counter–gradient terms deteriorated the lower tropospheric temperature. The areal distribution of rainfall associated with the depression is not predicted properly.

iii) The counter–gradient terms computed from functional relations marginally improved the rainfall, however, other predicted fields are found similar to those produced by Control

Run.

It may be concluded that counter-gradient terms in the PBL parameterization of the model have marginal impact in the prediction of monsoon circulation and rainfall irrespective of whether the centre of depression lies over the sea or over the land.

Authors wish to express their thanks to Shri D. R. Sikka, Director, Indian Institute of Tropical Meteorology, Pune, India for his keen interest in the study. They are also thankful to Dr. S. K. Mishra for useful suggestions. This work was done under Indo-Soviet Integrated Long-term Programme.

REFERENCES

Arakawa, A. and Mintz, Y. (1974), The UCLA Atmospheric General Circulation Model (with the participation of K. Katayama, J. W. Kim, W. Schubert, T. Tokioka, M. Schlesinger, W. Chao, D. Randall and L. Lord) Notes distributed at the Workshop, 25 March-4 April 1974, Department of Meteorology, University of California.

Deardorff, J. W. (1966), The counter-gradient heat flux in the lower atmosphere and the laboratory, *J. Atm. Sci.*, **23**: 503-506.

Deardorff, J. W. (1972), Theoretical expression for the countergradient vertical heat flux, *Journal of Geophysical Research*, **77(30)**: 5900-5904.

Kondo, J. (1975), Air-sea bulk transfer coefficients in diabatic conditions, *Bound. Layer. Meteor.*, **9**:91-112.

Krishnamurti, T.N., Cocke, S. Pasch, R. and Low-Nams. (1983), Precipitation estimates from raingauge and satellite observations; Rept. No. 83-7, Dept. Meteor., FSU, Tallahassee, Florida.

Kuo, H. L. (1974), *Further studies of the parameterization of the influence of cumulus convection on large-scale* flow, *J. Atm. Sci.*, **31**: 1232-1240.

Lykossov, V. N. (1990), K-theory of atmospheric turbulent planetary boundary layer and Boussinesq generalised hypothesis, *Sov. J. Numer. Anal. Math Modelling*, **5(3)**: 221-240.

Perkey, D. J. and Kreitzberg (1976), A time-dependent lateral boundary scheme for limited-area primitive equation model, *Mon. Wea. Rev.*, **104**: 744-755.

Priestley, C. H. B. and Swinbank, W. C. (1947), Vertical transport of heat by turbulence in atmosphere, *Proceedings of the Royal Society of London*, **189**: 543-561.

Sasaki, Y. K., Ray, P. S., Goerss, J. S. and Soliz, P. (1979), Inconsistent finite differencing errors in the variational Adjustment of horizontal wind components, *J. Met. Soc. of Japan*, **59(1)**: 88-92.

Singh, S. S., Vaidya, S. S. and Rajagopal, E. N. (1990), A limited area model for monsoon prediction, *Advances in Atmospheric Sciences*, **7(1)**: 11-126.