Large Scale Perturbations in Extratropical Atmosphere—Part II: On Geostrophic Waves

Y.L. McHall

Department of Earth, Atmospheric and Planetary Sciences, MIT, MA 02139, USA

Received May 21, 1992; revised October 9, 1992

ABSTRACT

We have examined, in Part I, the propagation mechanism and geostrophic property of classical Rossby waves in a non-divergent barotropic atmosphere. As we found that the non-divergent Rossby waves do not propagate in a hydrostatically equilibrium atmosphere, and do not manifest a good geostrophic property, an alternative large scale circulation pattern of geostrophic waves has been proposed (McHall, 1991a). The propagation mechanism and geostrophic property of these waves are examined in the present study.

I. INTRODUCTION

It is discussed in Part I that the classical Rossby waves in a non-divergent atmosphere do not possess a propagation mechanism and good geostrophic balance, so they cannot be applied to represent the large scale waves in the extratropical regions. Although we have some other Rossby wave patterns in which divergence is not ignored, the geostrophic property has not been examined. This property is essential for a theoretical perturbation model used for the study of large scale circulations, which are constrained to be geostrophic balance and hydrostatic equilibrium.

To ensure the geostrophic balance for large scale wave solutions, McHall (1991a) introduced the geostrophic perturbation equations. The wave solutions called the geostrophic waves have been used to explain the occurrences and distributions of the most fundamental large scale circulation patterns, such as the planetary stationary waves, blocks and stratospheric sudden warmings (McHall, 1991a, b, c, 1992a, b). The propagation mechanism and geostrophic property of the waves have not been discussed yet, but will be discussed in the present study.

II. β-EFFECT AND WAVE PROPAGATION

1. *A Simple Review*

The frequency of geostrophic waves discussed by McHall (1991a) is given by

$$\nu = \bar{u}k - \frac{k}{k^2 + k_T^2}\left(\beta + \frac{f^2}{a^2\beta}\right) \tag{1}$$

with

$$k_T^2 = \frac{\delta^2 f^2 \sigma_z m^2}{(1+\delta)^3 \sigma_y^2}, \tag{2}$$

where, δ denotes ageostrophic coefficient; σ_y and σ_z are the baroclinity and static stability parameters respectively. The second term on the right hand side of (1) represents the intrinsic frequency of the waves, while the first term gives the frequency produced by advection of mean fields. The corresponding phase speed reads

$$c=\bar{u}-\frac{1}{k^2+k_T^2}\left(\beta+\frac{f^2}{a^2\beta}\right). \tag{3}$$

Both the frequency and phase speed depend on the β-effect, Earth's curvature and mean temperature structure represented by k_T^2. When the effects of Earth's curvature and mean temperature structure are ignored, the dispersion relation becomes that of Rossby waves. Here, we discuss firstly the contribution of β-effect to wave propagation.

It was recognized widely that the dynamics of Rossby waves is associated with conservation of absolute vertical vorticity on a β-plane. As explained initially by Rossby (1940), the conservation of vorticity makes air parcels turn back in opposite curvatures on the northern and southern sides of wave trajectories respectively. According to this interpretation, the β-effect should increase wave frequency in mean westerly flows. But it is not true as shown by (1). Thus, the mechanism of large scale wave propagation related to β-effect should be re-studied.

Owing to the β-effect, the phase speed has a westward component relative to mean zonal flows. Platzman (1968) explained the westward drift of Rossby waves by using the diagram shown in Fig. 1a:

If we regard the variable Coriolis parameter as a field of East-West 'lines of force', we can say that by cutting these lines the transverse velocities in the wave create an alternating row of induced vorticities and corresponding induced velocities. The phase propagation that results always has a westward component, an anisotropy that stems from the fact that the gradient of Earth's vorticity points northward.

Whereas, Holton wrote (1979):

Rossby wave propagation can be understood in a qualitative fashion by considering a closed chain of fluid parcels initially aligned along a circle of latitude. ··· If the chain of parcels is subject to a sinusoidal meridional displacement then the induced perturbation vorticity will be positive (i.e., cyclonic) for a southward displacement and negative (anticyclonic) for a northward displacement as indicated schematically in Fig. 1b.

The meridional velocity field associated with the perturbation vorticity field advects the chain of fluid parcels southward west of the vorticity maximum and northward west of the vorticity minimum.

It is noted, however, that in the first case, the air parcels moving northward or southward

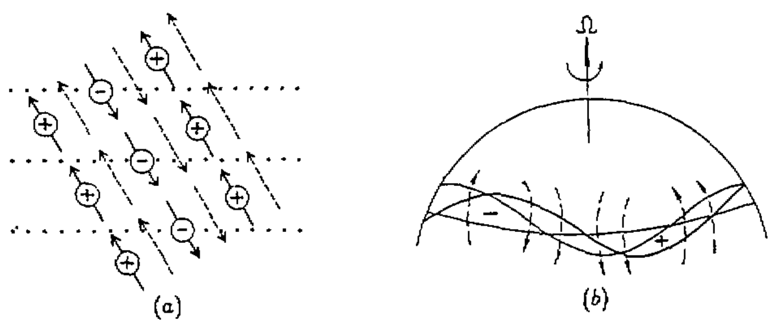

Fig. 1. Sketches for westward drift of Rossby waves relative to mean zonal flow given by Platzman (1968) and Holton (1979) in (a) and (b), respectively.

do not necessarily possess positive or negative vorticity, though they may produce the positive or negative relative vorticity. So distribution of induced vorticity may be different from that shown in Fig. 1a. While, the other explanation suggests that the wave lines are composed of the same air parcels when they propagate. It is true only for stationary waves. In the following, the β-effect on wave generation and propagation will be explained in an alternative way.

2. *Inertial Circles on f-Plane*

In classical physics, a mechanic wave is recognized as propagation of vibrations, other than displacement of medium mass. It propagates in at least two directions forwards and backwards relative to the medium. Its dynamic mechanism is associated mainly with linear processes. While, the geostrophic waves propagate in one direction only. These waves are an example of the kinematic waves discussed by Lighthill and Whitham (1955). The propagation of kinematic waves depends on advection of physical quantities, such as mass, temperature and vorticity, constrained to a special direction. The relation between Rossby wave propagation and vorticity advection was illustrated once by Rossby (1940).

The β-effect on wave propagation may be inspected by comparing the air motions on f- and β-planes. Without a horizontal pressure gradient and friction, the momentum equations are written as

$$\frac{du}{dt} = fv, \tag{4}$$

$$\frac{dv}{dt} = -fu. \tag{5}$$

The velocity components of air parcels on an f-plane can be given by

$$u = A\sin(ft + \gamma), \qquad v = A\cos(ft + \gamma),$$

where the constants A and γ are determined by initial velocities. The trajectories of the parcels are circles of radius A / f:

$$(x - x_0)^2 + (y - y_0)^2 = \frac{A^2}{f^2},$$

in which, x_0 and y_0 indicate an initial position. These circles will be referred to as the f-circles. Here, we do not call them inertial flow or inertial circles as other authors do (e.g., Holmboe et al., 1945; Holton 1979), because they are only a particular example of inertial circles. It is obvious that absolute angular momentum is conserved along the circles on an f-plane. The centers of f-circles do not move in a stationary medium, except when air parcels cross the equator.

If the f-circles are superimposed on a mean zonal flow, they will be advected by the mean flow. The combined movement of each parcel may produce a wave-like motion as shown by the line with circles in Fig. 2. In this figure, we have used

$$f = 2\Omega\sin 45^\circ, \qquad \gamma = 0, \qquad A = 15 \text{ m/s}$$

and $\bar{u} = 8$ m / s in a mean westerly flow, but $\bar{u} = -8$ m / s in a mean easterly flow.

The depicted motion is not considered as a classical mechanical wave which is propagation of vacillation but not the air parcel. The f-circles represent solely the motion of air parcels themselves, and may be considered as an example of the kinematic waves proposed by Lighthill and Whitham (1955). The propagation mechanism depends essentially on advection of mass field governed by a nonlinear process in a Eulerian representation.

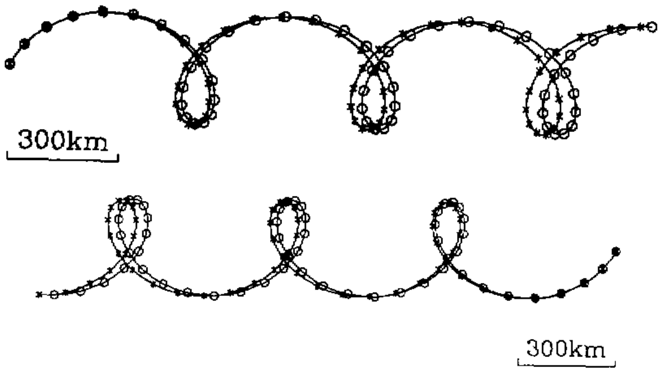

Fig. 2. The inertial circles in the mean westerlies a) and mean easterlies b). The circles and asterisks indicate f- and β-circles respectively. The time interval between two successive circles or asterisks is one hour.

In a mean zonal flow with velocity $\bar{u}$, the f-circles move at velocity $c_d = \bar{u}$. The displacement over a time period $2\pi / \nu$ is

$$L_{circ} = 2\pi \frac{c_d}{\nu} .$$

For convenience, we call L_{circ} the kinematic wavelength. As the mean zonal flow does not affect the frequency of f-circles, we have $\nu = f$, and so this distance is given by

$$L_{circ} = 2\pi \frac{c_d}{f} ,$$

which depends on mean zonal flow. This distance may be viewed as the zonal wavelength of the kinematic wave in the mean field without horizontal pressure gradient. If we define a circle number or a kinematic wavenumber as

$$k_{circ} = \frac{2\pi}{L_{circ}}$$

we see

$$\nu = c_d k_{circ} . \qquad (6)$$

This is similar to the relationship between wave frequency, phase velocity and wavenumber in the classical mechanical waves.

It has been made clear now that the idealized f-circles do not move westward relative to the mean flow over a period. This is not the case for the inertial circles on the Earth's spheres. As a particular example, we will discuss in the following the inertial circles on a β-plane.

3. *Inertial Circles on β-Plane*

On a β-plane, we may set $f = \beta_0 y$. Here, $\beta_0 = f_0 / y_0$, in which f_0 is the Coriolis parameter at $y = y_0$. So, (4) is replaced by

$$\frac{du}{dt} = \beta_0 y \frac{dy}{dt}$$

and follows that

$$u = u_0 + \frac{\beta_0}{2}(y^2 - y_0^2).$$

If y_0 is chosen at the point where $u_0 = 0$, we see that $u > 0$ for $|y| > |y_0|$ but $u < 0$ for $|y| < |y_0|$. Moreover, the magnitudes of u at the positions of $y_0 \pm \Delta y$ are different from each other. It implies that the air trajectories on the β-plane will not move in closed cycles as on an f-plane.

When we use

$$\frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt} = v\frac{dv}{dy},$$

(5) gives

$$v = \pm\sqrt{v_0^2 - \frac{\beta_0^2}{4}(y^4 + y_0^4) + \frac{\beta_0^2}{2}y_0^2 y^2 - \beta_0 u_0 (y^2 - y_0^2)}.$$

It is equivalent to the relationship of kinetic energy conservation:

$$v = \pm\sqrt{u_0^2 + v_0^2 - u^2}.$$

where, u_0 and v_0 measure the initial velocities at position y_0.

In the absence of a horizontal pressure force, an air trajectory of β-circle is represented in Fig. 3. Since the Coriolis force increases poleward, the trajectory curvature is not the same everywhere, but increases poleward too. Therefore, it draws unclosed circles on the β-plane referred to as β-circles, which are another example of inertial circles. Unlike the f-circles, the β-circles shift westward even if there are no mean horizontal flows.

When the β-circle is advected by a mean flow, it may produce a kinematic wave too. An example is depicted by the lines with asterisks in Fig. 2. The provided mean zonal flows are the same as those applied for the f-circles drawn previously. This motion is not a real mechanic wave also, but a kinematic wave on the β-plane. A report of the observed inertial circles in a deep current of ocean can be found in the study of Nan' nitti et al., (1964). The inertial circles in the oceans account for a significant amount of energy as shown by Warsh et al. (1971).

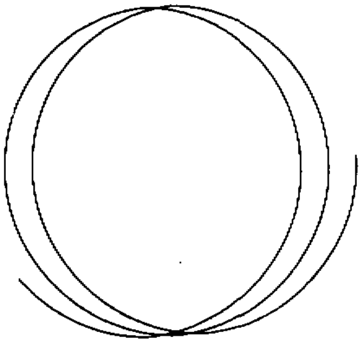

Fig. 3. The β-circles in a stationary medium.

If the β-circles move westward at velocity c_{dr} in stationary media, they will move at the velocity

$$c_d = \bar{u} + c_{dr} \tag{7}$$

in a mean zonal flow of velocity $\bar{u}$. So the drift velocity in a mean westerly flow decreases, but increases in a mean easterly flow with respect to the ground. This is shown clearly by comparing the f- and β-circles in Fig. 2. Consequently, from (6) and (7), the β-effect reduces the frequency of inertial circles in mean westerlies, and increases the frequency in mean easterlies. This is shown also by Fig. 2.

When there is no horizontal pressure gradient in the atmosphere, the β-circles in Fig. 2 have very short kinematic wavelengths. If the westward drift of the β-circles is estimated by

$$c_{dr} = -\beta_0 \frac{L_{circ}^2}{4\pi^2} \tag{8}$$

approximately, we find $c_{dr} \approx -8.86$ km / day for $L_{circ} = 500$ km at latitude 45°. This speed is very low as shown in Fig. 2. However, it increases greatly with wavelength.

III. PROPAGATION MECHANISM

The inertial circles discussed previously are associated with the motion of individual air parcels in an undisturbed medium. If all the air parcels in the medium move in the β-circles, a kinematic wave pattern in the whole medium may be produced. In this case, the pressure field is disturbed, and a vertical motion connected with horizontal convergence may be induced by the waves as a secondary circulation. These kinematic wave mechanisms are involved in Rossby waves and geostrophic waves. As these waves are associated with inertial motions in the atmosphere, their frequency depends substantially on meridional variation of horizontal inertial force. However, they are not the pure inertial waves since the wave mechanism depends on pressure force also. This kinematic wave dynamics is related to advection of physical quantities, such as mass and vorticity in a divergent atmosphere, and so is governed by nonlinear processes. The wave-like properties may be described approximately by linearized equations when produced perturbation amplitudes are relatively small. However, the wave solutions may still reflect the nonlinear property of the kinematic waves, because the propagation velocity has one direction only.

As the inertial circles drift westward on the spheres, these kinematic waves propagate westward relative to the mean zonal flows. It is difficult in mathematics to derive the analytical solution of the westward drift velocity for a β-circle in an undisturbed medium. While, for the waves produced by inertial motions in a zonal sysmetric pressure gradient field, the westward drift can be obtained readily. In this case, the left hand sides of (4) and (5) can be expanded into partial derivitives, and the derived westward velocity on a β-plane is the same as that of the classical Rossby waves and is also the c_{dr} given by (8). The pressure perturbations in the Rossby waves do not make any effect on wave frequency.

Apart from the β-effect discussed previously, the propagation mechanism of geostrophic waves is also related to the effect of the Earth's sphericity as shown by (3). This effect is greater than β-effect at high latitudes, so that it cannot be ignored there. When wave frequency is affected by β-effect alone, westward phase speed increases at higher latitudes. This however is not consistent with observations. Thus, the effect of the Earth's sphericity may be of significance for propagation of large scale waves.

The term including the Earth' s sphericity reflects the effect of horizontal divergence associated with the Earth' s curvature. If horizontal divergence of the geostrophic waves is presented, we may find that the terms involved in the divergence appear in the dispersion relation also but not in the classical Rossby waves. One of these terms depends on the Earth' s sphericity. As shown by the continuity equation on the spheres (referring to Part I), when air parcels move equatorward, they produce horizontal divergence and then reduce vertical vorticity, so that the air parcels will return in a clockwise sense. The reverse happens when air parcels move poleward. Therefore, the horizontal divergence produced by Earth' s curvature also reduces the eastward phase speed in a mean westerly flow. As the horizontal divergence increases poleward, the westward phase speed increases more rapidly at higher latitudes, so that the waves propagate eastward more slowly rather than more quickly in the mean westerly flow.

Another term which affects the wave mechanics is associated with the mean temperature structure represented by k_T^2. When an air parcel has a vertical component of motion in a hydrostatically stable atmosphere, it will oscillate around an equilibrium height. From (1) and (3), this effect is combined together with the contributions of the β-effect and Earth' s curvature. Because without the β-effect and Earth' s curvature, we may prove that air parcels in the waves will move on potential temperature surfaces, and therefore the motions will not be affected by vertical oscillations.

VI. STATIONARY WAVELENGTH

In a mean westerly flow, the zonal phase speed will be equal to or less than zero, when zonal wavelength of geostrophic waves equals or exceeds the stationary wavelength, evaluated by

$$L_s = \frac{2\pi}{\sqrt{\frac{\beta}{\bar{u}}\left(1 + \frac{f^2}{a^2\beta^2}\right) - k_T^2}}$$

derived from (3). Unlike the stationary wavelength of the classical Rossby waves given by (Rossby, 1939)

$$L_s = 2\pi\sqrt{\frac{\bar{u}}{\beta_0}},$$

it depends on the Earth' s sphericity and atmospheric thermal structure as well as the mean zonal flow. The stationary wavelength of the classical Rossby waves tends to infinitely large near the poles. This does not agree with observations. The large scale stationary circulation pattern in the polar regions of winter stratosphere is generally characterized by a wave 1 superimposed on the mean flows. This circulation has been simulated successfully by numerical models (Matsuno, 1970; Huang and Gambo, 1982; Lin, 1982). In the geostrophic waves, however, the stationary wavelength decreases poleward. It decreases also when baroclinity is increased.

The real number of the stationary wavelength depends on the condition

$$\sigma_y^2 \geqslant \frac{\delta^2 f^2 \sigma_z m^2 \bar{u}}{(1+\delta)^3 \left(\beta + \frac{f^2}{a^2\beta}\right)} \quad (\bar{u} > 0). \tag{9}$$

Here, we have used (2). As on isobaric surfaces

$$\sigma_y = \frac{R}{p}\frac{\partial \bar{T}}{\partial y},$$

this condition is replaced by

$$\left|\frac{\partial \bar{T}}{\partial y}\right| \geqslant \frac{\delta p}{(1+\delta)R}\sqrt{\frac{f^2 m^2 \sigma_z \bar{u}}{(1+\delta)(\beta + \frac{f^2}{a^2\beta})}}. \tag{10}$$

If baroclinity is lower than a certain limit in a mean easterly flow, stationary waves cannot exist. This is a possible reason for the absence of stationary waves in the stratosphere where temperature gradient is generally smaller than below.

Moreover, the stationary wavelength of geostrophic waves depends on baroclinity. The stationary waves in condition of higher baroclinity possess shorter wavelengths. Thus, the progressive waves may become stationary when they propagate into a highly baroclinic area, though the wavelengths are unchanged.

V. GEOSTROPHIC BALANCE

The geostrophic balance is a basic property of large scale circulations in the atmosphere. Any perturbation model proposed for the study of large scale circulations must possess this property. Since the planetary Rossby waves do not exhibit this property, especially at lower middle latitudes as discussed in Part I, the geostrophic waves have been proposed for study of the large scale circulations. The geostrophic property of these waves is examined in this section.

We consider, firstly, the Rossby number of the geostrophic waves. The magnitudes of Rossby number concerned with zonal and meridional components of momentum may be estimated from

$$R_{ox} \sim \frac{1}{fv}\frac{du}{dt}, \qquad R_{oy} \sim \frac{1}{fu}\frac{dv}{dt}$$

respectively. Since

$$\frac{du}{dt} \sim \bar{u}\frac{\partial u}{\partial x}, \qquad \frac{dv}{dt} \sim \bar{u}\frac{\partial v}{\partial x},$$

we have

$$R_{ox} \sim \frac{\bar{u}}{fv'_g}\frac{\partial u}{\partial x} \sim \frac{\bar{u}}{f}m, \qquad R_{oy} \sim \frac{1}{f}\frac{\partial v'_g}{\partial x} \sim \frac{k^2}{f^2}\varphi'.$$

Applying the typical scales

$$\bar{u} \sim 10^1\,\mathrm{m/s}, \quad f \sim 10^{-4}\mathrm{s}^{-1}, \quad \varphi' \sim 10^2\,\mathrm{m}^2/\mathrm{s}^2,$$
$$k \sim 10^{-6}\mathrm{m}^{-1}, \quad m \sim 10^{-7} - 10^{-6}\mathrm{m}^{-1},$$

we see R_{ox}, $R_{oy} \sim 10^{-2}$. It has been noted in Part I that a small Rossby number does *not* always suggest a good geostrophic balance. The geostrophic property of geostrophic waves is then examined further by comparing the streamlines and geopotentials in the waves.

The geopotentials of geostrophic waves are given by (McHall, 1991a)

$$\varphi' = \Phi e^{i(vt - kx + my + lp)}.$$

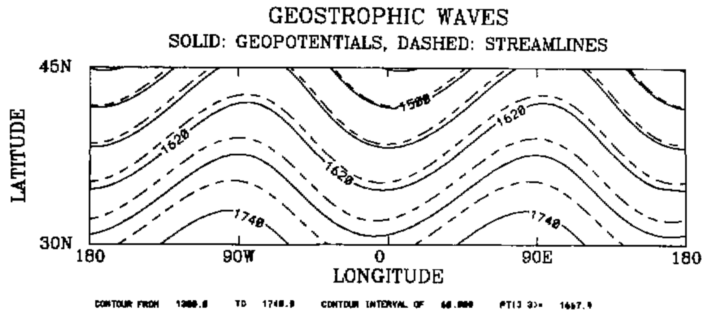

Fig. 4. Geopotentials (solid) and streamlines (dashed) of planetary geostrophic waves.

Also, the streamlines may be evaluated numerically from the geostrophic perturbation velocities

$$v'_g = \frac{k}{f}\Phi e^{i(vt - kx + my + lp)}, \qquad u' = \frac{m}{f}\Phi e^{i(vt - kx + my + lp)}.$$

Here, the geopotential perturbation amplitude is calculated from

$$\frac{1}{\Phi}\frac{d\Phi}{dy} = \frac{\beta + \dfrac{f^2}{a^2\beta}}{f(k^2 + k_T^2)}k^2.$$

Since $dy = ad\varphi$, it is replaced by

$$\frac{1}{\Phi}\frac{d\Phi}{d\varphi} = \frac{1}{(1 + \zeta\sin^2\varphi)\cos\varphi\sin\varphi}, \qquad \zeta = 4\delta^2\frac{\Omega^2 m^2 \sigma_z}{\sigma_y^2 k^2}.$$

If we set $\Lambda = \sin\varphi$, it becomes

$$\frac{1}{\Phi}\frac{d\Phi}{d\Lambda} = \frac{1}{\Lambda} + \frac{\Lambda}{(1+\zeta)(1-\Lambda^2)} - \frac{\zeta^2\Lambda}{(1+\zeta)(1+\zeta\Lambda^2)},$$

and follows that

$$\Phi = \Phi_0 \frac{\sin\varphi}{\left[\cos^2\varphi(1 + \zeta\sin^2\varphi)^{\zeta}\right]^{\frac{1}{2(1+\zeta)}}}, \tag{11}$$

in which, Φ_0 is a positive constant determined by the observed amplitude at a given latitude. At the equator where no Coriolis forces exist, geostrophic balance cannot hold and so the geostrophic waves do not occur.

Using the typical values

$$\Phi_0 = 700\ \mathrm{m}^2/\mathrm{s}^2, \quad \bar{u} = 15\ \mathrm{m/s}, \quad m = k, \quad \zeta = 0.005,$$

the calculated geopotential perturbations and streamlines of the planetary geostrophic waves with wavenumbers 2 are depicted in Fig. 4. It is shown that the streamlines are approximately parallel to geopotential contours. Although the wave amplitude, mean zonal flow and wavenumbers are similar to those of the Rossby waves depicted in Part I, geostrophic balance in geostrophic waves is significantly better than in Rossby waves.

VI. SUMMARIES

The geostrophic waves are an example of kinematic waves discussed by Lighthill and Whitham (1955), which propagate in one direction only. Unlike the classical mechanic waves, their propagation mechanism depends on horizontal advection of physical quantities towards a special direction. The advection is generally related to nonlinear processes. When the produced perturbation amplitudes are relatively small, the wave–like periodic motions may be described approximately by linear equations.

If the effects of the Earth's sphericity and atmospheric thermal structure are ignored, the geostrophic waves will become Rossby waves on a β–plane. Since the propagation of inertial motions is always to the west in a stationary medium, the geostrophic waves propagate westward relative to the mean zonal flow. Due to the Earth's sphericity, the phase speed decreases poleward in the mean westerlies at high latitudes. The propagation is slowed down by increasing baroclinity. This is a well known fact observed in the initial stage of baroclinic cyclogenesis. Since the vertical oscillation in geostrophic waves is affected by thermal stratification of the atmosphere, the wave frequency and phase speed depend on static stability of the atmosphere. This effect produced by thermal structure will not present in a flat atmosphere, bacause air parcels in geostrophic waves move on potential temperature surfaces in this atmosphere.

The geostrophic wave solutions are derived from geostrophic perturbation equations. Comparing the geostrophic perturbation equations with the perturbation equations used for Rossby waves, we find that the improvement in geostrophic property of geostrophic waves is due to application of the geostrophic perturbation equation

$$fv' = \frac{\partial \varphi'}{\partial y}.$$

It is important to note that although this equation is obtained by using the small–oscillation approximation (McHall, 1991a), the equation does not necessarily depend on this approximation, and may be derived more generally by scale analysis. Without using the small–oscillation approximation, we have $u' \sim v'$. Applying the previous typical scales together with

$$v' \sim 10^0 \ \text{m/s}, \quad t \sim 10^5 \ \text{s}, \quad \Delta x \sim 10^6 \ \text{m},$$

we find that the largest terms of zonal acceleration is one order smaller than the two terms in the previous equation. Usually, the zonal perturbation velocity is comparable with the meridional perturbation velocity in jet areas, so that the small–oscillation approximation is not available there. However, the geostrophic perturbation equation is still applicable for the perturbations in the areas. Thus, as proved by observations, the perturbations in jet areas are also in a good geostrophic balance.

The geostrophic waves may become stationary or propagate eastward in a mean westerly, if the wavelength equals or exceeds the stationary wavelength. Unlike the classical Rossby waves, the existence of stationary waves depends on baroclinity also. The stationary wavelength decreases at high latitudes and in the regions of high baroclinity. The stationary geostrophic waves cannot exist in a mean easterly or a weak baroclinic flow, but may exist in strong westerly flows.

In deriving the geostrophic wave solutions we did not use boundary conditions. These free geostrophic waves are also trapped by the mean easterlies. But unlike the forced Rossby waves described by Charney and Drazin (1961), the geostrophic waves are not trapped by a strong westerly. The existence of planetary scale waves in a strong westerly may be proved by observational analyses. At the earlier time, the rocket grenade data (Nordberg et al., 1965) and meteor wind data (Newell and Dickinson, 1967) indicate the presence of planetary eddies at least up to the mesopause. Since the time when satellite data are available, several planetary wave patterns in the middle atmosphere have been recognized, for example, the 5-day waves (Rodgers, 1976), 4-day waves (Venne and Stanford, 1979), 2-day waves (Rodgers and Prata, 1981) and 16-day waves (Hiroota and Hirota, 1985). It is not sure whether the present geostrophic wave model may be applied to study these waves. However, an attempt to explain the 4-day waves in the upper stratosphere has been made already (McHall, 1992b).

REFERENCES

Charney, J.G. and P.G. Drazin (1961), Propagation of planetary scale disturbances from the lower into the upper atmosphere, *J. Geophys. Res.*, **66:** 83-109.

Hiroota, T. and I. Hirota (1985), Normal mode Rossby waves observed in the upper stratosphere. Part II: Second antisymmetric and symmetric modes of zonal wavenumbers 1 and 2, *J. Atmos. Sci.*, **42:** 536-548.

Holmboe, J., G. E. Forsythe and W. Gustin (1945), *Dynamic meteorlogy*, John Wiley and Sons, Inc. & Chapman and Hall, Limited. 378 pp.

Holton, J. R. (1979), *An Introduction to Dynamic Meteorology* (Second Edition), Academic Press, 120-146.

Huang, R-H and K. Gambo (1983), The response of a hemispheric multi-level model atmosphere to forcing by topography and stationary heating sources in summer, *J. Meteor. Soc. Japan*, **61:** 495-509.

Lighthill, M.J. and G. B. Whitham (1955), On kinematic waves, I. Flood movement in long rivers, *Proc. R. Soc.*, **229:** 281-316.

Lin, B.-D. (1982), The behavior of winter stationary planetary waves forced by topography and diabatic heating, *J. Atm. Sci.*, **39:** 1206-1226.

Matsuno, T. (1970), Vertical propagation of stationary planetary waves in the winter Northern Hemisphere, *J. Atmos. Sci.*, **27:** 871-883.

McHall, Y.L. (1991a), Planetary stationary waves in the atmosphere—Part I: Orographical stationary waves, *Adv. Atmos. Sci.*, **8:** 211-224.

McHall, Y.L. (1991b), Planetary stationary waves in the atmosphere—Part II: Thermal stationary waves, *Adv. Atmos. Sci.*, **8:** 225-236.

McHall, Y.L. (1991c), Blocking distributions in the atmosphere, *Adv. Atmos. Sci.*, **8:** 327-338.

McHall, Y.L. (1992a), Nonlinear planetary wave instability and blocking, *Adv. Atmos. Sci.*, **9:** 173-190.

McHall, Y.L. (1992b), Wintertime stratospheric anomalies—Part I: Warm pools, *Adv. Atmos. Sci.*, **9:** 299-310.

McHall, Y.L. (1992c), Wintertime stratospheric anomalies— Part II: Sudden warmings, *Adv. Atmos. Sci.*, **9:** 311-322.

Nan'nitti, T., H. Akamatsu, and T. Nakai (1964), A further observation of a deep current in the east-north-east see of Torishima, *Oceanog. Mag.*, **16:** 11-110.

Newell, R.E. and R. Dickinson (1967), On the applications of a proposed global system for measuring meteor wind, *Pure Appl. Geophys*, **68:** 667-672.

Nordberg, W., L. Katchen, J. Theon and W. S. Smith (1965), Rocket observations of the structure of the mesosphere, *J. Atmos. Sci.*, **22:** 611-622.

Platzman, G.W. (1968), The Rossby wave, *Quart. J. R. Met. Soc.*, **94:** 225-248.

Rodgers, C.D. (1976), Evidence for the five-day wave in the upper stratosphere, *J. Atmos. Sci.*, **33:** 710-711.

Rodgers, C.D. and A.J. Prata (1981), Evidence for a traveling two-day wave in the middle atmosphere, *J. Geophys. Res.*, **86**: 9661–9664.

Rossby, C.G. (1939), Relation between variations in the intensity of the zonal circulation of the atmosphere and the displacements of the semi-permanent centers of action, *J. Marine Res.*, **2**: 38-55.

Rossby, C.G. (1940), Planetary flow patterns in the atmosphere, *Quart. J.R. Met. Soc.*, **66**: Supplement, 68–87.

Venne, D.E. and J.L. Stanford (1979), Observation of a 4-day temperature wave in I the polar winter stratosphere, *J. Atmos. Sci.*, **36**: 2016–2019.

Warsh, K.L., K.L. Echternacht and M. Carstang (1971), Structure of near-surface currents east of Barbados, *J. Phys. Oceanog.*, **1**: 123-129.