Topography and the Non–linear Rossby Wave in the Zonal Shear Basic Flow①

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ABSTRACT

Under semi–geostropical approximation, by means of phase angle function the non–linear ordinary differential equation is derived involving topography and zonal shear basic flow. Conditions for the existence of limited amplitude periodical and isolated wave solutions are directly obtained based on the qualitative theory of the ordinary differentical equation. Analysis is thus made of the influence of topography and zonal shear flow on the existence of wave solution. Finally, explicit wave solutions are determined by function approaching with the result that topography and zonal shear flow affect not only the existence but also the form of waves, indicating the non–linear features of waves and the effect of topography and shear basic flow on undulation.

I. INTRODUCTION

Since the pioneering work of Charney and Eliassen (1949) and Bolin (1950), great advances have taken place in the research into the effect of topography on undulation. Wu Rongsheng (1964) has studied its influence on the stability of barotropic and baroclinic fluctuations. Kasahara (1966) has pointed out that topography produces stationary wave fluctuations in the westerlies and only attenuative fluctuation in the easterlies. Lu Keli (1986,1987) discussed the effect of topography on barotropic Rossby wave and isolated wave respectively by the WKB method. Liu Shi Kuo and Tan (1988) has studied the influence of topography on the stability and solution of non–linear Rossby wave by means of the semi–geostropical model. Literature shows that under semi–geostropical approximation, the ordinary differential equation describing non–linear Rossby wave can be approximately reduced to Kdv equation by Taylor approximation.

In this study a non–linear ordinary differential equation involving zonal shear basic flow and topographical forcing is derived by means of phase angle function through semi–geostropical approximation in the non–linear shallow water wave model with topographical forcing. Instead of using Taulor approximation, research is made directly on the topological stucture of equation, thus the question of the existence of non–linear wave becoming the question whether there exists a periodical solution to the non–linear ordinary differential equation. Then, based on the geometric qualitative theory, the paper states the conditions for the closed phase orbit on a phase plane corresponding to the existence of periodical wave with limited or small amplitude and isolated wave, that is, non–linear Rossby wave. Thus the effect of topography and zonal shear basic flow on wave solution and the conditions for the initial phase disturbance are analyzed, thus obtaining explicit wave solution by function approaching. On this basis the influence of shear flow and topography on the

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nature of wave solution is discussed in much detail.

II. BASIC EQUATIONS

For β plane, homogeneous incompressible fluid is taken as the model atmosphere with h as the height of the free surface and h_B as that of the lower boundary. For $h_B \ll h$, the equations governing the fluid motion are

$$\begin{aligned} &(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})\ u-fv=-g\frac{\partial h}{\partial x},\\ &(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})\ v+fu=-g\frac{\partial h}{\partial y},\\ &(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})\ (h-h_B)+(h-h_B)\ (\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})\ =0. \end{aligned} \tag{1}$$

Let

$$u=\bar{u}(y)+u',v=v',h=\bar{h}(y)+h', \tag{2}$$

Where

$$\bar{u}=-\frac{g}{f_o}\frac{\partial\bar{h}}{\partial y}.$$

Substituting (2) into (1) and leaving out the superscript, we have

$$\begin{aligned} &(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})\ u-(f-\frac{\partial\bar{u}}{\partial y})v=-g\frac{\partial h}{\partial x},\\ &(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})\ v+fu=-g\frac{\partial h}{\partial y},\\ &(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})\ h+\frac{\partial\bar{h}}{\partial y}v\\ &+(\bar{h}+h)\ (\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})\ -\ (u\frac{\partial h_B}{\partial x}+u\frac{\partial h_B}{\partial y})\ =0. \end{aligned} \tag{3}$$

From the motion equation in (3), we get the vorticity equation

$$\begin{aligned} &(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})\ (\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y})\ +[(f_0-\frac{\partial\bar{u}}{\partial y})+\ (\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y})\ ]\ (\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})\\ &+(\beta-\frac{\partial^2\bar{u}}{\partial y^2})v=0. \end{aligned} \tag{4}$$

Under semi-geostropical approximation, the continuity equations in (4) and (3) can be written as

$$\begin{aligned} &(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})\ \xi_g+(\bar{\xi}_a+\xi_g)\ (\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})\ +\frac{\partial\bar{\xi}_a}{\partial y}v_g=0,\\ &(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y})\ h+(\bar{h}+h)\ (\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})\ -\frac{\partial h_B}{\partial y}v_g-\frac{\partial h_B}{\partial x}v_g=0, \end{aligned} \tag{5}$$

Where $\xi_g=\frac{\partial v_g}{\partial x}-\frac{\partial u_g}{\partial y}$ is the geostropical vorticity, $\bar{\xi}_a=f_o-\frac{\partial\bar{u}}{\partial y}$ the absolute vorticity and $-\frac{\partial\bar{\xi}_a}{\partial y}=-(\beta-\frac{\partial^2\bar{u}}{\partial y^2})$ the absolute vorticity gradient: $u_g=-\frac{g}{f_o}\frac{\partial h}{\partial y}$ and $v_g=\frac{g}{f_o}\frac{\partial h}{\partial x}$ in

$\frac{\partial \bar{\xi}_a}{\partial y} v$ and $\frac{\partial h_B}{\partial y} v$, $\frac{\partial h_B}{\partial x} u$ are replaced by v_g, u_g (Tan and Liu, 1988).

Suppose Eq. (5) has the following solution

$$u = U(\theta), v = V(\theta), gh = \varphi(\theta), \theta = kx + ly - rt, \tag{6}$$

Where θ is the phase angle, k and l are the wave numbers in the direction of x and y respectively, and r is the round frequency. Then substitute (6) into (5) and let topographical slope be constant. In view of the small varying intervaly y, it is necessary for the basic parameters to be slow variables. With the integration constant as 0, integration is done to θ , which gives

$$(-r + \bar{u}k)k_h^2 \varphi'' + f_o \bar{\zeta}_a (kU + lV) + (kU + lV)k_h^2 \varphi'' + k\frac{\partial \bar{\xi}_a}{\partial y} \varphi = 0,$$

$$-r\varphi + c_o^2(kU + lV) + \varphi(kU + lV) - \frac{kg}{f_o}\frac{\partial h_B}{\partial y}\varphi + \frac{lg}{f_o}\frac{\partial h_B}{\partial x}\varphi = 0. \tag{7}$$

From the above formula, we have the equation which satisfies the variable

$$\varphi'' = \frac{\frac{\partial \bar{\xi}_a}{\partial y}\varphi^2 + [f_o \bar{\xi}_a \ (c_x + \frac{g}{f_o}\frac{\partial h_B}{\partial y} - \frac{l}{k}\frac{g}{f_o}\frac{\partial h_B}{\partial x}) \ + c_o^2 \frac{\partial \bar{\xi}_a}{\partial y}]\varphi}{-k_h^2 \ (\bar{u} + \frac{g}{f_o}\frac{\partial h_B}{\partial y} - \frac{l}{k}\frac{g}{f_o}\frac{\partial h_B}{\partial x}) \ \varphi + c_o^2 k_n^2 (c_x - \bar{u})}, \tag{8}$$

Where $c_o = \sqrt{gh}$ is the shallow water phase speed, $k_h^2 = k^2 + l^2, c_x = r / k$ are the undulation phase speed. The superscript in the equation stands for the differentiation of phase angle θ.

Eq. (8) is the second order non-linear ordinary differential equation of non-linear Rossby wave under topographical forcing in the zonal shear flow. Obviously, Eq. (8) is the case as described in liu's work if the zonal shear flow ($\bar{u}$ = o) is not considered. In this study only general situations are dealt with and in discussing the stability and solution of undulation the second order non-linear ordinary differential equation is not taken, as is done in liu's work, as Taylor series approximation and thus the equation becomes Kdv equation. Instead, conditions for periodical wave and isolated wave solutions with limited amplitude in the second order non-linear differential equation as weall as their nature are directly studied on a strict basis. Conditions for stable undulation obtained by series approximation and for the variations of stable undulation in the original equation will be discussed elsewhere.

For $\partial \bar{\xi}_a / \partial y \neq 0$, Eq. (8) can be written as

$$\frac{d^2 \varphi}{d\tau^2} = \text{sgn}(r)\frac{\varphi^2 + A\varphi}{\varphi - B}, \tag{9}$$

where

$$A = \frac{f_0 \bar{\xi}_a (c_x + c_h)}{\frac{\partial \bar{\xi}_a}{\partial y}} + c_0^2, \quad B = \frac{c_0^2 (c_x - \bar{u})}{(\bar{u} + c_h)},$$

$$r = -\frac{\frac{\partial \bar{\xi}_a}{\partial y}}{[k_h^2(\bar{u} + c_h)]}, \quad \tau = |r|^{1/2}\theta,$$

$$c_h = \frac{g}{f_0}\frac{\partial h_B}{\partial y} - \frac{l}{k}\frac{g}{f_o}\frac{\partial h_B}{\partial x}, \quad sgn(r) = \begin{cases} 1 & r>0 \\ -1 & r<0 \end{cases}. \tag{10}$$

By transformation,

$$Q = \frac{(\varphi + A + B)}{(A + 2B)}, \tag{11}$$

which is written as

$$a = (A + B)B, b = A + 2B. \tag{12}$$

Thus Eq. (9) becomes

$$\frac{d^2 Q}{d\tau^2} = sgn(r)(Q + \frac{a}{b^2}\frac{1}{Q-1}), \tag{13}$$

which can be considered as the motion equation of unit mass carrier fluid conductor in the vicinity of infinite carrier fluid conductor. This is equivalent to the following differential system

$$\frac{dQ}{d\tau} = P, \frac{dP}{d\tau} = sgn(r)(Q + \frac{a}{b^2}\frac{1}{Q-1}). \tag{14}$$

On the phase plane as is defined by (Q, P), there are two balance points $A\pm \quad ((1 \pm A/b)/2, 0)$. Eq. (14) has initial integration

$$P^2 - sgn(r)(Q^2 + \frac{2a}{b^2}\ln|Q-1|) = E = P^2 + V(Q), \tag{15}$$

where E is the integration constant, which is determined from the initial disturbance and defined as entrophy. $V(Q)$ is the enstrophy function similar to geopotential energy. Then Eq. (15) is the energy equation as characterized by the motion equation(13). Thus the original problem is simplified to that of unit mass point in the potential field as defined by $V(Q)$. Calculating the enstrophy of the two balance points gives

$$E\pm = \frac{1}{2}(1 \pm \frac{A}{b}) - \frac{a}{b^2}(1 - 2\ln\frac{1 \mp \frac{A}{b}}{2}). \tag{16}$$

For $\frac{\partial \bar{\xi}_a}{\partial y} = 0$, Eq.(8) can be written as

$$\frac{d^2\varphi}{d\tau^2} = \frac{sgn(r_1)\varphi}{\varphi + \alpha}, \tag{17}$$

where

$$r_1 = \frac{f_o\bar{\xi}_a(c_x + c_h)}{-k_h^2(\bar{u} + c_h)}, \alpha = \frac{c_o^2(\bar{u} - c_x)}{\bar{u} + c_h}, \tau = |r_1|^{1/2}\theta. \tag{18}$$

Eq. (17) is equivalent to the following differential system($Q = \varphi$)

$$\frac{dQ}{d\tau} = P, \frac{dP}{d\tau} = \frac{sgn(r_1)\varphi}{(\varphi + \alpha)}. \tag{19}$$

On the phase plane defined by (Q,P), there is one balance point (0,0) whose initial integration is the energy equation

$$P^2 - 2sgn(r_1)(Q - \alpha\ln|Q + \alpha|) = E = P^2 + V(Q), \tag{20}$$

where E, $V(Q)$ has the same physical meaning as before. The enstrophy of balance point can be calculated from the above formula as

$$E_o = 2sgn(r_1)\alpha\ln|\alpha|. \tag{21}$$

III. CONDITIONS FOR FOPOGRAPHICAL ROSSBY WAVE IN ZONAL SHEAR FLOW

Features of the motion on the phase plane can be studied via energy integration, that is, Eq. (15) or (20) of enstrophy. Figs. 1 and 2 give the phase orbits on the planes $V(Q)$ and (P,Q) as defined by Eqs. (15) and (20), respectively. On this basis, conditions for the existence of topographical Rossby wave in the zonal shear flow can be analyzed.

For the basic westerly flow $\bar{u}>0$, from Eq. (10), when $\partial\bar{\xi}_a / \partial y \neq 0$, we have

$$sgn(r) = sgn[\frac{\frac{-\partial\bar{\xi}_a}{\partial y}}{(\bar{u} + c_h)}],$$

$$a = (\frac{f_o\bar{\xi}_a}{\partial\bar{\xi}_a / \partial y} + \frac{c_o^2}{\bar{u} + c_h})\frac{(c_x - \bar{u})(c_x + c_h)}{\bar{u} + c_h}, \tag{22}$$

and from Eq. (18), when $\partial\bar{\xi}_a / \partial y = 0$, we have

$$sgn(r_1) = sgn[-\frac{\bar{\xi}_a(c_x + c_h)}{\bar{u} + c_h}], \alpha = \frac{c_o^2(\bar{u} - c_x)}{\bar{u} - c_h}. \tag{23}$$

Based on the features of the motion on the phase plane, we have the following conditions for the existence of non-linear topographical Rossby wave in the zonal shear flow:

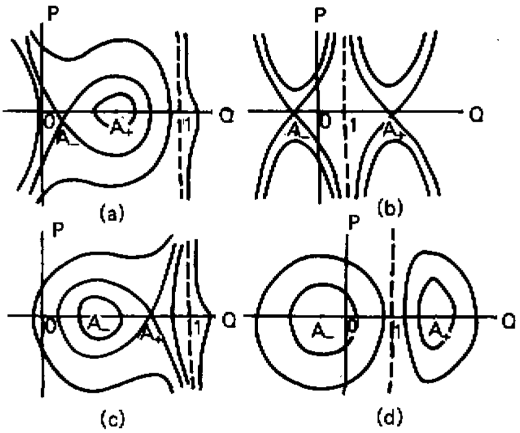

Fig.1. For $r\neq0$, and phase orbit. (a) sgn(r) = 1, a > 0; (b) sgn(r) = 1, a < 0; (c) sgn(r) = −1, a > 0; (d) sgn(r) = −1, a < 0.

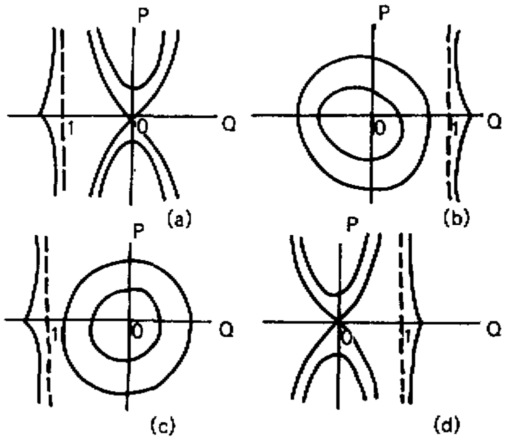

Fig.2. For r=0, and phase orbit. (a) sgn(r_1)=1, $\alpha>0$; (b) sgn(r_1)=1, $\alpha<0$; (c) sgn(r_1)=−1, $\alpha>0$; (d) sgn(r_1)=−1, $\alpha<0$.

(1) When $\dfrac{\partial\bar{\xi}_a/\partial y}{(\bar{u}+c_h)}<0$,

$$\left(\frac{f_o\bar{\xi}_a}{\partial\bar{\xi}_a/\partial y}+\frac{c_o^2}{\bar{u}+c_h}\right)\frac{(c_x-\bar{u})(c_x+c_h)}{\bar{u}+c_h}>0 \tag{24}$$

and when the enstrophy determined by the initial undulation satisfies $E_- < E \leqslant E_+$, the phase orbit is the closed curve around the central point A_+ or the boundary curve through the saddle point A_-. The former gives the periodical motion solution with limited or small amplitude (Fig. 1a) while the latter gives isolated wave solution ($E=E_+$) with limited amplitude. Otherwise, when $E>E_+$ or when Eq. (24) is not satisfied, we have

$$\left(\frac{f_o\bar{\xi}_a}{\partial\bar{\xi}_a/\partial y}+\frac{c_o^2}{\bar{u}+c_h}\right)\frac{(c_x-\bar{u})(c_x+c_h)}{\bar{u}+c_h}<0 \tag{25}$$

in which case there is no non-linear Rossby solution for any E (Fig.1b).

(2) When $\dfrac{\partial\bar{\xi}_a/\partial y}{(\bar{u}+c_h)}>0$, if Eq.(24) is satisfied, for $-E_+ < E \leqslant -E_-$, the phase orbits are closed curves around the center of A_- or boundary curves through the saddle point A_+ (Fig. 1c). In this case, we have wave solution with limited amplitude rather than isolated wave solution. Otherwise, for $E>-E_-$, the phase orbits are not closed and there are no boundary wave solutions. If Eq. (25) is satisfied, for any enstrophy value, phase orbits are two groups of closed curves with A_+ and A_- as their central points respectively (Fig. 1d). Thus, there is wave solution with limited amplitude rather than isolated wave solution.

(3)When $\partial\bar{\xi}_a/\partial y=0$, and

$$\bar{\xi}_a(c_x+c_h)(c_x-\bar{u})>0, \tag{26}$$

no non-linear Rossby wave solution exists for any enstrophy value (Figs. 2a and 2d). Only when

$$\bar{\xi}_a(c_x + c_h)(c_x - \bar{u}) < 0, \quad (27)$$

and $E \geqslant E_0$, are the phase orbits closed curves centering around the original point and there is wave solution with limited amplitude rather than isolated wave solution (Figs. 2b and 2c). For non-closed phase orbits with $E < E_0$, no boundary wave solution exists. It can be deduced that isolated wave solution exists only when E satisfies certain conditions and $\partial\bar{\xi}_a / \partial y \neq 0$, which satisfies Eq.(24).

If no consideration is gives to the zonal basic flow, then $sgn(r) = sgn(-c_h)$. Therefore, for the trailing waves on the southern and sestern topographical slopes ($l / k < 0$) or the leading waves on the eastern slope ($l / k > 0$), $c_h > 0$ and then $sgn(r) = -1$. On this occasion, wave solution with limited amptitude always exist while there is no constraint on wave speed c_x. As a result, the southern slope contributes to the stable existence of undulation while the western slope to the trailing waves and the eastern slope to the leading waves. For the leading waves on the northern and western slope and the trailing waves on the eastern alope, $c_h < 0$, and then $sgn(r) = +1$. Only when

$$(c_h + \beta R^2)c_x(c_x + c_h) > 0 \quad (28)$$

($R = c_0 / f_0$ is the Rossby radius of deformation) can there exist wave solution with limited amptitude. Therefore, the northern and western slopes do not always contribute to the stable existence of the leading waves and neither does the eastern slope to the trailing waves. That is, there is constraint on wave speed c_x.

If only topographical forcing is taken into account without considering the zonal basic flow and β effect, then $r = 0$, and only when $c_x(c_x + c_h) < 0$ can there be topographical periodic wave solution with limited amptitude. That is to say, topography has restraints on wave speed. For $c_h > 0$ and $-c_h < c_x < 0, 0 < c_x < -c_h$ if $c_h < 0$. Therefore, the trailing waves on the western slope, the leading waves on the eastern slope and the topographical waves on the southern slope all propagate westward ($c_x < 0$), while the leading waves on the western slope, the trailing waves on the eastern slope and the topographinсal waves on the nothern slope all travel eastward ($c_x > 0$). When the effect of the basic westerly flow $\bar{u} > 0$ is taken into account, Rossby waves generally spread eastward. And the northern slope speeds up the propagation of undulations while the southern slope slows them down. This is consistent with observational data. Therefore, east-west oriented terrain contributes to the formation of trough line or shear line. The western slope increases the speed of the leading waves and decreases that of the trailing waves, while the eastern slope plays an opposite part. That is to say, north-south oriented terrain increases the lehgth of the leading waves and decreases that of the trailing waves.

If the topographical waves are studied for one shearing in the zonal flow without considering β effect, wave solution with limited amptitude exists only when $\bar{\xi}_a c_x(c_x + c_h) < 0$. Obviously, in the inertially stable areas ($\bar{\xi}_a > 0$), the effect of topography is the same as in the case in which the zonal flow is not considered. But in the inertially unstable areas the results are different. For $c_h > 0$, $c_x > 0$ or $c_x < -c_h$; for $c_h < 0$, $c_x < 0$ or $c_x > -c_h$. Therefore, eastward propagation is found in undulations on the southern slope, the trailing waves on the western slope and the leading waves on the eastern slope, while westward transfer is observed in undulations on the northern slope, the leading waves on the western slope and the trailing waves on the eastern slope. Thus for homogeneous absolute vorticity, the effect of topography on waves depends not only on the features of the topography itself but also on the relative size of one shearing of the basic flow and planetary vorticity.

When β effect is taken into account with two shearings of the basic flow, the effect of topography on undulation is more complicated. For small slope, $<0(10^{-5})$ for example, $\bar{u}+c_h \approx \bar{u}$ with the results very similar to those when there is no topogtaphical forcing.①

For greater slope, $>0(10^{-3})$ for example, $\bar{u}+c_h \approx c_h . sgn(r) = sgn(-\frac{\partial \bar{\xi}_a}{\partial y}/c_h)$ Therefore, when c_h and $-\frac{\partial \bar{\xi}_a}{\partial y}$, that is, the absolute vorticity gradient, have opposite signs, $r<0$. In this case, topography always contributes to the stable existence of undulation and has no restraint on wave speed c_x. When they have the same sign, wave solution with limited amplitude exists only when Eq. (24) is satisfied. For the slope of $0(10^{-4})$, $\bar{u}\sim c_h$. In this case, for the trailing waves on the southern and western slopes and the leading waves on the eastern slope, $\bar{u}+c_h>0$. Thus, the subscript is only related to the absolute vorticity gradient. For the leading waves on the northern and western slopes and the trailing waves on the eastern slope, the effect depends on the relative values of u, c_h and absolute vorticity gradient.

To summarize, the existence of non–linear topographical wave solution in the zonal flow and shearing and its form (periodical or isolated) depend not only on the enstrophy decided by the initial disturbance phase but also on the features of the basic flow and its shearing, and the terrain itself.

IV. APPROXIMATE EXPLICIT SOLUTION OF NON–LINEAR ROSSBY WAVE

With the conditions for the existence of non–linear Rossby wave, the implicit expression of wave solution can be obtained from the energy equation

$$= \pm \int [E - V(Q)]^{-1/2} dQ \tag{29}$$

For the explicit solution, Taylor serial approximation has been used for reducing the equation to Kdv equation. However, in applying this method, attention should be paid to the premise of wave solution in the original problem, or else, there can be false elliptical cosine wave or isolated wave. Besides, the expansion of series will limit the range of the values of E and the conditions for the existence of wave solution may be altered. In this study, function approaching is used (Huang Sixum and Zhang Ming, 1988). Take Eq.(9), which corresponds to sgn$(r)=1$, $a>0$, for example. Obviously, for $E_- < E < E_+$, the phase orbits are closed curves centering around A_+. In this case, Q has boundary solution to τ. Approaching is done to $V(Q)-E=0$, by means of tri–polynomial, making sure that the polynomial and $V(Q)-E=0$ have the same number of zero points and zero values ($Q<1$). Therefore, from Eq. (29), stable boundary wave solution of φ can be obtained by integration

$$\varphi = [f_0 \bar{\xi}_a (c_x + c_h) / \frac{\partial \bar{\xi}_a}{\partial y} + c_o^2 + \frac{2c_o^2(c_x - \bar{u})}{\bar{u}+c_h}] Q_2 + (Q_1 - Q_2)$$

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$$Cn^2\sqrt{\frac{Q_1-Q_3}{6}[-\frac{\partial\bar{\xi}_a}{\partial y}/(\bar{u}+c_h]\frac{\theta}{k_h}]}$$

$$-(f_0\bar{\xi}_a/\frac{\partial\bar{\xi}_a}{\partial y}+\frac{c_o^2}{\bar{u}+c_h})(c_x+c_h) \tag{30}$$

where Q_i is the three solutions (< 1) of the skip equation

$$-(Q^2+\frac{2a}{b^2}\ln|Q-1|)=E \tag{31}$$

and $Q_1>Q_2>Q_3$. Eq. (30) is elliptical cosine wave.

For $E=E_+$, the phase orbits are bounday curves through the saddle point A_-. When Q, which corresponds to $\theta=0$, satisfies $Q_2<Q(\tau=0)\leqslant Q_l$, φ has isolated wave solution

$$\varphi=[f_0\bar{\xi}_a(c_x+c_h)/\frac{\partial\bar{\xi}_a}{\partial y}+c_o^2+\frac{2c_o^2(c_x-\bar{u})}{\bar{u}+c_h}][Q_2+(Q_1-Q_2)$$

$$\sec^2h\sqrt{\frac{Q_1-Q_3}{6}[-\frac{\partial\bar{\xi}_a}{\partial y}/(\bar{u}+c_h]\frac{\theta}{k_h}]}$$

$$-(f_0\bar{\xi}_a/\frac{\partial\bar{\xi}_a}{\partial y}+\frac{c_o^2}{\bar{u}+c_h})(c_x+c_h) \tag{32}$$

where Q_1 is the single root of Eq. (31) under the above conditions, while $Q_2=Q_3$ is the dual root. For $Q(\tau=0)<Q_2$ and $E>E_+$, the phase orbits are open curves and there is no stable boundary wave soultion.

Since the original problem has periodical solution as stated in the previous section, it can be obtained by function approaching. Therefore, elliptical cosine wave or isolated wave solution can be viewed as approximate explicit solution of limited amptitude periodical wave solution. It can be deduced from the approximate solution that the influence of topography and zonal shear flow on non-linear wave can be found not only in the existence of non-linear wave, but also the other features, such as wave speed, wave length, and amptitude are related to the zonal flow and its shearing,and topographicl slopes, indication the influence of non-linear wave, topography and the basic flow on undulation. Similar methods can be applied for obtaining approximate explicit expression of wave solution in many cases where conditions for the existence of periodical or isolated wave solution are satisfied.

V. CONCLUDING REMARKS

Under semi-geostropical approximation, the non-linear ordinary differential equation involving topography and zonal shear flow is derived by using the phase angle function. Based on the qualitative theory of the ordinary differential equation, conditions for the existence of periodical and isolated wave solutions with limited amptitude. It is proved by function approaching that the explicit wave solutions are elliptical cosine wave or isolated wave solutions. Results show that topography and zonal shear flow influence not only the existence but also the form of of waves, which indicates the non-linear features of waves and the effect of topography and basic flow on undulation.

REFERENCES

Bolin, B. (1950), On the influence of the earth's orogoraphy on the general character the westerlies, *Tellus*, **2**: 184-195.

Charney, J. G., and A. Eliassen. (1949), A numerical method for predicting the pertur bations of the middle latitude westerlies,*Tellus*, **1**: 38-54.

Kasahara, A. (1966). The dynamical influende of orogoraphy on the large scale motion of the atmosphere, *J. A. S.*, **3**: 259-271.

Liu, S. K., and Tan, Ben Kui. (1988), Nonlinear Rossby waves forced by topography, *Applied Mathes and Mechanics*, **3**: 229-240.

Lu, K. L. (1986), The large-scale orography and the stability of Rossby wave in the barotropic atmosphere, *Acta. Met. Sinica.*, **44**: 275-281.

Lu, K. L. (1987), The effects of orography on the solitary Rossby wave in a barotropic atmosphere, *ibid*, **45**: 267-273.

Tan, B. K., and Liu, S. K. (1988), Nonlinear rossby wave in the geophysical fluid, *Acta Scientiarum Naturalium Universitatis Pekinensis*, **3**: 341-349.

Wu, R. S. (1964), On the influences of mountain range on the instability of disturbances, *Acta. Meteorologica Sinica*, **34**: 11-19.(in Chinese)