The Propagation of Disturbances Excited by Low–Frequency Oscillations in the Tropics

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ABSTRACT

The propagation of disturbances excited by low–frequency oscillations in the tropics is investigated by applying the theory of wave packet dynamics. For simplicity, a linearized barotropic model is adopted and the zonal circulation is taken as basic current. Suppose that the disturbances or waves are superimposed on jet–like westerly basic current and excited by the forcing in the tropics. We have (1) only the eastward propagating ($m>0$, $n>0$ and $\sigma>0$) low–frequency disturbances and the stationary ($\sigma=0$) waves can propagate into the middle and high latitudes in the Northern Hemisphere; the others, such as the westward propagating low–frequency wave ($m>0$, $n<0$, $\sigma<0$) and the high–frequency waves, are restricted only in the vicinity of source region; (2) a stationary wave ($\sigma=0$) reaches a given latitude even more quickly than some low–frequency ones ($\sigma>0$) due to the fact that the group velocity of stationary wave is larger; (3) there is a whole wave train excited by the forcing in the tropics and extended into the middle and high latitudes, if the amplitude of the source is independent on time, especially, the low–frequency wave ($\sigma>0$) is of travelling type propagating along the ray; (4) if the source lasts only for an interval of time, namely, its amplitude also has the character of low–frequency oscillation, the excited wave train is not always a whole one, but is restricted in the vicinity of source region in the beginning, extended from the source region to the middle and high latitudes in its saturated stage, after that it gradually becomes weaker and weaker and is detectable only in some area at high latitude, and eventually disappears. Undoubtedly, case (4) is closer to the reality, even though case (3) gives a more impressive wavy pattern.

Key words: Wave packet, Low–frequency oscillation, Quasi–stationary source, Non–stationary source

I. INTRODUCTION

Low–frequency oscillation and low–frequency disturbance in the atmospheric general circulation are some important objects in the short term climatic variabilities. Especially, the problem on the waves excited by the tropical low–frequency oscillation and propagated into middle–high latitudes attracts more and more scientists' attention. The disturbances associated with low–frequency oscillations are some major systems for the medium–long range weather evolution and short term climatic anomaly, as well as some important sources generating abnormal planetary waves and regional climatic anomalies in the middle–high latitudes. For example, the onset date of seasonal transition in monsoon region of East Asia and the abnormal Meiyu are all correlated with the low–frequency oscillations in the tropics. Many observational studies and numerical simulations in these aspects have been systematically described and published (see, for example, Ye, Zeng and Guo, 1991; Tao et al., 1987, 1988; Huang, 1986, 1990; Zeng et al., 1990; Li, 1992; Wu et al., 1992).

Meteorologists usually apply the theoretic results obtained for the stationary wave train to the interpretation of disturbance excited by the tropical low–frequency oscillation and propagated into middle–high latitudes. In these works the basic current is zonal and steady, the tropic source is stationary, hence the excited wave train is stationary, and the ray and amplitude are obtained by using the group velocity and wave action. The Chinese as well as others (for example, Hoskins, 1981) dealt with this problem, namely in such a way. These concepts and methods give rather compact and clear results. However, no picture of two–dimensional and three–dimensional structure has been given so far. On the other hand, few investigators (Simmons et al., 1983; Zhang, 1990) discussed the non–stationary problem. In their works, the basic current is a non–zonal but climatological mean (steady) one, and the normal modes as well as their instability properties are computed. Their results give wave trains similar to the observed teleconnection patterns. It is not difficult to imagine that such computation is not simple, and every normal mode presents only a standing wave, hence the concept of group velocity can not be simply and directly applied, and it is not easy to obtain a simple and clear picture of wave propagation by such method. The theory has to be developed further.

Zeng et al. (1982; 1983; 1986) have developed systematic theory of wave packet dynamics and made qualitative analysis of the correspondent equations. In the theory, the basic current can be zonal or non–zonal, steady or non–steady; and the disturbances can be stationary or non–stationary. Although Zeng restricted himself by the qualitative analysis, however on the basis of his method it is possible to develop a simple and clear method for computing and representing the propagation and structure of low–frequency disturbances. Applying Zeng's method to a barotropic quasi–geostrophic model, the author (Lu, 1992) obtained whole two–dimensional structure of stationary wave packet superimposed on various zonal and steady basic currents, and found some new and very simple and clear laws governing the structure and propagation of such waves, for example, the line connecting the maximum (positive centre) and minimum (negative) of the disturbance does not coincide with the wave ray, but goes across it with an angle due to the non–orthogonality of the ray and isophase line. In the present paper we will extend our research to the problem of spatial structure and temporal evolution of non–stationary wave packet by the same method. In so doing we will be able to explore the whole dynamical process of low–frequency disturbance excited by the tropical source and propagated into middle–high latitudes.

II. MATHEMATIC PROBLEM ON THE PROPAGATION OF DISTURBANCE

The model adopted in this paper is the same as in the previous paper (Lu, 1992), i.e., the barotropic quasi–geostrophic model linearized with respect to a zonal and steady basic current.

Our interest is in the propagation of disturbances from the tropics to the middle–high latitudes, therefore we will not discuss that part of the solution which is determined by the initial condition.

Suppose that there is a wave packet (disturbance characterized by oscillation of motion) as follows:

$$\Psi' = (x, y_0, t) = \Psi_0(X, T) \cos \alpha(x, y_0, t), \tag{1}$$

and that the associated solution in the whole domain (including tropics and middle–high latitudes) is given by

$$\Psi' = (x, y, t) = \Psi(X, Y, T) \cos \alpha(x, y, t), \tag{2}$$

it is required that

$$\lim_{Y \to Y_0} \Psi(X, Y, T) = \Psi_0(X, T), \tag{3}$$

where $(X, Y, T)=\varepsilon(x,y,t)$, $0<\varepsilon\ll 1$ (small parameter), Ψ is the wave amplitude, and α is the phase angle from which the frequency σ and two components of the wave number (m, n) along x and y directions respectively are determined as

$$\sigma \equiv -\frac{\partial\alpha}{\partial t}, \quad m \equiv \frac{\partial\alpha}{\partial x}, \quad n \equiv \frac{\partial\alpha}{\partial y}. \tag{4}$$

Under our conditions, σ and m are both constants, independent on (x, y, t), and n is only a function of y.

Giving σ, m and (1), the spatial structure and the temporal evolution of the disturbance excited by the tropical forcing [determined by (1)] can be analyzed if the solution (2) satisfying (3) can be found out.

III. SONE CHARACTERISTICS OF WAVE PROPAGATION

According to the results obtained by the theory of wave packet, we have

$$(\sigma - m\bar{u})(m^2 + n^2 + 1) + \bar{\beta}m = 0, \tag{5}$$

(see, for example, Lu, 1992), where $\bar{u}(y)$ is the non–dimensional basic current, and $\bar{\beta}$ is the derivative of non–dimensional potential vorticity with respect to the non–dimensional y, i.e.,

$$\bar{\beta} = 1 - \frac{\partial^2 \bar{u}}{\partial y^2} + \bar{u}. \tag{6}$$

Denoting

$$S \equiv \frac{\bar{\beta}}{\left(\bar{u} - \frac{\sigma}{m}\right)}, \tag{7a}$$

we have

$$n(y) = \pm\sqrt{S - (m^2 + 1)}. \tag{7b}$$

It is clear from (7b) that n is real as $S(y) \geqslant m^2 + 1$, where the structure of the disturbance indeed is wavelike. $n(y_c) = 0$ if $S(y_c) = m^2 + 1$, hence y_c is a reflex point. In the region where $n^2 < 0$, i.e. $S(y) < m^2 + 1$, the intensity of the disturbance decays rapidly as $|y - y_c|$ becomes larger. This means that disturbance indeed is restricted from y_0 to y_c.

Let the basic current be given by

$$\bar{u}(y) = U_0 + U_1 y + \frac{1}{2} U_2 y^2, \tag{8}$$

where $U_0=0.35$, $U_1=1.2$, and $U_2=-0.6$; and take $y_0=0.2$ (near the Equator $y=0$). $\bar{u}(y)$ is given in Fig.1. Taking $m=1$, and $\sigma=0$, 0.3142, and 0.5, which is correspondent to the dimensional period ∞, 69.4 days, and 44 days respectively, we compute $n(y)$ and the two components of group velocity $\vec{C}_g$,

$$C_{gx} = \frac{\partial\sigma}{\partial m} = \bar{u} - \frac{\bar{\beta}(n^2 + 1 - m^2)}{(m^2 + n^2 + 1)},$$
$$C_{gy} = \frac{\partial\sigma}{\partial n} = \frac{2\bar{\beta}mn}{(m^2 + n^2 + 1)}. \tag{9}$$

They all are shown in Fig.1.

The wave rays are determined by C_{gx} and C_{gy} by the following ordinary equation and the initial condition

$$\begin{cases} \dfrac{dx_r}{dy} = \dfrac{C_{gx}(y)}{C_{gy}(y)}, \\ X_r(y_0) = x_{r0}, \end{cases} \tag{10}$$

where r is an index (parameter) denoting a ray passing the points (x_{r0}, y_0), and the ray–curve consists of the set of geometrical point (x_{rk}, y_k), where $X_{rk} \equiv x_r(y_k)$.

The interval of time $t_e(y)$ required for the propagation of disturbance (signal) from y_0 to y is given by

$$t_e(y) = \int_{y_0}^{y} \frac{dy'}{C_{gy}(y')}. \tag{11}$$

While, the wave amplitude Ψ can be determined by the equation of wave action. Without forcing and dissipation we have,

$$\left(\frac{\partial}{\partial T} + \vec{C}_g \cdot \nabla\right)A = -A\nabla \cdot \vec{C}_g, \tag{12}$$

where A is the wave action, i.e.,

$$A \equiv |\Psi|^2(m^2+n^2+1)/(2\bar{\beta}). \tag{13}$$

From equation (12) we have the conservation of AC_{gy} during its propagation along the wave ray, i.e.,

$$(AC_{gy})\Big|_{\substack{x=x_{rk},\\ y=y_k,\\ t=t_{ek},}} = (AC_{gy})\Big|_{\substack{x=x_{r0}\\ y=y_0\\ t=0,}} \tag{14}$$

where $t_{ek} \equiv t_e(y_k)$. Once A is determined from (14), we have the correspondent Ψ,

$$\Psi\Big|_{\substack{x=x_{rk},\\ y=y_k,\\ t=t_1,}} = \frac{(2\bar{\beta}A)^{\frac{1}{2}}}{m^2+n^2+1}\Big|_{\substack{x=x_{rk},\\ y=y_k,\\ t=t_1,}} = \Psi_0\Big|_{\substack{x=x_{r0,}\\ t=t_1-t_{ek}}} \times \left(\frac{m^2+n_0^2+1}{m^2+n_k^2+1}\right)\cdot\left(\frac{n_0}{n_k}\right)^{\frac{1}{2}}, \tag{15}$$

where $n_0 \equiv n(y_0)$, $n_k \equiv n(y_k)$.

Taking $m=1$ and $A(x_{r0}, y_0, t) \equiv 1$, the curves $t_e(y)$, $A(y)$ and $\Psi(y)$ corresponding to $\sigma=0$, 0.3142, and 0.5 are given in Fig.2. It should be pointed out that for a given y some low–frequency wave reaches it even slower than the stationary one ($\sigma=0$) due to the larger t_e.

The results obtained by our computation clearly show that in our case of jet–like and steady westerly basic current the eastward propagating low–frequency waves ($m>0$, $n>0$ and $\sigma>0$) with period larger than 40 days are all of penetrating types, they can propagate from the tropics ($y=y_0$) into the middle–high latitudes in the Northern Hemisphere due to $C_{gy}>0$, and their amplitude reaches its maximum at the latitude of jet core.

Another computation with westward propagating waves ($m>0$, $n<0$, $\sigma<0$) or high–frequency waves ($|\sigma|>0.6$) shows that $y_c - y_0$ is very small for the high–frequency waves, and that $\Psi \to 0$ (due to $S \to \infty$ and $|n| \to \infty$) at some y which is close to y_0. Therefore, the westward propagating low–frequency waves or the high–frequency waves can not penetrate into middle latitudes in the Northern Hemisphere, but all are restricted in the vicinity of source region in the tropics.

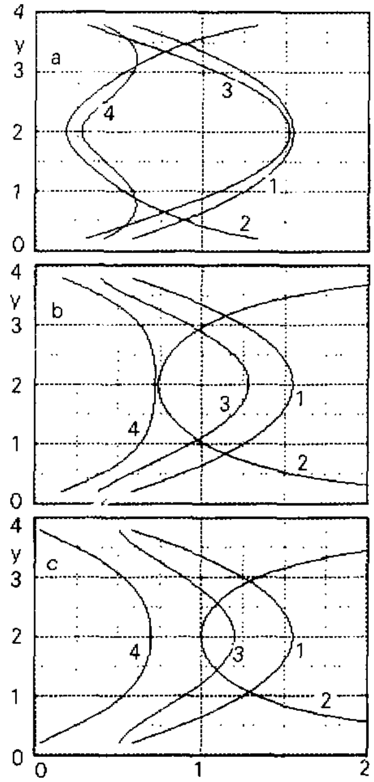

Fig. 1. The basic current $\bar{u}(y)$, wave number $n(y)$ and the two components of group velocity C_{gx} and C_{gy}. 1—$\bar{u}(y)$, 2—$n(y)$, 3—$C_{gx}(y)$, 4—$C_{gy}(y)$. (a) $\sigma=0$, (b) $\sigma=0.3142$, (c) $\sigma=0.5$.

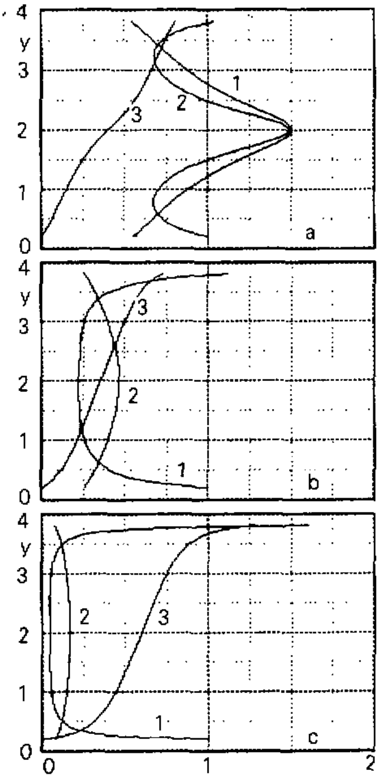

Fig.2. Propagation characteristics of waves superimposed on the basic current given in Fig.1. 1—$A(y)$, 2—$\Psi(y)$, 3—$t(y)$. (a) $\sigma=0$, (b) $\sigma=0.3142$, (c) $\sigma=0.5$.

IV. WAVE TRAIN EXCITED BY QUASI-STATIONARY SOURCE

We call a source as quasi-stationary if its $\Psi_0(X,T)$ is independent of t, i.e., it generates waves, but the amplitude does not change with time. In this case, let,

$$\Psi_0(X,T)=\Psi_0(x)\Phi(t), \tag{16}$$

we have $\Phi_0(t)=1$. Therefore for a given y both A and Ψ are established (independent of time) as $t>t_e(y)$, and the established disturbance of penetrating type is represented by a whole wave train.

Taking basic current (8), $m=1$, and

$$\Psi_0(x)=0.5\left\{e^{-0.64x^2}+e^{-(x+2)^2}+e^{-(x-2)^2}+\frac{1}{1+\left(\frac{x}{2}\right)^2}\right\},\quad(-\infty<t<\infty) \tag{17}$$

the established wave trains corresponding to $\sigma=0$, 0.3142, and 0.5 respectively are given in Figs. 3–4. Generally speaking, the characteristics are the same as given in the previous paper

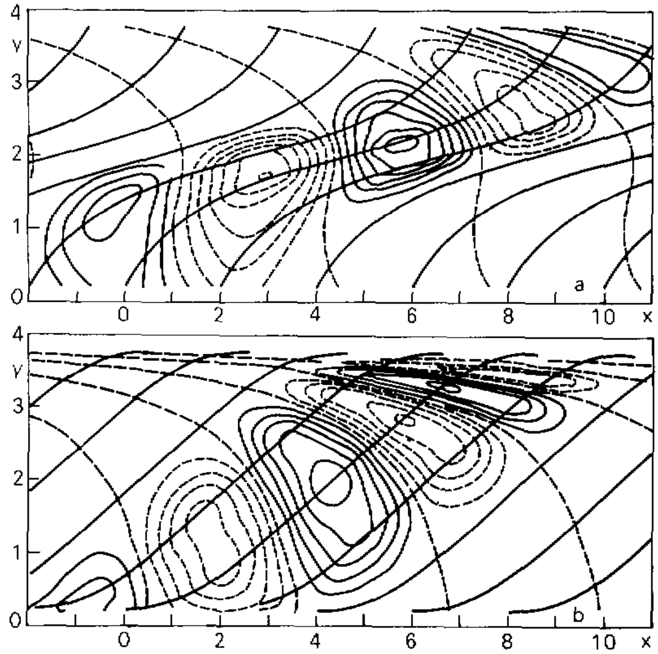

Fig.3. The established wave train excited by quasi–stationary source in tropics. The rays are also given. $t=0$. (a) $\sigma=0$, (b) $\sigma=0.5$.

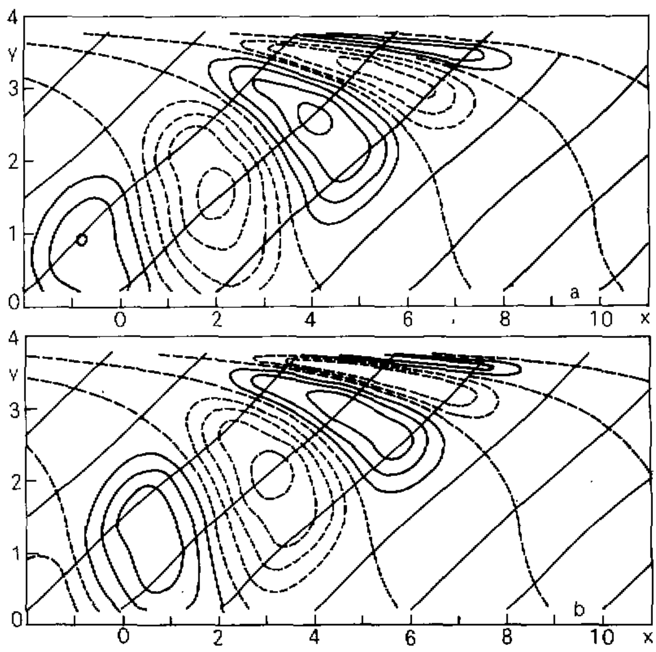

Fig.4. The established wave train excited by quasi–stationary source with $\sigma=0.3142$ in tropics. (a) $t=0$, (b) $t=\pi/2\sigma$.

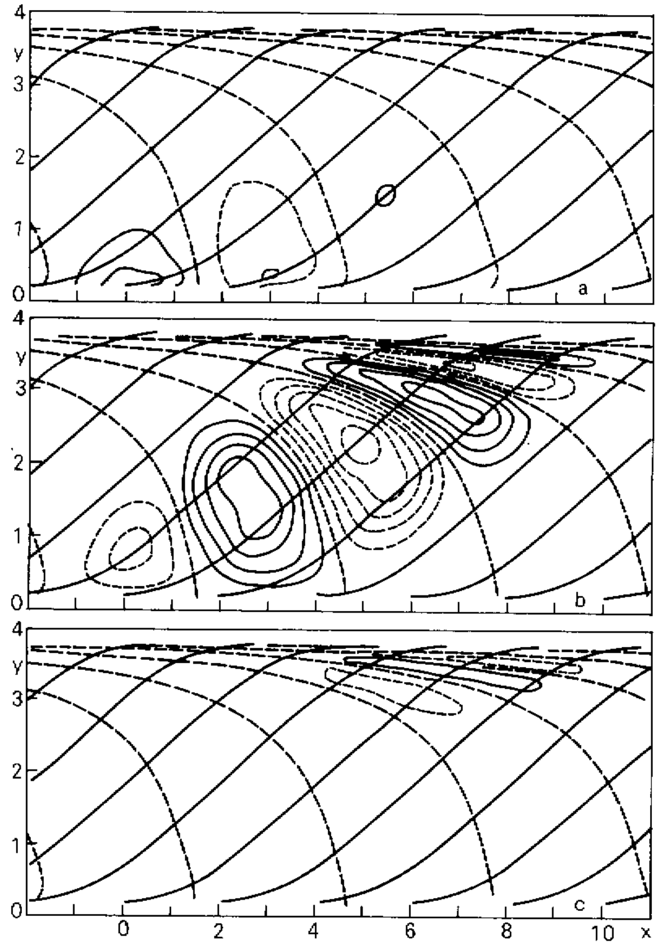

Fig.5. Propagation of disturbances excited by non–stationary source with $\sigma = 0.5$ in tropics. (a) $t = \tilde{t}$, (b) $t = \tilde{t} + \pi / \sigma$, (c) $t = \tilde{t} + 2\pi / \sigma$.

(Lu, 1992), i.e., (a) the disturbance propagates along the ray, but the line connecting the maximum (or positive) and minimum (or negative) centres of the disturbance goes across the ray with an angle; and (b) during the propagation from the tropics to the middle–high latitudes the wave amplitude increases, then reaches its maximum at the latitude where the jet core is located, and after that decreases. However, the low–frequency wave ($\sigma \neq 0$) differs from the stationary one ($\sigma = 0$), it is travelling, hence at every point (x, y) Ψ' is oscillating with a non–dimensional frequency σ.

V. PROPAGATION OF DISTURBANCE EXCITED BY NON–STATIONARY SOURCE

A source is called as non–stationary if $\Psi_0(X, T)$ is time dependent. Taking (16), $\Phi_0(t)$ is no longer a constant but varying with t in this case.

Taking $m = 1$ and the same basic current as in section IV, but $\Psi_0(X, T)$ varies with t, for example, $\Psi_0(X)$ is given by (17), and $\Phi_0(t)$ is given by

$$\Phi_0(t)=e^{-\mu|t-\tilde{t}|^2},\ (-\infty<t<\infty), \tag{18}$$

hence we have

$$\psi'(x,y_0,t)=\Psi_0(x)\Phi_0(t)\cos(mx+n_0y_0-\sigma t), \tag{19}$$

where μ and $\tilde{t}$ are two parameters. This means that source works for some interval of time, and is practically switched off outside this interval.

The computed $\psi'(x,y,t)$ with $\sigma=0.5$, $\mu=2(\pi/\sigma)^{-2}$, and $\tilde{t}=n_0y_0/\sigma$ is given in Fig.5. It can be seen that in the initial stage the wave is detectable only in the vicinity of source region; after some interval of time, before the source is not too weakened, there is a whole wave train extended from the tropics to middle–high latitudes; and as the source is switched off, then the amplitude of the disturbance decays in the vicinity of source region, but the disturbance can be detected only in some area in high latitudes.

Taking the same value of μ and $\tilde{t}$ as before, but computing n and Ψ by Eqs.(7) and (15) with $\sigma=0$, we obtain the picture of propagation of the stationary (quasi–stationary in fact) wave excited by the non–stationary source (Fig.6). It is clear that due to the energy dispersion, such low–frequency disturbance excited by the tropical low–frequency source can also penetrate into middle–high latitudes, and its characteristics are also similar.

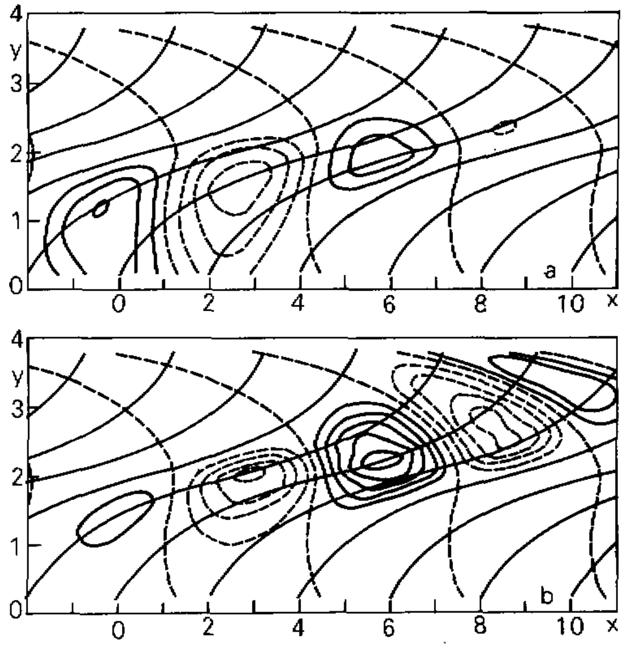

Fig.6. Propagation of disturbances excited by non–stationary source with $\sigma=0$ in tropics. (a) $t=\tilde{t}$, (b) $t=\tilde{t}+2\pi$ (the value $\tilde{t}$ is the same as in Fig.5).

In fact, the tropical source is not a quasi–stationary one, therefore the case discussed in section V is more close to the reality. Our analysis shows that even in the non–stationary case, application of theory of wave packet dynamics to the analysis can also give a clearly physical picture.

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