Improving Numerical Weather Prediction in Low Latitudes by Optimizing Diffusion Coefficients

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ABSTRACT

The horizontal diffusion coefficients of the operational model (T42L9) in numerical weather prediction are optimized by the steepest descent search of multi-dimensional optimization. In order to improve prediction accuracy in low latitudes, the optimum horizontal diffusion coefficients are chosen, with changing variation of the basic diffusion coefficient with the passage of time, and later forecasts are also made better. In view of the averages of forecast verifications of 9 cases, the forecasts with optimum diffusion coefficients are an improvement on operational forecasts. It means that the forecasts are got much better with optimum values of some important parameters by optimization in numerical weather prediction.

Key words: Multi-dimensional optimization, Steepest descent search, Optimum horizontal diffusion coefficients, Optimum values of parameters

I. INTRODUCTION

Some important parameters are frequently employed in numerical weather prediction and simulation. Recently some parameters are introduced in various parameterizations. Sometimes these parameters are estimated values of the actual atmosphere, such as the horizontal diffusion coefficients and so on. If the structure function is taken as Gauss function in the optimum interpolation scheme, the correlation broadness needs optimizing. The critical frequency, over which we wish to eliminate high-frequency gravity oscillations, also needs optimizing in nonlinear normal mode initialization. In general, these parameters can not be evaluated by any theoretical formula, they can be determined by experimental calculation. When a numerical modeling was being designed and a numerical simulation was being carried out, scientists make efforts to choose better parameters by experimental calculation. These processes that the parameters are artificially determined are subjective and blind. When many parameters need determining, scientists attend to some parameters and lose sight of others. In addition, are the parameters determined artificially optimum? Is the CPU time the least? In other words, how to find the optimum values of many parameters by minimum experiments? This is an important and difficult problem. At first, numerical weather prediction is an initial value problem. Secondly, an appraisal of numerical weather prediction has many norms (multi-objective problem). It is hoped that the correlation coefficient is higher, the root mean square error is lower and the skill-score is also lower. The multi-objective problem in optimization is a difficult problem in the field of optimization in the world. Both above problems exist in determining parameters artificially.

Because of the complexity of atmospheric motion, there will be many parameters to be optimized in numerical weather prediction and numerical simulation. According to our experience of optimizing tests, if the objective function approaches minimum, there is a proper proportion between many parameters, the proper proportion can not be obtained in determining them artificially. So the accuracy of forecast or simulation with optimized parameters is significantly improved.

It is difficult to forecast the subtropical circulation system in numerical weather prediction. The accuracy of numerical weather prediction in low latitudes is much lower than that in middle and high latitudes in most countries, and so does our operational forecast model (T42L9) in national meteorological center. Recently many researchers reviewed numerical weather prediction and simulation in low latitudes, for example as Kanamitsu (1985), and Krishnamurti (1988). And many researchers studied the formation and track forecast of the tropical storms. In this paper the optimum horizontal diffusion coefficients were chosen by the steepest descent search in multi–dimensional optimization to improve numerical weather prediction in low latitudes.

II. STEEPEST DESCENT SEARCH IN MULTI–DIMENSIONAL OPTIMIZATION

There are many methods for multi–dimensional optimization. The steepest descent search is adopted to seek optimum parameters in this paper. The critical idea of the steepest descent search is successively to seek negative gradient direction of the objective function until the minimum of the objective function (the corresponding parameters are optimum values) is found, then a number of tests for optimization is the least by such a search.

Assuming n parameters $(x_1, x_2, \cdots, x_n)$ to be optimized and the ranges of these parameters are as the followings:

$$a_i \leqslant x_i \leqslant b_i \quad (i=1,2,\cdots,n),$$

the parameters $x_1, x_2, \cdots, x_n$ constitute an n–dimensional Euclidean space from which a point $(x_1^1, x_2^1, \cdots, x_n^1)$ and n points of

$$(x_1^1+\delta_1, x_2^1, \cdots, x_n^1), (x_1^1, x_2^1+\delta_2, \cdots, x_n^1), \cdots, (x_1^1, x_2^1, \cdots, x_n^1+\delta_n)$$

are chosen in the domain of x, the superscript indicates a number of searches, the superscript 1 indicates the first search. One test is made on each point of the $n+1$ points and $n+1$ values of the objective function are obtained as

$$z^1, z_1^1, z_2^1, \cdots, z_n^1.$$

It is assumed that the objective function has one minimum only in the domain of x. Consequently on the n straight–lines of

$$x_i^1 + \frac{z^1 - z_i^1}{\delta_i} t^1 (i=1,2,\cdots,n),$$

the parameter t^1 is optimized by one–dimensional search technique to seek the minimum value of z^1. The range of t^1 is determined by the following inequalities:

$$\text{(i)} \quad t^1 \geqslant 0,$$

$$\text{(ii)} \quad a_i^1 \leqslant x_i^1 + \frac{z^1 - z_i^1}{\delta_i} t^1 \leqslant b_i^1 (i=1,2,\cdots,n). \tag{1}$$

When the minimum value of z^1 is found at $t^1 = t_0^1$, the first search has finished. Then let

$$x_i^{k+1} = x_i^k + \frac{z^k - z_i^k}{\delta_i} t_0^k \, (i = 1,2,\cdots,n;\ k = 1,2,\cdots)$$

which is used to replace x_1^k for the same repeated tests. The $(k+1)'$th search will be carried out. The z^k value gets smaller and smaller along with increase in search until a distance between the points of $(x_1^{k+1}, x_2^{k+1},\cdots,x_n^{k+1})$ and $(x_1^k,x_2^k,\cdots,x_n^k)$ in the n–dimensional Euclidean space is shorter than a control error ε. A distance in the n–dimensional Euclidean space is defined by

$$d = \max_{i=1,2,\cdots,n} \left| x_i^{k+1} - x_i^k \right|$$

or

$$d = \Big(\sum_{i=1}^{n} (x_i^{k+1} - x_i^k)^2 \Big)^{1/2} = t_0^k \Big(\sum_{i=1}^{n} (\frac{z^k - z_i^k}{\delta_i})^2 \Big)^{1/2}.$$

For optimizing one–dimensional parameter t^k, the golden section search in one–dimensional search is adopted in this paper. The range of t^k, is determined by the corresponding inequality (1), i.e.,

$$t_a^k \leqslant t^k \leqslant t_b^k.$$

The golden section search consists of the following steps:

(1) The first trial point t_1^k is on the τ point of the interval $[t_a^k,\ t_b^k]$, i.e.,

$$t_1^k = t_a^k + (t_b^k - t_a^k)\tau,$$

where $\tau = \dfrac{\sqrt{5}-1}{2} = 0.6180339887$ is the golden section ratio, it is from the equation $\tau^2 + \tau - 1 = 0$. A value T_1^k of the objective function is obtained by a test.

(2) The second trial point t_2^k is taken as the symmetric point of the point t_1^k with respect to the middle point $\dfrac{a+b}{2}$, i.e.,

$$t_2^k = t_a^k + t_b^k - t_1^k = t_a^k + (t_b^k - t_a^k)\tau^2.$$

Then the second value T_2^k of the objective function corresponding to t_2^k is obtained by the test again.

(3) According to the values of T_1^k and T_2^k, a new interval is chosen under the following three cases:

(i) If $T_1^k < T_2^k$, choose the interval $[t_2^k, t_b^k]$.

(ii) if $T_1^k > T_2^k$, choose the interval $[t_a^k, t_1^k]$.

(iii) if $T_1^k = T_2^k$, choose the interval $[t_2^k, t_1^k]$.

(4) In the cases of (i) and (ii), steps (2) and (3) are repeated and in the case of (iii), steps (1), (2) and (3) are repeated on the remaining intervals until the last remaining interval is smaller than a control error ε_t . If $t_b - t_a = 1$, then a number of tests which are performed to

make the last remaining interval be less than the control error ε_t are about

$$m \sim \lg\frac{1}{\varepsilon_t}.$$

(5) After m times of tests, the last remaining interval is less than the control error ε_t , the smallest one among T^k_m, T^k_{m-1} and T^k_{m-2} is taken and the corresponding parameter t^k is recognized as the optimum parameter t^k_0.

More detailed explanation about why the above mentioned two methods are capable of minimizing the tests for seeking optimum parameters can be found in the work of Hua (1981). Himmelblau (1972) encoded a program of the golden section search in FORTRAN, in order to accelerate convergence he modified the ordinary golden section search in accordance with many objective functions, for his considered objective functions the program accelerates convergence, but for other objective functions the program can not accelerate and even slows down convergence.

For the steepest descent search if we hope that a distance between $(x^{k+1}_1, x^{k+1}_2, \cdots, x^{k+1}_n)$ and $(x^k_1, x^k_2, \cdots, x^k_n)$ is less than a control error ε, a number of seeking negative gradient direction is about $l \sim \lg\frac{1}{\varepsilon}$. n tests for n parameters and m tests $(m \sim \lg\frac{1}{\varepsilon_t})$ for optimizing one–dimensional parameter t^k are performed in each seeking negative gradient direction. A total of tests which are performed in the steepest descent search is about

$$s \sim (\lg\frac{1}{\varepsilon})(\lg\frac{1}{\varepsilon_t} + n).$$

III. OBJECTIVE FUNCTION AND OPTIMIZING THE HORIZONTAL DIFFUSION COEFFICIENTS

The objective function selection is a key problem for optimizing some important parameters by multi–dimensional optimization in order to improve numerical weather prediction. Numerical weather prediction in low latitudes is improved for our purpose. The statistical scores of objective verification in low latitudes must be higher when an objective function has been adopted in optimization. We have selected the following three objective functions:

(i) $z = RMSE_{30} / (RMSE_0 \times COR_{30})$
(ii) $z = RMSE_{30} / RMSE_0 - COR_{30})$
(iii) $z = -COR_{30}$

where $RMSE_{30}$ means the root mean square error of 500 hPa height in the zone between 0°N and 30°N latitudes. $RMSE_0$ means the first forecast value of $RMSE_{30}$. COR_{30} means the anomaly correlation coefficient of 500 hPa height in zone between 0°N and 30°N latitudes. The minimum of the objective function is found and the corresponding parameters are optimum values. The results of optimization are almost the same using these three objective functions by optimizing experiment. The result is slightly better using $z = -COR_{30}$ for the objective function.

If we make five–day forecast, it would be better to optimize the horizontal diffusion coefficients by each day. We have done such optimization experiments for a barotropic primitive equation model, it is found that the result of forecast is a little improvement and the optimization experiment takes too much CPU time. Therefore we optimize the parameters for the first day forecast only. According to our experience of optimizing tests, if the objective function approaches minimum, there is a proper proportion between many parameters. If the many parameters have the proper proportion, the results of later forecast are also quite good. The proper proportion can not be obtained in determining them artificially. The horizontal diffusion coefficients to be optimized are for vorticity, divergence, temperature, moisture and two parameters depending on wavenumbers (Simmons, 1987; Liu and Yan, 1991). The values of these six parameters in operational forecast model (T42L9) are used for initial values of optimization. The control error ε and the increment δ for each parameter are 0.5, the control error ε_t for parameter t^k is 0.04 in the Euclidean space. After these three control errors are determined, in general it takes us CPU time of making 29 days forecast to optimize six parameters. On average the test was done only 4.8 times for each parameter. If the forecast model will be not changed, these six parameters will not need changing and hold good for all time. Because the values of sea surface temperature, snow depth and so on are monthly mean in physical processes, it would be better to optimize the parameters for each month.

A problem of the current operational forecast model (T42L9) is that the mean heights of the isobaric surfaces are being forecasted lower and lower. We get the basic diffusion coefficient decreased as time goes on, it is prevented that the mean heights of isobaric surfaces descend as time goes on. With the initial data of the operational objective analysis at 1200 GMT 6 June 1992, the horizontal diffusion coefficients of vorticity, divergence, temperature, moisture and two parameters depending on wavenumbers were optimized. We optimized the horizontal diffusion coefficients for the first day forecast only, and six optimized horizontal diffusion coefficients are obtained.

The basic diffusion coefficient K_0 is modified artificially. At the beginning of forecast, $K_0 = 7.0 \times 10^{15}$, it is the same as operational model. Table 1 shows the variation of the basic diffusion coefficient with the passage of time. It is worthy of note that K_0 descends two orders of magnitudes during 24–48 hours, and descends two orders of magnitudes again during 48–72 hours. The reason is that descent of the mean heights of the isobaric surfaces is not too much.

Table 1. *The Variation of the Basic Diffusion Coefficient with the Passage of Time*

Time	00–24	24–48	48–72	72–96
K_0	7.0×10^{15}	5.0×10^{13}	1.0×10^{11}	1.0×10^{12}

IV. THE RESULT OF THE EXPERIMENT

We used optimized horizontal diffusion coefficients and improved basic diffusion coefficient to make 4–day forecast at 1200 GMT on 1,2,3,4,5,6,7,8,10 in June 1992 (the data are not read out from a tape on 9 June 1992). The averages of forecast verifications of 9 cases are listed in Table 2 for the anomaly correlation coefficients and the root mean square errors at 500 hPa height. In Table 2, "NEW" indicates that the new optimized horizontal diffusion coefficients are used to make forecast, "OLD" indicates that the old operational model is taken to do forecast, "%" represents a percentage of the forecast improvement. A calculation of the percentage is the following:

(i)% = 100 × (NEW−OLD) / |OLD| for the anomaly correlation coefficients

(ii) % = 100 × (OLD−NEW) / OLD for the root mean square error.

If the percentage is positive, it means that the forecast with optimum coefficients is better than operational forecast. Conversely, if the percentage is negative, the forecast with optimized coefficients is worse than the operational forecast. We want to explain that numbers are rounded off automatically by computer in Table 2. When the percentage is computed, numbers are not rounded off, so some percentages which are calculated directly by numbers of Table 2 may be different from percentages listed in Table 2.

Table 2. The Averages of Forecast Verification of 9 Cases

		24 h	48 h	72 h	96 h
Anomaly correlation coefficient at 500 hPa height					
00°N–30°N	NEW	.650	.507	.443	.328
00°N–30°N	OLD	.590	.498	.410	.317
00°N–30°N	%	10.3	1.7	8.2	3.6
20°N–90°N	NEW	.895	.789	.684	.572
20°N–90°N	OLD	.890	.786	.673	.559
20°N–90°N	%	.6	.4	1.7	2.4
00°N–90°N	NEW	.788	.672	.585	.468
00°N–90°N	OLD	.762	.671	.569	.465
00°N–90°N	%	3.3	.1	2.8	.7
R.M.S. Error at 500 hPa height		24 h	48 h	72 h	96 h
00°N–30°N	NEW	11.7	15.7	17.9	19.7
00°N–30°N	OLD	13.8	16.2	19.1	21.6
00°N–30°N	%	15.4	3.1	6.4	8.6
20°N–90°N	NEW	26.9	43.6	58.0	70.1
20°N–90°N	OLD	24.7	43.6	61.3	75.0
20°N–90°N	%	−8.8	.0	5.4	6.6
00°N–90°N	NEW	21.6	33.9	44.1	52.9
00°N–90°N	OLD	20.9	34.0	46.6	56.6
00°N–90°N	%	−3.4	.4	5.3	6.6

We analyzed the experimental results from three aspects.

(1) Verification item: The anomaly correlation coefficient is better than the root mean square error, because "NEW" is all better than "OLD" in three zonal belts for 4–day forecast. For the root mean square error, "NEW" is worse than "OLD" for the first day forecast in two zonal belts of 20°N–90°N and 0°N–90°N. This is related to the chosen objective function, because we chose the anomaly correlation coefficient in zonal belt of 0°N–30°N as the objective function, the improvement of the anomaly correlation coefficients is much better.

(2) Zonal belt: From the results at three zonal belts, the zonal belt of 0°N–30°N is the best, 0°N–90°N is the second. For the root mean square error the zonal belt of 0°N–90°N is evidently better than 20°N–90°N. The reason is that the zonal belt of 0°N–90°N includes the

zonal belt of 0°N–30°N. The root cause is also the chosen objective function.

(3) Daily verification: From daily forecast, the first day forecast is better than later for the anomaly correlation coefficients; the fourth day forecast is better than early for the root mean square error. The second day forecast is worse than the others except for the root mean square error at zonal belts of 20°N–90°N and 0°N–90°N. The reason is that the basic diffusion coefficient has decreased in two orders of magnitudes and too much faster.

Because we optimized the horizontal diffusion coefficients for only the first day forecast and chose the anomaly correlation coefficient in the zone of 0°N–30°N as the objective function, the verification results of the first day forecast in 0°N–30°N are improved well for both anomaly correlation coefficients and the root mean square error. But the root mean square error increased by 8.8 per cent in the zone of 20°N–90°N at the first day forecast. The reason is that the numerical model (T42L9) is not normal. The heights of the isobaric surfaces are being forecasted higher and higher in low latitudes by the model T42L9, but lower and lower in high latitudes. When the forecasting height is normal in low latitudes, the forecasting height is lower than normal in high latitudes, thus the root mean square error has increased in middle and high latitudes. The anomaly correlation coefficient of the first day forecast increases by 0.6 per cent in middle and high latitudes, it is explained that forecasted positions of the trough and ridge are improved in middle and high latitudes.

Generally speaking, in view of the averages of forecast verifications of nine cases, the forecast with optimized horizontal diffusion coefficients are obviously improved. Only the horizontal diffusion coefficients were optimized in this paper, it is deduced that numerical weather prediction will be improved evidently if other important parameters will be optimized by optimization.

V. CONCLUDING REMARKS

The optimization is utilized in numerical weather prediction and simulation in order to improve forecast and simulation. This research just begins. There are many problems which need to be solved. For example, there is a problem in need of improvement in this paper, i.e., how to select a more suitable basic diffusion coefficient after the second day forecast to make better the forecasts thereafter. How is the optimization utilized in the fields of the objective analysis, the quality control of observation data, initialization and so on? These problems need to be studied. European Centre for Medium–Range Weather Forecasts performed an experiment of variational analysis in which the minimization of the cost function was required in later 1991. It is explained that the optimization is paid attention to in the field of numerical weather prediction. The author (Liu and Yan, 1991) solved a computational instability of the model T42L9 at that time by optimization in 1988, and then an accuracy of the model T42L9 was improved in the Northern Hemisphere by optimization again. There is no need to change the scheme and the programs of the forecast system if the optimization is utilized in improving forecast by optimizing important parameters of the forecast system. After setting up a forecast system it is recommended to optimize all important parameters of the forecast system before the forecast system is put into operation so that the forecast system runs in optimum condition. If the forecast system is already put into operation, the results of forecast are improved by optimization of the forecast system, manpower and expenses are thrown into not too much.

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