Study on Atmospheric Travelling Wave Solutions and Review of Its Present Developments①

Huang Sixun (黄思训) and *Zhang Ming* (张 铭)
Laboratory of Numerical Modelling for Atmospheric Sciences and Geophysical Fluid Dynamics, Chinese Academy of Sciences, Beijing 100080

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ABSTRACT

The scientific achievements of travelling waves in a barotropic atmosphere are introduced, including i) the existence conditions of periodic solutions (wavetrain solutions) and solitary wave solutions (pulse solutions), together with the solution finding methods and a series of related problems, ii) seeking solutions of monotonous wave (wave front) and of nonmonotonous travelling wave (oscillatory wave) by using phase plane shooting technique and iii) progress in the study of travelling wave solution at home and abroad. The investigation of travelling wave solutions in recent years has been found in mathematics, physics, chemistry, biology and other sciences. Over the past decade the problem has been the subject of much interest and become an important area of research. So it is no doubt of great significance to investigate the travelling wave solutions and thereby explain phenomena of weather.

Key words: Barotropic atmosphere, Wavetrain, Wave front, Travelling wave solution (TWS), Pulse solution, Nonmonotonous travelling wave solution

I. INTRODUCTION

In a sequence of daily weather maps, one can sometimes see a trough ridge system propagates eastward at extratropics with little change in the pattern for days and some of the meso-scale squall lines move at a rough uniform speed with no great changes in intensity or structure in several hours. These systems can be regarded as a wave motion with an invariant flow pattern and constant velocity in a limited period. In general, these systems are not considered as small disturbance, neither treated by a linearized method. They are viewed as non-dispersive solution to the nonlinear equations for the motion in the atmosphere or the travelling wave solution (TWS) mathematically. It is known that atmospheric waves are generally dispersive with the flow field deformed because of Coriolis force even if linear models are used. But for special wave patterns, the non-dispersive solution of nonlinear wave is really available during which the nonlinear effect produced by wave steepness is in equilibrium with dispersive and dissipative effects, leading to an invariant wave pattern. As such, the exploration of non-dispersive solutions to nonlinear waves is significant theoretically and practically. In the late 1950s, by introducing the phase angle function Kuo (1959) investigated the non-dispersive solutions in the quasi-geostrophic framework, which is called the eternal wave. In terms of the singular perturbation method, Zeng (1979) obtained the special solution of non-dispersive slow wave to the primitive equations, followed by a detailed discussion and assumed that there maybe exists a special solution of the fast type. Liu et al. (1982a, 1982b)

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reported their work in this area. Huang et al. (1985), Huang (1985, 1987, 1992a). Huang and Zhang (1986, 1987, 1989a, 1989b, 1989c, 1990a) and Zhang and Huang (1987, 1988) documented their efforts on the periodic and solitary wave solutions of nonlinear wave together with the general rules and some new ideas. Later on, Huang (1988, 1991), Huang and Zhang (1991), Huang et al. (1992) and Zhang (1992) investigated solutions of monotonous travelling waves (wave front type) and nonmonotonous wave (oscillatory travelling type) in the atmosphere. The investigation of atmospheric monotonous and nonmonotonous travelling wave helps explain some of the weather facts.

Now we present below the review of the study of TWS of a barotropic atmosphere and some methods in use by domestic and foreign meteorologists.

II. ASPECT OF WAVETRAIN AND PULSE SOLUTIONS

For a barotropic atmosphere, expect the simplicity of equations, most important, the boundary conditions are not considered. In terms of non-dispersion characteristics we introduce the phase angle function

$$\theta = x \cdot v - |c|t \ ,$$

where $v = (v_1, \ldots, v_n)$ is the unit vector in the c direction and $x = (x_1, \ldots, x_n)$, leading to $u(x, t) = U(\theta)$. If U is a periodic function of θ, then we have the wavetrain solution or the periodic solution in meteorological context. If U is not constant and $U(+\infty) = U(-\infty)$, then we have the pulse solution or the solitary wave solution. The introduction of the phase angle function causes the change from the nonlinear partial differential equations into the ordinary ones followed by the discussion of the nature around the equilibrium and singular points therein, resulting in a range of analytic expressions of nonlinear wave in the atmosphere (e.g., periodic solutions, solitary wave solution and their intermittent types). The reader is referred to references (Huang et al., 1985; Huang, 1985, 1987, 1992a; Huang and Zhang, 1986, 1987, 1989a, 1989b, 1989c, 1990a and Zhang and Huang, 1987, 1988). We have focus on the following:

(1) The concepts of pseudo-energy and its influence function are established. A constant available almost in each of the models is a conservative quantity, i.e., pseudo-energy, which possesses some properties of energy, with other dimension than that for energy. Generally the pseudo-energy of the nonlinear gravity inertial wave is in close relation to the usual form of energy, but the pseudo-energy of the nonlinear Rossby wave to the energy of potential vorticity. Analysis of the pseudo-energy and its influence function can help to determine the existence condition and extent of all the sorts of wave solutions. In discussing the orbit around the equilibrium point of the phase diagram the concept of generalized solution (piecewise continuous solution) is introduced, revealing the sudden change of the phase trajectory so as to obtain the discontinuous condition. It will undoubtedly help us understand the structure of a squall line and its genesis mechanism. The decay of the system will give rise to a discontinuous solitary wave solution, which is, in turm, compared with a real squall line development, showing that both have much in common. Generally, it is difficult for the solitary wave solution to exist. It should be pointed out that when the pseudo-energy influence function produces tiny changes, because of an external source, there will be solitary wave solution, different from the KdV equation. Because of the complexity of the nonlinear problem, it is hardly possible to seek an analytic solution. Instead, Taylor expansion was used to obtain the approximate solution more often than not. This method, however, has certain limitations. The approximate solution has been derived by the function-fitting of the influence function

curve.

(2) Establishment of the wave velocity formula and introduction of criterion M. The existence conditions and analytic expressions of wave solutions are obtained by use of the pseudo-energy and its influence function, and then by means of the relation between roots and coefficients of the equation of pseudo-energy influence function and inequality satisfied by pseudo-energy, a non-dimensional quantity M is introduced, which indicates the ratio of the external parameters to wave amplitude. For a small disturbance, whose wave amplitude is limited and the external parameters are given, we find $|M| \to +\infty$, and for a finite amplitude wave, we have $|M| > \frac{2}{3}$, which gives us a clue to make the existence condition of a periodic solution in the form of $|M| > \frac{2}{3}$. For $|M| \to \frac{2}{3}$, we get a solitary wave solution. Thereupon we establish a nonlinear wave velocity formula, suggesting that wave velocity c bears relation with amplitude, characteristic divergence (or characteristic vorticity) β, f and M on a diagnostic basis. During the change from the nonlinear wave velocity expression to the linear one, both are close to each other for $|M| \gg 1$ and different greatly for $|M| < 1$, so that the issue is unable to be treated within the linear framework. Thus, $\frac{2}{3} < |M| < 1$ is considered to be the domain of nonlinear effect, meaning that the nonlinear effect is innegligible; when $|M| \to \frac{2}{3}$, the system produces a solitary wave, which is thought to be the limit of nonlinear effect. And the related M is referred to as the criterion M, which serves as a measure of nonlinear and linear effect.

(3) The possibility of Taylor expansion along with the associated problems. For nonlinear wave, it is highly difficult to obtain an analytic solution, and for this reason, the Taylor representation is done of it, dropping high-order terms for an approximate solution. But Taylor expansion brings about a series of problems. First, the amplitude of the solution to the original problem is changed. Second, the extreme value point is away from the origin, thereby making the approximate and analytic solution differ greatly. Third, in considering the solitary wave solution by use of Taylor expansion, the allowable condition of the original Taylor expansion is damaged thereby. It is under such circumstances that we propose the approximate method for pseudo-energy influence function, so that a lot of similarities between the approximate and analytic solutions is preserved. On the other hand, the possibility of Taylor expansion is demonstrated. It should be noted that the Taylor expansion obeys certain conditions and if a periodic solution results from Taylor expansion, it does not mean that the original system must involve such a solution. So we have to explore the condition under which the requirements for the periodic solution existence do not affect the Taylor expansion of the problem. Finally we have to demonstrate the possibility of Taylor expansion in a strict sense together with its applicability.

III. ASPECTS OF MONOTONOUS AND NONMONOTONOUS TRAVELLING WAVE

It is known that sharp rise of air pressure occurs after the passage of a squall, and is mathematically called a discontinuous jump and often compared to the water jump in hydromechanics as well as the shock wave in aerodynamics. Tepper (1955) is the first to address the pressure surge of squalls in this way, and since then, many scientists have applied this idea in their shallow water models and in an attempt to solve specific problems (Houghton, 1969; Li, 1976). Generally, the theoretical results coincide with observations.

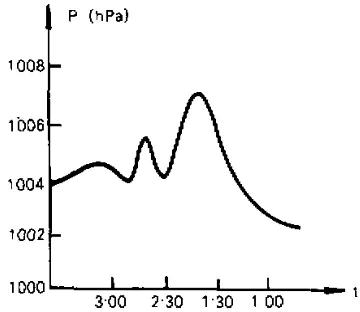

Fig.1. The autographic pressure curve of a squall passing over Suzhou at 0100 BST, 28 April, 1983.

However, the variation of air pressure during a squall passage is by no means so simple as a jump. The feature has a subtle structure, as shown in Fig.1. Besides, there is a remarkable oscillation after the sharp rise and obviously, such a structure cannot be perfectly described if the surge is simply looked upon as a discontinuity. To demonstrate this, we consider one–dimensional shallow water model with dispersive and dissipative effects included (Huang and Zhang, 1991). The model is in the form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g^{*}\frac{\partial h}{\partial x}+\frac{1}{3}\eta\frac{\partial^{3}h}{\partial x\partial t^{2}}-\nu\frac{\partial^{2}u}{\partial x^{2}}=0\ , \tag{1}$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+h\frac{\partial u}{\partial x}=0\ , \tag{2}$$

where u is the wind velocity in the x direction, h the altitude of the fluid at the second layer, $g^{*}=g\frac{\rho_2-\rho_1}{\rho_2}$ the reduced acceleration of gravity, $\frac{1}{3}\eta\frac{\partial^3 h}{\partial x\partial t^2}$ the dispersive term and $-\nu\frac{\partial^2 u}{\partial x^2}$ the dissipative term. Suppose the intensity of the two layers of incompressible fluid to be ρ_1 and ρ_2 ($\rho_2>\rho_1$), Separately. We shall now employ the shooting technique to examine the existence of monotonous (wave front) and nonmonotonous (oscillatory type) travelling wave solution. Set

$$\theta=-(x-ct)\ ,\quad u=U(\theta)\ ,\quad h=h_0+H(\theta)\ ,\quad Z=\frac{H}{h_0}\ ,\quad F=\frac{c_0^2}{c^2}\ ,\quad c_0=(g^{*}h_0)^{\frac{1}{2}},$$

and then (1) and (2) change into

$$\frac{1}{3}\eta h_0\ddot{Z}+\frac{\nu}{c}\frac{\dot{Z}}{(1+Z)^2}+(F-\frac{1}{1+Z})Z+\frac{Z^2}{2(1+Z)^2}=0\ . \tag{3}$$

Also, set

$$\xi=\frac{\sqrt{3}\,\theta}{\sqrt{\eta h_0}}\ ,\quad m=\sqrt{\frac{3}{\eta h_0}}\frac{\nu}{c}\geqslant 0$$

and "$\cdot$" $=\frac{d}{d\theta}$, "$'$" $=\frac{d}{d\xi}$.

and (3) becomes

$$Z'' + \frac{m}{(1+Z)^2} Z' + \frac{F(1+Z)^2 - \frac{1}{2}(1+Z) - \frac{1}{2}}{(1+Z)^2} \cdot Z = 0 \ , \tag{4}$$

which is then changed into the following equivalent nonlinear ordinary differential systems

$$Z' = P \ , \tag{5}$$

$$P' = -\frac{m}{(1+Z)^2} P - \frac{F(1+z)^2 - \frac{1}{2}(1+Z) - \frac{1}{2}}{(1+Z)^2} Z \ , \tag{6}$$

Clearly, the equilibrium points of systems (5) and (6) are (0,0) and $(Z_1, 0)$, where $Z_1 = \frac{1+\sqrt{8F+1}}{4F}$.

Then (0,0) is found to be the saddle point, and $(Z_1, 0)$ falls into three cases:

1°) For $0 < m^2 < m_0^2 \triangleq 4Z_0(FZ_0^3 - 1)$, where $Z_0 = 1 + Z_1$, $(Z_1, 0)$ is the stable focus.

2°) With $m = 0, (Z_1, 0)$ is the center.

3°) When $m^2 \geqslant m_0^2$, $(Z_1, 0)$ is the stable node.

Case 1: $m = 0$ (i.e., $\nu = 0$, meaning no viscosity) indicates the realization of the link between unstable and stable manifold of the saddle point, i.e., homoclinic orbit. This case gives a solitary wave solution, centered around the $(Z_1, 0)$ point, corresponding to a periodic solution, together with the expression of the analytic solution.

Case 2: $m^2 \geqslant m_0^2$, in which the link between saddle and node points is achieved by the shooting technique. With $m_0^2 = 4Z_0(FZ_0^3 - 1)$, meaning greater viscosity, a monotonous travelling wave solution is available.

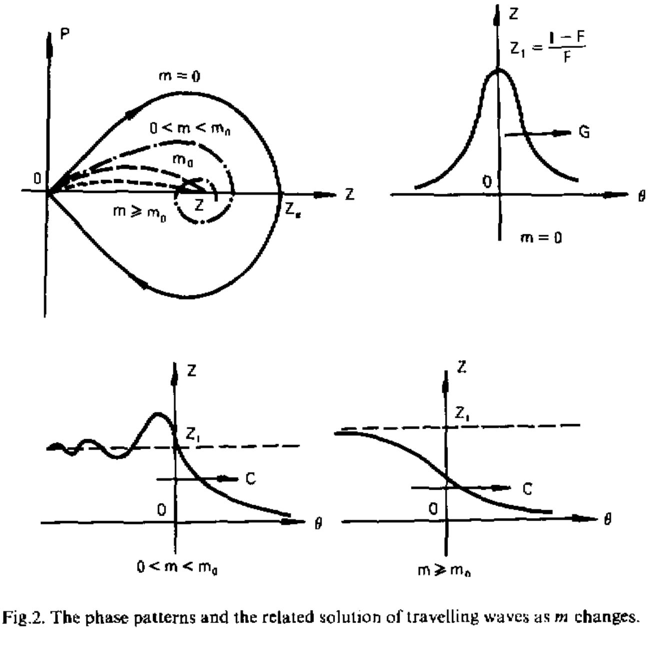

Fig.2. The phase patterns and the related solution of travelling waves as m changes.

Case 3: $0 < m^2 < m_0^2$ (for weaker viscosity than m_0) helps to achieve the link between the saddle and focus points, suggesting the existence of an oscillatory travelling wave solution.

The above discussion shows that with m = 0, corresponding to $\nu = 0$ for no dissipation, there exists a solitary wave solution in the system, with its maximum amplitude $\hat{H} = h_0 \frac{1-F}{F}$, and the velocity fo the wave $c = [g^*(\hat{H} + h_0)]^{\frac{1}{2}}$, in which case the greater the amplitude, the higher the velocity; for a weak dissipation $0 < m < m_0$, there exists an oscillatory travelling wave solution. When the external parameter h_0 is fixed, for travelling waves of the same phase velocity, the smaller the viscosity (the smaller the m), the greater the maximum altitude of the disturbance Z_β and its gradient (Fig.2). When m increases to $m \geqslant m_0$, or the viscosity $\nu^2 \geqslant \eta \frac{h_0 c_0^2}{3} \cdot 4Z_0(Z_0^3 - \frac{1}{F})$, a monotonous travelling wave solution occurs, with the maximum height of the disturbance fixed at $Z_t = \frac{1-F}{F}$, and its qradient being smaller and smaller until it approaches zero (when $m \to +\infty$) as a function of the increase of viscosity.

Further, the possibility is investigated of the existence of nonlinear Rossby travelling wave solution with Rayleigh friction included and with static flow as the background in the semi-geostrophic approximation (Huang and Xiang, 1992). The model consists of

$$\frac{Du_g}{Dt} - fv = -\frac{\partial \varphi}{\partial x} - \mu u_g \ , \tag{7}$$

$$\frac{Dv_g}{Dt} + fu = -\frac{\partial \varphi}{\partial y} - \mu v_g \ , \tag{8}$$

$$\frac{D\varphi}{Dt} + (c_0^2 + \varphi)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \ , \tag{9}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$, and $f = f_0 + \beta y$, with f and β constant, $\varphi = g^* h$ is the geopotential disturbance, $c_0^2 = \bar{\varphi} = g^* \bar{h}$ is the velocity of gravity wave in a static fluid and u_g and v_g are the geostrophic winds in two dimensions, which satisfy

$$u_g = -\frac{1}{f_0}\frac{\partial \varphi}{\partial y} \ , \qquad v_g = \frac{1}{f_0}\frac{\partial \varphi}{\partial x} \ . \tag{10}$$

Set

$$u = U(\xi), \quad v = V(\xi), \quad \varphi = \Phi(\xi), \quad \xi = kx + ly - \nu t \ ,$$

$$\lambda = (k^2 + l^2)^{-\frac{1}{2}}, \quad Z = \frac{\Phi}{c_0^2}, \quad A = -\frac{\lambda^2 \beta k}{\nu} > 0, \quad m = \frac{\mu}{\nu}, \quad \theta = -\left(1 + \frac{f_0^2 \nu}{\beta k c_0^2}\right),$$

where, with $\nu < 0$ we have $A > 0$ and $m < 0$. Our focus will be on the case $\theta > 0$, in which (7), (8) and (9) are changed into

$$Z'' - m(1+Z)Z' + A(Z - \theta)Z = 0 \tag{11}$$

where the prime "$'$" $= \frac{d}{d\xi}$. In dealing with the likelihood of travelling wave solutions, we employ the shooting technique in combination with the method for a degenerate equation, achieving the proof that monotonous and nonmonotonous travelling wave solutions exist in this system, and with the aid of a set of selected parameters, the analytic expression of the monotonous travelling wave solution is found. When $m = \overline{m}_0 = -2\sqrt{\frac{A}{\theta+2}}$, we have

$$\Phi = c_0^2\theta[1 + e^{-\sqrt{\frac{A}{\theta+2}}\theta(kx+ly-\nu t)}]^{-1} . \tag{12}$$

From the above analysis one can see that in the semi-geostrophic model, there does exist a nonlinear slowly travelling wave solution, whose structure is related to friction. For $\mu = 0$, i.e., without friction, the system has only a periodic solution and a solitary wave solution; for $0 < \mu < \mu_0$, indicating smaller friction, the system has an oscillatory travelling wave solution; for $\mu \geqslant \mu_0$, it has a monotonous travelling wave solution with μ_0 satisfying

$$\mu_0^2 = \frac{4k^2}{k^2+l^2} c_0^2 \cdot \frac{\beta^2}{f_0}\left(1 - \frac{c_0^2}{c_x^2} \cdot \frac{\beta}{f_0^2}\right), \tag{13}$$

where $c_x = -\frac{\nu}{k}$, $c_x^*\left(= \beta\frac{c_0^2}{f_0^2}\right)$ is the critical wave velocity, so that $c_x \geqslant c_x^*$. Now we shall discuss the limiting relations among the parameters.

Case 1: With k, l, ν and c_0^2 fixed, and β and f_0 variable, the results are as follows. At high latitudes $\varphi \sim \frac{\pi}{2}$; at tropics $\varphi \sim \varphi_2 = \arcsin[\frac{r\sqrt{r^2+4} - r^2}{2}]^{\frac{1}{2}}$, where $r = \frac{c_0^2}{2\Omega R c_x}$, Ω, is the earth's rotating angular velocity and R is the radius of the earth, during which case μ_0 is small enough for a monotonous travelling wave solution; and at midlatitutes $\varphi \sim \varphi_0$, where $\varphi_0 = \arcsin\frac{1}{\sqrt{x_0}}$, in which x_0 is an unique real root of the equation $\frac{x}{x-1} = \frac{r^2}{4}(4x-1)^2 (x > 1)$, and in this case μ_0 becomes great enough for μ to satisfy $\mu < \mu_0$, leading to an oscillatory travelling wave solution. It is predictable that for midlatitude when a front passes by some physical quantities will exhibit oscillatory damping and reach a constant in the end.

Case 2: Suppose k, l, β, f_0 and c_x are fixed. Then we only consider the relation between μ_0^2 and c_0^2. Denote $c_0^* = f_0\sqrt{\frac{c_x}{\beta}}$ as the limit of c_0 and we find that for c_0 close to 0 or c_0^*, μ_0 is quite small, meaning that the system tends to give a monotonous travelling wave solution; for c_0 close to $\frac{c_0^*}{2}$, μ_0 is largest, causing the system tends to produce an oscillatory travelling wave solution.

Case 3: Suppose k, l, β, f_0 and c_0^2 are fixed, and we have $c_x \geqslant c_x^*$, when c_x approaches c_x^*, μ_0 is very small so that it is possible for the system to produce a monotonous travelling wave solution; for $c_x \gg c_x^*$, μ_0 is great enough to cause an oscillatory travelling wave solution.

In the two examples we demonstrate the possible existence of solutions to monotonous and nonmonotonous travelling waves in the real atmosphere. Up to now, the problem of atmospheric nonlinear wave is known to have been just attacked. It is believed that the research into the travelling wave solution will no doubt lead to the discovery and interpretation of a wide range of complex weather phenomena.

IV. DEVELOPMENT OF TWS RESEARCH

It is known that many processes in chemistry, physics, biology and atmosphere display oscillation with the disturbance propagating at a finite velocity. Interestingly, the travelling wave solution (TWS) in the form $u(x,t) = U(x - ct)$ (c being constant) shows exactly the double property. It is of much importance to study the existence, uniqueness and stability of TWS appearing in many physical models and to find the analytic solutions under the certain conditions.

Fisher (1936) established the mathematical model describing the evolution of dominant gene that takes the form

$$u_t = u_{xx} + f(u) \ , \tag{14}$$

where $f(u) = u(1 - u)$, leading Fisher to guess that Eq.(14) had the wave front solution of $c^* = 2$. Kolmogorov et al. (1937) first proved the existence of the wave front solution by use of a phase plane technique. Later on, Roth et al. (1975) studied the generalized Fisher equation in the form of $f(u) = u(1 - u)(u - a)$ where $0 < u < 1$, indicating that there did exist solution to monotonous and nonmonotonous TWS, the latter including oscillatory and trigger types.

In recent years study of solutions to TWS in various scopes of learning has been widely undertaken with the extension from a single equation to a system from semi-linear to nonlinear and even the degenerate equations. The methods used are other than the phase plane technique. The main methods are outlined below.

1. *The shooting technique*

It is based on the qualitative theory of ordinary differential equations and analysis of the conditions for stable and unstable manifolds at singular points, when combined with topology, the shooting can reveal the existence of TWS and make an estimate of the wave velocity c. For phase plane problems Poincare-Bendixson theorem can be used for semi-linear, nonlinear and degenerate equations (Huang, 1988; Hadeler 1975; Ye et al. 1990; Zhang et al., 1990; Engler, 1985; Wang et al*., Wang 1992). For a problem in a 3D phase space, where the above theorem does not hold, we have to tackle it in combination of the shooting and the topological method for the TWS (Huang, 1991). When the shooting is employed with the singular point stable and unstable manifolds considered, the issue of 4D phase space should be further tackled with the aid of the related results of the ordinary differential equations or a topological technique in order to find the existence of a TWS (Field 1979; Tyson, 1976; Troy, 1980).

2. *The upper and lower solution method*

Berestycki (1983, 1985) is the first to treat the wave front solution (WFS) as the problem of the upper boundary value in a bounded domain when the latter approaches its limit at infinity. Subsequently Ye et al. (1990a) indicated the existence of the WFS by this method with their further study demonstrating the existence of the strictly monotonous TWS (see Refs. Kozjakin et al., 1987, Wang, 1988; Ye et al., 1989).

3. *Singular perturbation method (Asymptotic analysis method)*

Therewith Fife (1977, 1979) investigated the existence of the WFS. Later Hosono and Mimura (1982), Hosono (1987), Wu* and Huang* made further study in this respect and

Wu* documented in terms of the Mimura technique (1982), the TWS to a cross-diffusion system with small parameters that is degenerated and nondegenerated.

4. *Conley index method*

It is the extension of Morse index technique. In recent years, the Conley index method has been developed in both theoretical and practical aspects, especially in dealing with the existence of TWS (Gardner, 1984). When this method is used, the solution to the problem is viewed as semi-flow in the Banach space so as to be dealt with in a topological method. The main idea is based on the homotype invariability of Conley index to transform the equations "with the index kept constant" into a system of standard equations, of which the Conley index is easy to find, thereby demonstrating the existence of TWS (Gardner, 1984; Smoller, 1983).

5. *Method for analytic solutions*

In general, it is quite difficult to find an analytic solution to a nonlinear equation. The following are the major methods used in seeking TWS in many frontiers.

i) Technique for the form to be determined. It is for the form of a TWS to be determined based on the properties of the equations. The problem is transformed into a system of nonlinear algebraic equations, where from the expression of an accurate solution is achieved with the aid of a set of suitable parameters selected (Ye et al., 1990a; Wang et al**., Ye and Li, 1990; Huang 1988; Huang et al., 1992; Wang, Z.Y., 1988).

ii) Painleve analysis approach. This method was reproposed by Ablowitz et al. in the 1970s. It is conjectured a nonlinear partial differential equations is IST soluble if and only if every nonlinear ordinary differential equations obtained by a reduction method is of P type or of the type after variables transformation, scil. all solution to the ordinary differential equations have no movable critical points. Mcleod (1983) derived a number of results under quite common assumptions. Later, Weiss et al. (1983) extended this analysis approach to nonlinear partial differential equations. Guo and Chen (1989, 1991, 1985; Chen et al*,**) showed that this method can produce an auto-Backlund transformation for non-trival solutions although these equations are not of P type nor integratable.

iii) The algebraic method. With the phase angle function introduced, partial differential equations are changed algebraically into ordinary differential equations and put in the matrix form $A(U)\frac{dU}{d\theta}=B(U)$. The sufficient and necessary condition for solving $\frac{dU}{d\theta}$ is rank $[A(U),B(U)]=$ rank[A(U)]. From this, we get a group of algebraic conditions and by using some special techniques, an analytic solution can be obtained (Sacholev et al., 1990).

In addition, it is an important topic to explore the nonlinear stability, asymptotic stability and decay estimate of TWS, of which the last includes the algebraic decay order $O(|t|^{-\alpha})$ (where $\alpha>0$) and the exponential damping rate $e^{-\beta t}$ (where $\beta>0$) as $t\rightarrow+\infty$. There is a variety of techniques, e.g., of nonlinear operator semi-group, energy estimate, maximum principle, spectrum analysis of linear problems and many others. Because of space limitation, the contributions are not enumerated here.

V. CONCLUDING REMARKS

In the preceding sections we have outlined the research on travelling wave solutions in the barotropic atmosphere and current achievement of this issue at home and abroad. At present, the study of the problem is just beginning. Quite a lot of problems remain unattacked. The examples may be the stability of travelling wave solutions, the estimate for maximum and minimum velocities and the regularity of travelling wave solutions, etc.. It is believed that such a study will help to elucidate and discover other nonlinear problems of the atmosphere, and thus contribute to atmospheric sciences.

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