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Predictability of the 500 hPa Height Field

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ABSTRACT

The dimensions of attractors and predictability are estimated from phase space trajectories of observed 500 hPa height over the Northern Hemisphere. As a first estimate the dimensions of attractors are about 11.5 and the doubling time of the initial error is 6 to 7 days for original data. But the former is shorter and the latter is longer for low frequency data set.

To verify if the predictability estimated by this method and by general circulation model is identical, the doubling time of the initial error of a model data set by both methods is estimated. It is shown that the predictability obtained from phase space trajectories is overestimated to sufficient small initial error. But it is underestimated to the time being equal to the climatological RMS error.

Key words: Predictability, Attractor, 500 hPa height

I. INTRODUCTION

It is well known that prediction of instantaneous weather patterns at sufficiently long range is impossible because of the instability of the atmosphere with respect to perturbations of small amplitude. It results in the research of atmospheric predictability.

In general, there are two methods to investigate predictability. The first one is to use numerical models. Most estimates of this method are based on numerical integrations of systems of equations of varying degrees of complexity, starting from two or more rather similar initial states. Predictability is defined by the rate of amplification of small errors in terms of a doubling time. This method is straight and has been extensively adopted. The results, however, depend greatly upon the model itself. Are such estimates identical with real atmospheric predictability? Scientists have been looking for another method, i.e., analysing real observational data, to investigate atmospheric predictability. Lorenz (1969) tried to find some similar initial states in real observations and to examine their time evolution. It is estimated, however, that more than one-hundred-year long data set is required for finding two near states including wind, temperature and some other elements in the whole troposphere. Later, Fraedrich (1986, 1987) studied the rate of divergence of initially close pieces of trajectories evolving from attractors by using a data set of the time evolution of the weather or climate system. This method is easy to practise (see Yang et al., 1990; Zheng et al., 1992). Their estimates of predictability are longer than those of general circulation models (GCM).

The attractor dimensions and predictability of 500 hPa height field over the Northern Hemisphere in the summer season are estimated firstly in terms of Fraedrich's method in next section. To compare these results with the estimates of GCM, we discuss the doubling time of initial error of a model data set by using both methods in Section III. The summary and conclusions are given in Section IV.

II. ESTIMATING ATMOSPHERIC PREDICTABILITY ON ATTRACTORS

1. *Method and Procedure*

By following Fraedrich, the estimates of attractor dimensions are based on distance distributions of pair of points on the single variable trajectory evolving in phase spaces in which the attractor is embedded. Predictability is defined by the divergence of initially close pieces of trajectories and estimated by the cumulative distance distributions. The principle and method are described in detail by Fraedrich (1986, 1987). Here the procedure is only briefly introduced as follows.

Now consider the time evolution of a single state variable

$$X_i = X(t_0 + i\Delta t) \ , \qquad i = 1,2\cdots M, \tag{1}$$

where t_0 denotes the initial observational time, Δt is the time interval, M is the sample size or the length of time series. Let us define a time lag, τ, reconstruct a m-dimensional phase space, R^m , and renew the original dynamical system, i.e.,

$$\begin{gathered} X_m(t_i) = \{X(t_i), \quad X(t_i + \tau)\cdots X[t_i + (m-1)\tau]\} \ , \\ i = 0, 1, 2 \cdots, M - (m-1)\tau / \Delta t \ . \end{gathered} \tag{2}$$

Here m is called embedding dimension. Consider a pair of points, $X_m(t_i)$ and $X_m(t_j)$, in this m-dimensional phase space of time-lagged coordinates which is large enough to embed the attractor, they are a distance r_{ij} apart which depends on the phase-space dimension m,

$$r_{ij}(m) = |X_m(t_i) - X_m(t_j)| \quad \text{with} \quad |t_i - t_j| > \tau \ .$$

The number $N_m(l)$ of such pairs, whose distance is smaller than the prescribed threshold l, is formally determined by

$$N_m(l) = \sum_{i,j=1}^{M} \theta(l - r_{ij}(m)) \ , \tag{3}$$

where $\theta(\alpha)$ is the Heaviside-function with $\theta(\alpha) = 0$ or 1, if $\alpha < 0$ or > 0. M is the total number of points. The related cumulative distribution $C_m(l)$ is normalized by the total of M^2 pairs of points

$$C_m(l) = N_m(l) / M^2 \ . \tag{4}$$

It represents essentially the probability of which the distance of two points on the attractor in the phase space R^m is less than l. For $M \to \infty$ and $l \to 0$, there is a scaling law

$$C_m(l) \sim l^{d\infty} e^{-m\tau h} \ . \tag{5}$$

Here d_∞ is called the attractor dimension or correlation dimension which can be deduced from the linear slope of the distribution in a $\ln[C(l)]$ versus $\ln(l)$ diagram

$$d_\infty = \ln[C_m(l_2) / C_m(l_1)] / \ln[l_2 / l_1] \ , \tag{6}$$

if the dimension m is high enough, ($m > m_\infty$) that the attractor is embedded in the phase space of m_∞ time- shifted coordinates. m_∞ is called embedding saturation dimension, and generally m_∞ is chosen to be $2d_\infty + 1$ to d_∞^2. h in Eq.(5) is the sum of positive Lyapunov characteristic exponents, a measure of the mean divergence of pieces of trajectories. It can be estimated by the following formula

$$h \sim \frac{1}{\tau_k} \ln[C_m / C_{m+k}] \ . \tag{7}$$

The inverse value, $P = 1 / h$, defines a mean time scale up to which predictability may be possible, if e-folding volume expansion is considered. The doubling time of volume expansion, therefore, can be calculated by $P' = P \cdot \ln 2$.

2. *Data*

The data set used in this study consists of 15 years (1967–1981) of the spherical harmonics for the Northern Hemisphere 500 hPa height field (only symmetric harmonics are available for obvious reasons). This data set was provided by Dr. Roads of the Scripps Institution of Oceanography. The summer season defined to be the 123–day period from 15 May to 14 September is studied in this paper. So the total sample size is $M = 1845$

To investigate if the low frequency fluctuation has longer predictability, a low–pass filter (Blackmon, 1976) through which the wave of more than 10–day periods can only be passed is used to filter the original data. We also extract the summer season in the filtered data set. Therefore, another time series of $M = 1845$ is obtained. According to the definition of spherical harmonics, the fields of 500 hPa height in physical space, a grid of 16 Gaussian latitudes and 64 equally–spaced longitudes, can be easily transformed.

3. *Results*

The dimensions of attractors as a function of the embedding dimension at (30°N, 180°E) for low frequency data are shown in Fig.1. The saturation dimensions of attractors d_∞ are 8.4–10.5. Fig.2 shows the cumulative distribution functions $C_m(l)$ for a sequence of embedding dimensions ($m = 4,6,8,\cdots,30$, increasing from left to right). According to $m > m_\infty$ we choose $m = 26$–30 to calculate the predictability. The attractor dimensions d_∞ and the

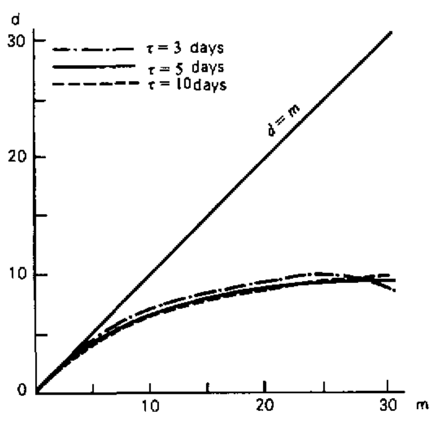

Fig.1. Dimensionality of the attractor as a function of the embedding dimension at (30°N, 180°E). The thin, heavy and dotted curves represent the lag τ = 3,5 and 10 days, respectively.

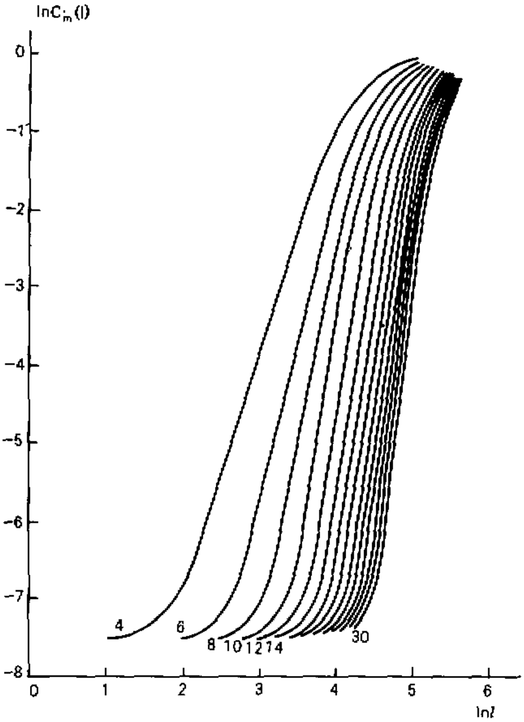

Fig.2. Cumulative distribution function $C_m(l)$ for a sequence of embedding dimensions with $\tau = 5$ days for low frequency data.

predictability of e-folding phase space volume expansion for original and low frequency data at (30°N, 180°E) and (60°N, 180°E) are listed in Table 1. So the conclusions can be summarized as follows:

(1) The results for middle and high latitudes show almost no difference.

(2) The dimensions of attractors of the unfiltered data are larger than those of low frequency data. And the low frequency variable leads to greater predictability.

(3) The dimensions of attractors show not too much difference when $\tau = 3 - 10$ days.

(4) The time scales of predictability increase with τ. If $\tau = 3$ days, the time scales of e-folding (or doubling) volume expansion are equal to 8 to 10 days (or 5 to 7 days) for original data, but equal to 11 to 14 days (or 7 to 10 days) for low frequency data.

Table 1. Estimates of Attractor Dimensions (d_∞) and Time Scales of Predictability for the Original and Filtered Data

		$\tau = 3$ days		$\tau = 5$ days		$\tau = 10$ days	
		d_∞	P (days)	d_∞	P (days)	d_∞	P (days)
(30°N, 180°E)	Original	10.9–11.4	8.3–9.2	11.1–11.6	17.9–19.2	10.9–11.5	23.6–24.4
	Filtered	8.4–8.8	10.8–14.4	8.5–9.0	21.5–24.6	9.7–10.5	29.8–32.3
(60°N, 180°E)	Original	12.0–12.6	8.7–10.2	11.6–12.6	14.6–17.5	10.5–11.8	23.9–25.2
	Filtered	8.5–8.9	11.3–12.2	9.7–9.9	19.3–22.9	9.7–10.6	28.0–31.5

III. PREDICTABILITY OF MODEL DATA

The error doubling time obtained above and those estimated based on the attractors by others (see Fraedrich, 1987; Yang et al., 1990; Zheng et al., 1992) are longer than the doubling time of small errors revealed by GCMs. Is this discrepancy caused by the different methods? A model data set will be used to estimate the error doubling time by both methods in this section.

The model data set used here is the stream-function integrated by a quasi-geostrophic two level spectral model and triangularly truncated at 21-wavenumber, which also is provided by Roads. The doubling time of small errors of this model data set has been analyzed in detail by Roads (1987). For simplicity, the barotropic component of the stream-function is only studied here. The total sample size is $M = 3600$.

1. *Estimating Model Weather on Attractors*

Table 2. The Attractor Dimensions (d_∞) and Time Scales of Predictability P and P' at (30°N, 180°E) of Model Data

τ (days)	d_∞	P (days)	P' (days)
3	12.5–13.2	8.2–8.6	5.7–6.0
5	13.1–13.8	12.6–14.9	8.7–10.3
10	13.3–14.1	25.2–26.8	17.5–18.6

The estimates of attractor dimensions (d_∞) and time scales of predictability (P and P') of model data are listed in Table 2. The predictability time scale, P, is a measure of the mean e-folding phase space volume expansion, and $P' = \ln 2 \cdot P$ for volume doubling. The attractor dimensions, $d_\infty = 12.5 - 14.1$, are larger than those of real data mentioned above. The larger embedding dimensions, therefore, are needed for calculating the predictability. Here we use $m = 30 - 34$. Being similar to the results of real data, the dimensionality of attractors has not too much difference for $\tau = 3 - 10$ days. The time scales of predictability increase with τ. When τ increases from 3 to 10 days, the error doubling time increases from about 6 to 18 days.

2. *Error Doubling Time Estimated by Model*

Due to Roads (1987) has estimated the predictability of this data set, we will directly use his results here. His procedure estimating predictability is briefly introduced as follows. In every 30 days of the 3600-day control run, a prediction of 60 days was made for an ensemble total of 119 forecasts. The prediction was made with an initial state identical to that of the control run except for waves with two-dimensional wavenumbers greater than or equal to 15. For these small scale waves, the amplitude was set to zero which created, in effect, a random initial error. By comparing both forecasting results, one can estimate the growth of small error.

As shown in Fig.3 which is quoted from Fig.9(a) in Roads' paper, the initial error quickly grows. For the barotropic component the initial RMS doubling time, i.e., RMS error increases from 0.00032 to 0.00064, is about two days. This growth rate decreases with increasing error. By day 12 the RMS error is comparable to the RMS of the climatological variance. At this point the RMS doubling time is about 12 days. As the error approaches the

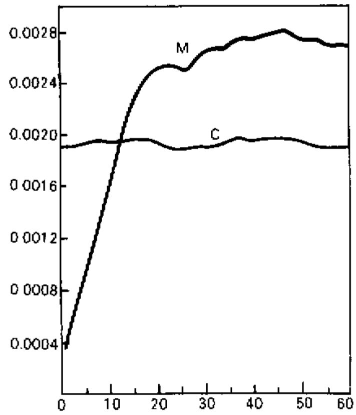

Fig.3. RMS error, E, growing with integrating time. Curve M is the day to day prediction results for the model; C denotes the results for climatological prediction.

saturation limit the doubling time approaches infinity. In other words, the growth rates are different with the magnitudes of initial error. If the initial error is small (large), the growth is quick (slow). Of course, the prediction being less than climatological error is only useful.

According to the estimates on the attractor, the doubling time is about 6 days for $\tau = 3$ days (see Table 2). It is equivalent to the doubling time of the moderate initial error (0.0008) estimated by model. Predictability estimate on the attractor, therefore, is overestimated for the sufficient small initial error. But it is underestimated to the time being equal to the climatological RMS error for $\tau = 3$ days. If τ is chosen to be more than 5 days, the estimate of predictability on the attractor is too high. It seems that the time lag τ can not be chosen too large for preventing the predictability from too overestimates. τ can not be chosen too small either, otherwise the trajectory can not fill the phase space (Fraedrich, 1986). To choose $\tau = 3$ days seems suitable.

IV. SUMMARY AND CONCLUSIONS

The dimensions of attractors and predictability are first estimated from phase space trajectories of the 500 hPa height over the Northern Hemisphere. It is shown that the effect of time lag, τ, on the attractor dimensions is negligible, if τ is chosen to be more than or equal to 3 days. The dimensions of attractors for the original data set are about 11.5, but they become about 8.5 for the low frequency data. Predictability, however, depends on the lag time. The larger the τ, the longer the time scale estimating predictability is. When $\tau = 3$ days, the error doubling times are about 6 to 7 days for the original data, and they increase to 8 to 10 days for the low frequency data. It is in agreement qualitatively with the results of Yang et al. (1990) and with the facts that the time-averaged forecasts show greater skill than the instantaneous long range forecasts.

The estimates of attractor dimensions here are larger than those of Yang et al. (1990) and Zheng et al. (1992). The reason probably is that the data of the whole Northern Hemisphere rather than a single station are used. So the more independent numbers of variable simulating dynamics will be required. The length of the data set used here, $M = 1845$, is short for the attractor dimensions obtained. In terms of an independent estimate of attractor dimensionality by empirical orthogonal functions described by Fraedrich (1986), however, the accurate modes associated 95% confidence interval are roughly equal to 10 for our data set. So the sample size used here is marginal.

To verify if the predictability estimated by the doubling phase space volume expansion and by the model error doubling time is identical, we compare the estimates of both methods by using a model data set. The results show that both estimates are different from each other. If lag time is not chosen too large, the predictability estimated from phase space trajectories is equivalent to that of moderate initial error. It is overestimated for sufficient small initial error, but underestimated to the time being equal to the climatological RMS error. If proper modification is taken into account, the method of attractors is still a useful tool to estimate real weather predictability.

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