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Vertical Structure of Beta Gyres and Its Effect on Tropical Cyclone Motion①

Tian Yongxiang (田永祥) and *Luo Zhexian* (罗哲贤)
Nanjing Institute of Meteorology, Nanjing 210044

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ABSTRACT

Recently, Fiorino and Elsberry (1989) proposed a concept on beta gyres using a numerical model in a quasi-geostrophic barotropic framework, and analysed the horizontal structure. In this paper, the authors extend the findings of Fiorino et al. to a baroclinic atmosphere applying a quasi-geostrophic, three-layer model, and analyse the horizontal and vertical structures of beta gyres and their relation to tropical cyclone motion.

Key words: Tropical cyclone, Beta gyers, Translation, Baroclinc model, Numerical simulation

I. INTRODUCTION

In the mid 1980s, Chen (1985) showed in his review that the typhoon's structure is an important factor influencing its translation. Later, a series of numerical experimental findings (Chan et al., 1987; Fiorino et al., 1989; Li and Zhu, 1990; Luo, 1991) and reviews (Dong, 1987) about the typhoon's structure and translation have come out. Particularly, the beta gyres and the quasi-uniform flow posed by Fiorino et al. (1989) have received widespread attentions because of their close correlation to the typhoon's movement.

The conclusions obtained by Fiorino et al. (1989) in the quasi-geostrophic barotropic framework are limited to the horizontal structure of beta gyres alone. In this paper, we extend the findings of Fiorino et al. (1989) to a baroclinic atmosphere, and analyse the horizontal and vertical structures of beta gyres and the relations between the structure of tropical cyclone at various model levels and its translation. This extension is evidently significant for gradually applying the ideas and conclusions from simply theoretical models to the operational forecast of typhoon tracks.

II. BRIEF DESCRIPTION OF THE MODEL

We assume that $p = p_0 = 100$ hPa and $p = p_6 = 1000$ hPa are the upper and lower boundary of model atmosphere, respectively. And the model atmosphere is equally divided into three discrete layers with the pressure interval $\Delta p = 300$ hPa (Fig.1).

The quasi-geostrophic vorticity equation and thermodynamic equation can be written as

$$\frac{\partial}{\partial t}\nabla^2\psi = -J(\psi, \nabla^2\psi + f) + f_0\frac{\partial\omega}{\partial p} , \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial p}\right) = -J\left(\psi, \frac{\partial\psi}{\partial p}\right) - \frac{\sigma}{f_0}\omega , \tag{2}$$

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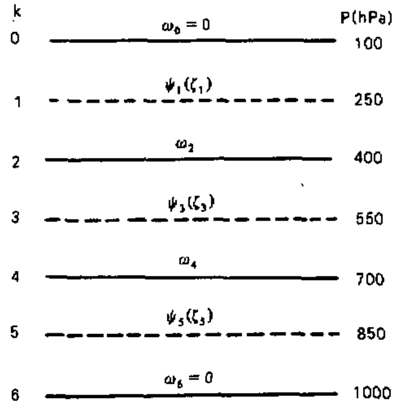

Fig. 1. Arrangement of variables in the vertical for the quasi-geostrophic three-layer model.

where ψ is the streamfunction, $\omega = dp / dt$ the vertical velocity; f the Coriolis parameter, and $f = f_0$ at $\varphi = \varphi_0$ (φ_0 being the latitute at the median of the beta-plane); and σ the static stability parameter. We apply the vorticity equation (1) at the model levels 1, 3 and 5, yielding

$$\frac{\partial}{\partial t}\nabla^2\psi_k = -J(\psi_k, \nabla^2\psi_k + f) + \frac{f_0}{\Delta p}(\omega_{k+1} - \omega_{k-1}), \quad k = 1,3,5 \ . \tag{3}$$

We write the thermodynamic equation (2) for the model levels 2 and 4 to obtain

$$\frac{\partial}{\partial t}(\psi_{k-1} - \psi_{k+1}) = -J(\psi_k, \psi_{k-1} - \psi_{k+1}) + \frac{\Delta p}{f_0}\sigma_k\omega_k \ , \quad k = 2,4 \ , \tag{4}$$

where ψ_2 and ψ_4 must be found by linearly interpolating between their neighboring levels. Deriving from formulas (3) and (4) we can get a quasi-geostrophic Omega equation, i.e.

$$\nabla^2\omega_k - \frac{f_0^2}{\sigma_k(\Delta p)^2}(2\omega_k - \omega_l) = \frac{f_0}{\sigma_k\Delta p}\{-J(\psi_{k-1}, \nabla^2\psi_{k-1} + f) + J(\psi_{k+1}, \nabla^2\psi_{k+1} + f) + \nabla^2[J(\psi_k, \psi_{k-1} - \psi_{k+1})]\}, \quad k = 2,4 \ , \tag{5}$$

here we specify $\omega_l = \omega_{k+2}$ for $k = 2$, and $\omega_l = \omega_{k-2}$ for $k = 4$.

We assume that

$$\psi_k(x,y,t) = \bar{\psi}_k(y) + \psi'_k(x,y,t) \ . \tag{6}$$

Substituting (6) into the equations (3) and (5), respectively, we get a closed set of equations in the dependent variables ψ_k and ω_k for the quasi-geostrophic three-layer baroclinic model, i.e.

$$\frac{\partial}{\partial t}\nabla^2\psi_k = -J(\psi_k, \nabla^2\psi_k) - \beta\frac{\partial\psi_k}{\partial x} + \frac{f_0}{\Delta p}(\omega_{k+1} - \omega_{k-1}) \ , \quad k = 1,3,5, \tag{7}$$

$$\nabla^2\omega_k - \frac{f_0^2}{\sigma_k(\Delta p)^2}(2\omega_k - \omega_l) = \frac{f_0}{\sigma_k\Delta p}\{-J(\psi_{k-1}, \nabla^2\psi_{k-1}) + J(\psi_{k+1}, \nabla^2\psi_{k+1}) - \beta(\frac{\partial\psi_{k-1}}{\partial x} - \frac{\partial\psi_{k+1}}{\partial x})$$

$$+\nabla^2[J(\psi_k,\psi_{k-1}-\psi_{k+1})]\},\quad k=2,4\ , \tag{8}$$

here we have let the basic current be zero everywhere and neglected the primes.

At 550 hPa, the initial vorticity of tropical cyclone is given by (Luo, 1991)

$$\zeta_3(r)=\frac{2V_m}{r_m}[1-\frac{1}{2}\left(\frac{r}{r_m}\right)^b]\exp\{\frac{1}{b}[1-\left(\frac{r}{r_m}\right)^b]\}\ , \tag{9}$$

where $r=\sqrt{(x-x_0)^2+(y-y_0)^2}$ is the radius, (x_0,y_0) the coordinates of tropical cyclone centre, V_m the maximum wind speed, r_m the distance between the vortex centre and the location where V_m appears, and b the factor which determines the shape of the vortex. Solving the Poisson equation

$$\nabla^2\psi_3(x,y)=\zeta_3(x,y) \tag{10}$$

in terms of $\zeta_3(x,y)$, we get the initial perturbation streamfunction $\psi_3(x,y)$ (called as the streamfunction for short, hereafter). We can specify the initial streamfunction at 250 hPa and 850 hPa applying the relationships

$$\begin{aligned}\psi_{k-2}(x,y)&=\psi_k(x,y)+c\psi_k(x,y)\ ,\quad k=3,\\ \psi_{k+2}(x,y)&=\psi_k(x,y)-c\psi_k(x,y)\ ,\quad k=3,\end{aligned} \tag{11}$$

The model parameters are chosen as follows: $\varphi_0=25°\text{N}$; $\sigma_k=0.032\ \text{m}^2\text{s}^{-2}\ \text{hPa}^{-2}$ for k = 2,4, the corresponding vertical lapse rate of temperature $\gamma=0.6℃\ /\ 100\ \text{m}$; $V_m=25\ \text{ms}^{-1}$, $r_m=100$ km, $b=1$; and $c=0.44$.

The model equations (7)–(8) are solved on a square domain in the beta–plane with sides of 2500 km length. The grid spacing is uniform with a horizontal grid increment of 50 km. The literal boundary conditions are $\psi_k=0$ along the northern and southern boundaries, and cyclic continuity in the east–west direction. The vertical boundary conditions are given by

$$\begin{aligned}\omega_0&=0\quad \text{at}\quad p=100\ \text{hPa},\\ \omega_6&=0\quad \text{at}\quad p=1000\ \text{hPa}.\end{aligned} \tag{12}$$

The Jacobian term is evaluated using the finite difference form developed by Arakawa (1966). The equations (7)–(8) are solved applying super–relaxation iteration mathod. The time step of numerical integration is 10 min. The total integrating time is 6 model days.

In the process of numerical calculating, the total streamfunction $\psi_k(x,y,t)$ is divided into an axially symmetric component $\psi_{ks}(x,y,t)$ and an asymmetric one $\psi_{ka}(x,y,t)$, and the quasi–uniform flow between the anticyclonic and cyclonic beta gyres is calculated. The computing procedures are outlined as follows.

1. *The Dividing Procedure of Streamfunction*

First, we determine the tropical cyclone centre O_k in terms of the distribution of *streamfunction* $\psi_k(x,y,t)$, and establish a polar coordinate system with a radial increment of 50 km and an azimuthal increment of 7.5 deg, its coordinate origin being at O_k. Then, interpolating from the model grid to the polar coordinate grid, we get $\psi_k(r,\theta,t)$. And we find the symmetric component at each polar coordinate grid point from $\psi_k(r,\theta,t)$ via

$$\psi_{ks}(r,t)=\frac{1}{N}\sum_{n=1}^{N}\psi_k(r,\theta_n,t)\ , \tag{13}$$

where N denotes the number of grid points in the polar coordinates along a circumference at radius r within the calculating domain ($N_{\max} = 360^\circ / 7.5^\circ$). Third, the symmetric component at each model grid point $\psi_{ks}(x,y,t)$ is specified by interpolating from $\psi_{ks}(r,t)$. Finally, the asymmetric component on the model grid $\psi_{ka}(x,y,t)$ is found by substracting $\psi_{ka}(x,y,t)$ from the total streamfunction $\psi_k(x,y,t)$.

2. *The Calculating Procedure of the Quasi–uniform Flow*

First, a circular area A_k at radius $r = 300$ km around the tropical cyclone centre is chosen (Fiorino et al., 1989). Then, the asymmetric wind speed components at all the model grid points within A_k are found from $\psi_{ka}(x,y,t)$ via

$$u_{ka} = -\frac{\partial \psi_{ka}}{\partial y} \ , \quad v_{ka} = \frac{\partial \psi_{ka}}{\partial x} \ . \tag{14}$$

And the asymmetric wind speed / direction is calculated by using

$$V_{ka} = \sqrt{u_{ka}^2 + v_{ka}^2} \ , \quad \theta_{ka} = t_g^{-1}\left(\frac{v_{ka}}{u_{ka}}\right) \tag{15}$$

respectively. Finally, algebraically averaging $V_{ka}(x,y,t)$ and $\theta_{ka}(x,y,t)$ in the circular area A_k, we obtain $\overline{V}_{ka}(t)$ and $\overline{\theta}_{ka}(t)$. The $\overline{V}_{ka}(t)$ and $\overline{\theta}_{ka}(t)$ are referred as the mean speed and mean direction of the quasi–uniform flow, respectively.

III. THE RESULTS OF NUMERICAL EXPERIMENTS

1. *The Vertical Structure of Beta Gyres*

Starting from an initially circular symmetric vortex (zero asymmetric component), the asymmetric component is growing as the integration goes on. After 48 hours of integration, the structure of beta gyres clearly appears not only at 550 hPa but also at 250, 850 hPa (Fig. 2).

(1) Comparing Fig. 2b (550 hPa) to the results of Fiorino et al. (1989)

In Fig. 2b, the distance d between the two beta gyre centres is 900 km, and the straight line connecting the two centres does not lie in the east–west direction but at 10 deg intersecting angle with the x axis. Referring to the corresponding results at $t = 12$ h and $t = 24$ h of Fiorino et al. (see Figs. 7b, 7c therein), we see that the distance d is 960 and 1040 km, and that the intersecting angle α is 12 and 13 deg, respectively. Notice that the findings of Fiorino et al. are for 500 hPa, and that Fig. 2b in our study is for 550 hPa. Considering the increase in d and α with height, we think that these two findings are quite consistent. In other words, the situation of beta gyres in the mid troposphere in the baroclinic case is similar to the one in the barotropic case. An obvious difference between the two findings appears in the area of the quasi–uniform flow between beta gyres. In the barotrpic case, contours there are quite smooth (see Figs. 7b, 7c therein). However in the baroclinic case, contours there are more fluctuating (Fig. 2b).

(2) The difference in horizontal structures of beta gyres between different model levels

(i) The distance $d$between the two beta gyre centres depends on altitude. It is 700, 900 and 1100 km at 850, 550 and 250 hPa, respectively (Fig. 2).

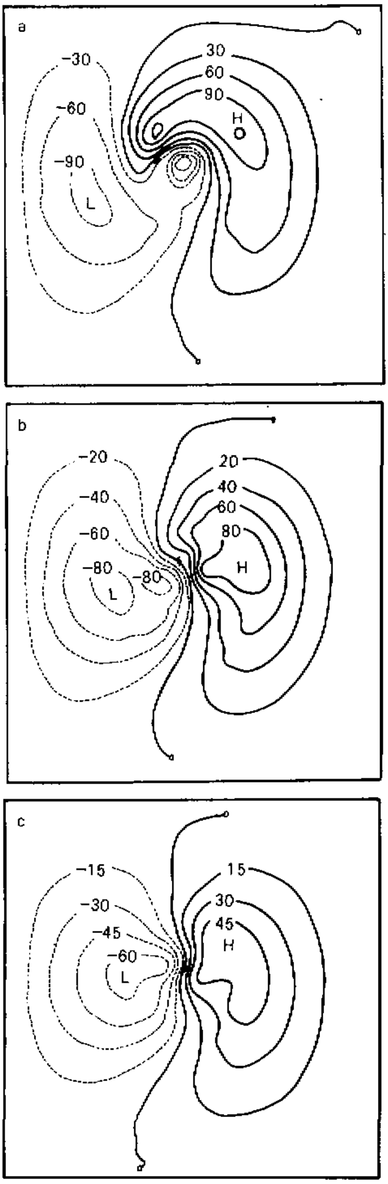

Fig. 2. The asymmetric streamfunction ψ_a at 48 h for (a) 250 hPa; (b) 550 hPa; and (c) 850 hPa. Contour intervals are 30, 20 and 10 (10^4 m^{2}s^{-1}), and the maximum positive values of ψ_a are 120, 97 and 60 (10^4 m^{2}s^{-1}) and the minimum negative values are −97, −94 and −67 (10^4 m^{2}s^{-1}), respectively.

(ii) The relative position of the two beta gyres varies with height. The intersecting angle α between the straight line connecting the beta gyre centres and the x axis can be taken to describe the relative position of beta gyres. The intersecting angle α is 18, 10 and 23 deg for 850, 550 and 250 hPa, respectively (Fig. 2).

(iii) The flow pattern in the quasi-uniform flow area changes with altitude also. The contours in the area at 850 hPa are slightly curved. At 550 hPa, this curve for contours is more evident, while the small-scale gyres occur at 250 hPa (Fig. 2).

(3) The vertical allocation of beta gyres

It can be inferred from Fig. 2 that the vertical axis of both cyclonic and anticyclonic beta gyres is tilted. The vertical axis of cyclonic beta gyre tilts southwestward from 850 to 250 hPa. The vertical axis of the anticyclonic beta gyre tilts southeastward from (850 to 550 hPa and northward from 550 to 250 hPa.

The spatial structure of the beta gyres at $t = 48$ h has been analyzed. The temporal variation of beta gyres will be discussed as follows.

2. *The Evolution of Beta Gyre Intensity*

As shown by Fiorino et al. (1989), during the first 48 hours of integration, the beta gyre amplitude grows with time (see Fig. 8a therein). In the baroclinic case, the variable tendency of beta gyre amplitude at various model levels increases with time also (Fig. 3). The difference between the two findings is that the amplitude grows with time in a monotonic / fluctuating manner in the barotropic / baroclinic case (Figs. 3a, 3b, curve m).

It can be found from Fig. 3 that the time variation of beta gyre amplitude is not synchronous for different model levels. At 850 hPa, for example, the amplitude of the cyclonic beta gyre attains its maximum at $t = 48$h, while at 550 hPa, the beta gyre amplitude keeps growing after $t = 48$h (Fig. 3a). Another example is that the amplitude of anticyclonic beta gyre achieves its maximum at $t = 108$, 96 and 84 h for 850, 550 and 250 hPa, respectively (Fig. 3b).

Another discrepancy in the time variation of beta gyre amplitude for different model levels is that the evolution at low level is fairly smooth and the one at high level is slightly fluctuating (Fig. 3). This discrepancy is consistent with the one of the contour configuratin in the area of the quasi-uniform flow between different model levels in Fig. 2.

3. *The Relationship between the Quasi-uniform Flow and Tropical Cyclone Translation*

As indicated by Fiorino et al. (1989), the translation direction of tropical cyclone at 500 hPa closely correlates to the mean direction of the quasi-uniform flow. After $t = 48$h, the tropical cyclone is moving consistently about 5 deg to the left of the mean direction of the quasi-uniform flow (see Fig. 11b therein). In the baroclinic case, such a correlation remains at 550 hPa. After $t = 48$ h, the tropical cyclone moves also to the left of the mean direction of the quasi-uniform flow, and maximum deviation is in less than 6 deg (Fig. 4b). And at 250 and 850 hPa, this relationship is still valid except that it moves to the left or right of the mean direction of the quasi-uniform flow, sometimes (Figs. 4a, 4c).

In the barotropic case, the translation direction of tropical cyclone varies with time monotonically. The tropical cyclone moves singly to the northwest only (Fiorino et al., 1989). In the baroclinic case, the translation direction of tropical cyclone changes with time in a fluctuating manner (Fig. 4). Nevertheless, there exists still a strong correlation between the trans-

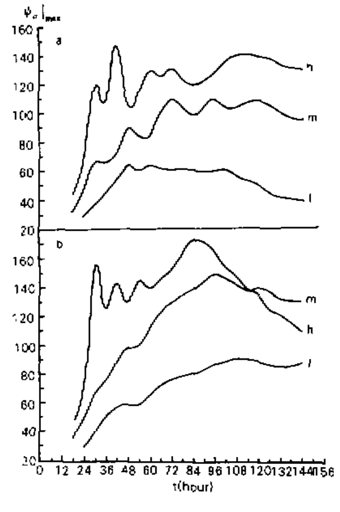

Fig. 3. Time variation of beta gyre amplitude $|\psi_a|_{max}$ (10^4 m^{2}s^{-1}) for (a) the cyclonic beta gyre and (b) the anticyclonic beta gyre, and the model level 850, 550 and 250 hPa labelled by *l*, *m* and *h*, respectively.

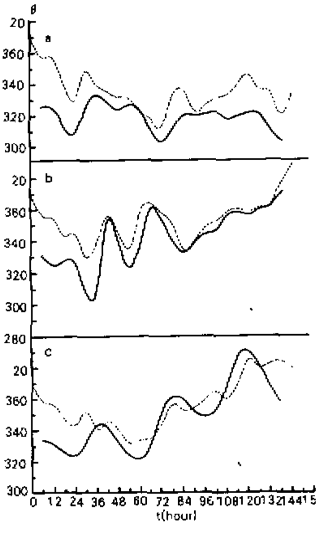

Fig. 4. Time variations of the translation direction of tropical cyclone (solid) and the mean direction of the quasi-uniform flow (dashed) in unit of deg for (a) 250 hPa, (b) 550 hPa, and (c) 850 hPa.

lation direction of tropical cyclone and the mean direction of the quasi–uniform flow.

However, it should be pointed out that the baroclinic model developed in our study is a quite simple one. Some important physical processes like cumulus convection etc. are not included in the model. The vertical transports of physical quantities are considered only partially. Therefore, it causes some differences between the whole structure of tropical cyclone in the model atmosphere and that in the real atmosphere. For instanse, there exist certain differences in the translation direction of tropical cyclone between different model levels (Fig. 4). This is a deficiency of our model. An improvement in the results is likely to be expected when other physical processes are incorporated in the model.

As shown by Fiorino et al. (1989), the translation speed of tropical cyclone at 500 hPa closely also correlates with the mean speed of the quasi–uniform flow, both of them increase with time monotonically (see Fig. 11a therein). In the baroclinic case, the translation speed of tropical cyclone at 550 hPa is associated with the mean speed of the quasi–unform flow. But the correlation between them is weaker than that gotten by Fiorino et al. (1989). In our experiment, the mean speed of the quasi–uniform flow is larger than the translation speed of tropical cyclone. Moreover, both of them vary with time in a fluctuating way (Fig. 5).

Additionally, such a correlation between the speeds at 250 or 850 hPa is also weaker than that at 500 hPa obtained by Fiorino et al. (1989).

IV. CONCLUDING REMARKS

Scientifically understanding of typhoon's movement will provide a physical basis for its track prediction. It is quite necessary to carry out researches theoretically and numerically by using a simple quasi–geostrophic barotropic model. Recently, a considerable amount of new outcome has been obtained. On the other hand, the real atmosphere conditions or the models for typhoon operational forecast are highly complicated. Not all the results from simple model are valid in any complex case. Identifying these results using complicated models is quite difficult. Therefore, it is perhaps an effective approach to identify them by means of models whose complexity increases gradually. This paper is a preliminary attempt in such a aspect.

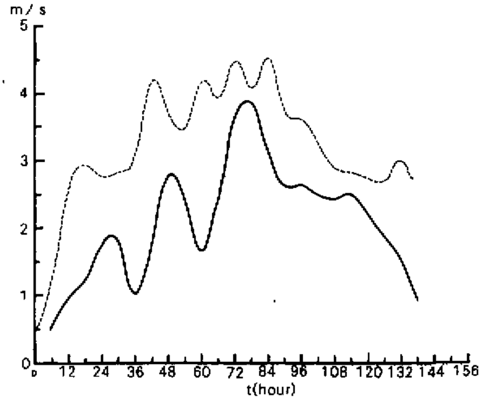

Fig. 5. Time variation of the translation speed of tropical cyclone (solid) and the mean speed of the quasi–uniform flow (dashed) at 550 hPa.

Recently, Fiorino and Elsberry (1989) proposed the concepts on the beta gyres and the quasi-uniform flow in the quasi-geostrophic barotropic framework. This study has received widespread attentions because of its potential applicabiliy to the operational forecast of typhoon track. We have designed a quasi-geostrophic three-layer baroclinic model and verified the relevant conclusions of Fiorino et al. (1989) with it. The results of numerical experiments indicate that the strong correlation between the translation direction / speed of tropical cyclone and the mean direction / speed of the quasi-uniform flow remains / weakens in the baroclinic case. The cause of this result may be in insufficient consideration of the oscillation of typhoon track in the barotropic model.

In the barotropic model, the correlation between tropical cyclone motion and quasi-uniform flow is very close. But both of them vary with time monotonically. In the baroclinic model, this strong correlation between them weakens. However, their variation with time is not monotonic but in a fluctuating manner. A similar phenomenon was found in the study on an abrupt change of seasonal circulations (Luo, 1987). For example, the equilibrium states presenting around the abrupt change of seasonal circulations in the barotropic atmosphere are all in steady states. However, these steady states become periodic states with a flutuating form when the atmospheric baroclinity is included in the model.

The typhoon dynamics in the baroclinic atmosphere is quite complicated. It involves many physical processes. The baroclinic model developed in our study is very simple. Some important physical processes are not included in the model. Moreover, the numerical experiments in our study are the ones only for a part of model parameters. For instance, the parameter c in Eq.(11) describes the features of vertical structure for the initial perturbation vortex. In future, we shall choose different kinds of model parameters to investigate the vertical structure of beta gyres and its effect on tropical cyclone motion.

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