

A Forecasting Model of Vector Similarity in Phase Space for Flood and Drought over the Huanghe-Huaihe-Haihe Plain in China

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ABSTRACT

To represent well the characteristics of temporal and spatial distributions, chart of 3-dekad moving total precipitation is proposed in this paper first. Then this kind of chart is expanded in terms of Chebyshev polynomial at irregular grids, and the quantitative representation of precipitation is got. Finally the Chebyshev coefficients are forecasted by using the forecasting method of vector similarity in phase space proposed by Zhou (1992). Using above mentioned procedures temporal and spatial distributions of precipitation over the Huanghe-Huaihe-Haihe Plain in China are forecasted.

Key words: 3-dekad moving total precipitation, Chebyshev expansion, Forecasting method, Phase space

1. INTRODUCTION

It is said that among the losses caused by various kinds of natural disasters, the one caused by meteorological disasters is the first to be considered. Furthermore, floods and droughts account for the most part of loss caused by meteorological disasters. In China, the floods and droughts over the Huanghe-Huaihe-Haihe Plain are the most serious. The statistical result shown in Table. 1 for the areas suffered from droughts (Bi, 1990) is the evidence for the above statement. And the floods and droughts over the Huanghe-Huaihe-Haihe Plain will be discussed.

Table 1. Areas of Disasters

Region	A(%)	B(%)
HHH	46.5	50.5
MLOC	22.0	19.2
NE China	11.6	11.4
S China	5.2	4.4
SW China	10.0	9.4

HHH-the Huanghe-Huaihe-Haihe Plain.

MLOC-the Middle and Lower Reaches of the Changjiang River.

A-Ratio of regional drought area related to the total drought area over the whole country.

B-Ratio of the disaster area induced by regional drought related to the total disaster area hit by drought over the whole country.

Most previous forecasting models dealt with only the mean precipitation in a region or the rainfall for a representative station. The spatial distribution of precipitation was usually omitted. In this paper the Chebyshev expansion at irregular grids proposed by Zhou et. al. (1982, 1985) is used to describe the distribution of rainfall. Most previous models treated only the precipitation in a period (e.g., monthly or seasonal rainfall) and it was difficult to clarify its temporal variation. The time series of Chebyshev coefficients are used to describe the temporal variation of rainfall distribution in this paper.

The theory on chaos is a new branch of modern sciences. Its application has been paid much attention. Zhou (1992) proposed a new forecasting model of vector similarity in phase space. This method is used to predict the distribution of precipitation over the Huanghe-Huaihe-Haihe Plain in China.

II. REPRESENTATION FOR CHARACTERISTICS FOR TEMPORAL AND SPATIAL DISTRIBUTIONS OF PRECIPITATION—CHARTS OF 3-DEKAD MOVING TOTAL PRECIPITATION

To represent well the characteristics for temporal and spatial distributions of precipitation, charts of 3-dekad moving total precipitation are proposed here, which is obtained from the 10-day precipitation data set at every station. The first 3-dekad moving total precipitation is the total precipitation got from the 35th dekad of last year to the first dekad of this year, the second 3-dekad moving total precipitation is got from the 36th dekad to the second dekad, the third 3-dekad precipitation is that during the first 3 dekads, ..., the 36th 3-dekad moving precipitation is that from the 34th to the 36th dekad.

The advantages of the 3-dekad moving total precipitation are as follows:

- 1) The original data are the ten-day precipitation so that they contain much information than monthly rainfall does.
- 2) The variability of the total 3-dekad precipitation is less than that of ten-day precipitation.
- 3) Owing to the limit of calendar months, the physical characteristics of many rainfall processes can not be detected by monthly rainfall. The same problem occurs for ten-day and pentad rainfall. But all the rainfall processes can be detected well if 3-dekad moving precipitation is adopted.
- 4) Its sample interval is ten days. So the temporal variation of rainfall can be described in detail.

III. QUANTITATIVE REPRESENTATION OF TEMPORAL AND SPATIAL DISTRIBUTIONS OF RAINFALL—CHEBYSHEV EXPANSION AT IRREGULAR GRIDS

Charts of 3-dekad moving rainfall over the Huanghe-Huaihe-Haihe Plain are expanded in terms of Chebyshev polynomials at irregular grids (Zhou et al., 1985). The following formulae are used

$$\tilde{R}(i, j, t) = \sum_{k=0}^{K_0} \sum_{s=0}^{S_0} A_{ks}(t) \varphi_{k,j}(i) \psi_s(j), \quad (1)$$

$$A_{ks}(t) = \sum_{i=1}^{J_0} \sum_{j=1}^{I_2(j)} R(i, j, t) \varphi_{k,j}(i) \psi_s(j), \quad (2)$$

$$(t = 1, 2, \dots, T_0)$$

where (i, j) denotes ordinal of the stations, t represents time, the sample interval is ten days. $R(i, j, t)$ is the percentage of rainfall anomaly at station (i, j) , $\tilde{R}(i, j, t)$ is its fitting, $\varphi_{k,j}(i)$

is the value of normalized Chebyshev polynomial at grid i , and k is the order and j denotes the row, $\psi_s(j)$ is the normalized Chebyshev polynomial of order s at grid j , K_o and S_o are truncation orders of Chebyshev polynomials, and A_{ks} is Chebyshev coefficient.

According to the theory of Zhou (1990), the distribution of rainfall can be expressed by the linear combination of different patterns $\varphi_{k,j}(i)\psi_s(j)$ and Chebyshev coefficient A_{ks} , the weight of the pattern.

The first several Chebyshev coefficients can be interpreted as follows.

A_{00} —Trend of rainfall anomaly in a region. A_{00} with large positive value represents plentiful rainfall in the region, A_{00} with negative sign and large absolute value denotes deficient rainfall, and A_{00} close to zero indicates that the rainfall is normal.

A_{01} —Positive A_{01} indicates that the rainfall is below normal in the northern part of the region and above normal in the southern part. Negative A_{01} represents that the precipitation is above normal in the northern part and below normal in the southern part. Fig.1 (a) shows the pattern corresponding to $\varphi_{0,j}(i)\psi_1(j)$.

A_{10} denotes the difference of rainfall anomalies between western part and eastern part of the region. Fig.1(b) is the pattern corresponding to $\varphi_{1,j}(i)\psi_0(j)$.

A_{02} is the weight of rainfall distributions with above normal anomaly in the northern and southern parts and below normal anomaly in the middle part in the region as shown in Fig.1 (c).

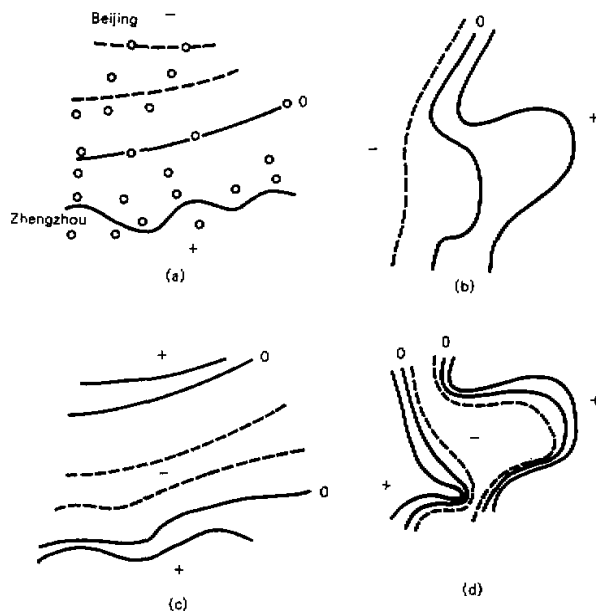


Fig. 1. Patterns corresponding to $\varphi_0, f^{(0)}\psi_1(j)$, $\varphi_1, f^{(1)}\psi_0(j)$, $\varphi_0, f^{(0)}\psi_2(j)$, $\varphi_2, f^{(2)}\psi_0(j)$.

(a) $\varphi_0, f^{(0)}\psi_1(j)$ (b) $\varphi_1, f^{(1)}\psi_0(j)$

(c) $\varphi_0, f^{(0)}\psi_2(j)$ (d) $\varphi_2, f^{(2)}\psi_0(j)$

Beijing (116°28'E, 39°48'N) Zhengzhou (113°39'E, 34°43'N)

O—Station Location

A_{20} represents the difference among rainfall anomalies in the western, middle and eastern parts in the region as shown in Fig.1(d).

The rest Chebyshev coefficients possess more complicated characteristics (See Zhou, 1990).

We can forecast the distributions with Eq.(1) if the values of Chebyshev coefficients are obtained.

IV. FORECASTING MODEL OF VECTOR SIMILARITY IN PHASE SPACE

Zhou (1992) proposed a forecasting model of vector similarity in phase space and applied it to the forecast of southern oscillation. Here we use it to forecast the rainfall distributions over the Huanghe-Huaihe-Haihe Plain in China.

The outline of the forecasting model is as follows.

Suppose we have a univariate time series,

$$\begin{aligned} X(t_i) &= X(t_0 + i\Delta t), \\ i &= 0, 1, 2, \dots, N-1, \end{aligned} \quad (3)$$

where $X(t_i)$ represents some above mentioned Chebyshev coefficients, t_0 is the initial moment, Δt is the sample interval, N is the total sample number. Introducing a time lag τ , an m -dimensional phase space is reconstructed, i.e. (Packard et al., 1980),

$$X_m(t_i) = \{X(t_i), X(t_i - \tau), X(t_i - 2\tau), \dots, X(t_i - (m-1)\tau)\}. \quad (4)$$

The dynamic system is located at phase point $X_m(t_i)$ at t_i and it will be located at phase point $X_m(t_{i+1})$ at t_{i+1} , i.e.

$$X_m(t_{i+1}) = \{X(t_{i+1}), X(t_{i+1} - \tau), X(t_{i+1} - 2\tau), \dots, X(t_{i+1} - (m-1)\tau)\}. \quad (5)$$

The forecasting problem is just to find the position of phase point $X_m(t_{i+1})$. Owing to that $\tau \geq \Delta t$ and $\tau/\Delta t$ is an integer, only $X(t_{i+1})$ in the expression of $X_m(t_{i+1})$ (Exp.(5)) needs to be found.

The procedures of forecast are as follows:

(1) Evaluating the distances between point $X_m(t_i)$ and other points in phase space and selecting the minimum distance D_i corresponding to point $X_m(t_b)$.

$$D_i = \left\{ \sum_{l=0}^{m-1} [X(t_i - l\tau) - X(t_b - l\tau)]^2 \right\}^{\frac{1}{2}}. \quad (6)$$

At next moment t_{i+1} , point $X_m(t_i)$ will reach point $X_m(t_{i+1})$ and point $X_m(t_b)$ will reach point $X_m(t_{b+1})$ and the correspondent distance between $X_m(t_{i+1})$ and $X_m(t_{b+1})$ is D_{i+1} . Supposing $D_i = D_{i+1}$ we can get a quadratic equation for unknown number $X(t_{i+1})$ and evaluate two values for $X(t_{i+1})$, y_1 and y_2 .

(2) A phase point corresponds to a vector from original point to that point, then points $X_m(t_i)$, $X_m(t_b)$, $X_m(t_{i+1})$, $X_m(t_{b+1})$ correspond to vectors $V_m(t_i)$, $V_m(t_b)$, $V_m(t_{i+1})$, $V_m(t_{b+1})$. Evaluating the angle θ_i between $V_m(t_i)$ and $V_m(t_b)$ and angle θ_{i+1} between $V_m(t_{i+1})$ and $V_m(t_{b+1})$ and supposing $\theta_i = \theta_{i+1}$ we can get another quadratic equation for unknown number $X(t_{i+1})$ and evaluate two values for $X(t_{i+1})$, y_3 and y_4 .

(3) Replacing $X(t_{i+1})$ in Exp. (5) with y_1 , y_2 , y_3 , y_4 respectively we get four points $X_m^{(1)}(t_{i+1})$, $X_m^{(2)}(t_{i+1})$, $X_m^{(3)}(t_{i+1})$, and $X_m^{(4)}(t_{i+1})$. Evaluating the distances D_j between $X_m^{(3)}(t_{i+1})$ and

$X_m(t_{h+1})$ and D_4 between $X_m(t_{h+1})$ and $|D_{j_0} - D_i| = \min_j |D_j - D_i|$, if $j_0 = 3$ we select y_3 as predicted value of $X(t_{i+1})$, otherwise we select y_4 .

Corresponding to $X_m^{(1)}(t_{i+1})$ and $X_m^{(2)}(t_{i+1})$ we have $V_m^{(1)}(t_{i+1})$ and $V_m^{(20)}(t_{i+1})$. Then we evaluate angle θ_1 between $V_m^{(1)}(t_{i+1})$ and $V_m^{(1)}(t_{h+1})$, θ_2 between $V_m^{(2)}(t_{i+1})$ and $V_m(t_{h+1})$ and $|\theta_h - \theta_i| = \min_j |\theta_j - \theta_i|$, if $j_0 = 1$ we select y_1 as predicted value of $X(t_{i+1})$, otherwise we select y_2 .

V. THE FORECAST OF TEMPORAL AND SPATIAL DISTRIBUTIONS OF RAINFALL OVER THE HUANGHE—HUIHE—HAIHE PLAIN IN CHINA

The data of ten-day rainfall at 23 stations over the Huanghe—Huaihe—Haihe Plain in China from January 1959 to February 1992 are used. The 3-dekad moving total rainfall is evaluated. The distribution of stations is shown in Fig. 1(a). Using the forecasting method shown in Section IV, we get the forecast of rainfall from March 1993 to February 1994. The forecast is shown in Table 2.

Table 2. Forecast of Chebyshev Coefficients

Y	1993																	
M	3			4			5			6			7			8		
TD	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
A_{00}	-26.0	-11.6	-0.8	-4.9	-6.1	5.3	4.2	12.6	18.8	4.4	5.3	0.3	-2.2	-3.9	-4.5	2.0	2.7	3.8
A_{01}	-5.2	-1.4	2.6	4.7	-7.1	3.7	1.6	4.8	-5.2	-5.1	-0.5	0.5	3.3	-0.4	-0.6	-0.9	0.5	-2.1
A_{02}	-2.3	-4.5	-4.9	0.9	1.1	-0.7	0.8	-1.3	-1.0	-1.3	-2.8	-5.0	1.3	2.0	0.3	-1.2	1.1	1.6
A_{10}	1.0	3.9	0.9	-2.2	-1.2	4.3	-1.4	3.9	-3.8	-6.1	-4.0	1.9	-1.6	-0.2	0.4	3.5	2.3	0.9
A_{20}	1.6	-1.3	1.5	-4.5	-0.9	-2.3	1.7	-1.5	-2.3	-1.0	-1.0	-2.1	-1.8	1.0	-0.4	-3.1	-3.1	-2.4

Continued Table 2.

Y	1993												1994					
M	9			10			11			12			1			2		
TD	25	26	27	28	29	30	31	32	33	34	35	36	1	2	3	4	5	6
A_{00}	-1.8	-1.6	2.4	1.5	-5.2	-1.9	2.0	-4.1	-2.2	4.5	-7.1	3.9	-14.2	7.1	-1.7	-8.3	-16.0	-7.7
A_{01}	-2.6	0.2	2.1	7.4	-0.4	6.6	3.7	1.3	-10.7	8.1	8.9	-6.0	-12.2	-11.7	7.7	-2.4	12.3	12.6
A_{02}	0.3	-10.6	3.5	-2.8	4.0	-0.7	-1.8	-0.9	5.6	-4.9	3.4	4.5	-1.0	-0.9	-5.5	1.0	3.2	-8.6
A_{10}	-1.1	-2.3	1.0	0.2	-0.7	-0.7	1.4	-0.6	-5.4	-4.7	3.4	1.3	-2.9	7.3	3.1	-0.6	0.4	3.5
A_{20}	0.7	0.9	-1.3	-0.8	-2.6	-3.2	5.7	0.3	7.6	4.3	6.0	-0.2	-3.5	0.8	-6.8	-4.6	3.5	4.8

Y—Year M—Month TD—Ten days

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