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Study of Effects of Beta Term and Nonlinear Advection on the Structure of Tropical Cyclones①

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ABSTRACT

The effects of the Beta term on the typhoon structure are examined within the linear framework in terms of an analytical method of 2–D Fourier representation and numerical experiments by a Beta–plane quasi–geostrophic barotropic model. Results show that the joint effects of the difference of Rossby phase velocities and the dispersion of typhoon energy keep the maximum wind velocity reasonably evolving rather than irrestrictively increasing. On the one hand, the nonlinear advection accelerates typhoon vortex damping, and on the other, the high pressure system formed downstream due to energy dispersion makes it easy to maintain.

Key words: Typhoon structure, Energy dispersion, Nonlinear advection

I. INTRODUCTION

Recently, Chan and Williams (1987) have drawn wide attention by presenting the formative causes of the stream function asymmetrical structure (concentrated in the east, dispersed in the west) in typhoon region in terms of the phase velocity of harmonics and by the expansion of the initial symmetrical typhoon stream function with the 2–D Fourier series. However, the linear mechanism would make the stream line concentrate in the east and the maximum wind velocity increase irrestrictively with time. This is unreasonable to some degree. We suppose, besides the machanism mentioned by Chan and Williams (1987), there might be some other physical processes which prevent the irrestrictive increase of wind velocity. This is what we are going to discuss in the present paper. On the basis of it, the effects of the Beta term and nonlinear advection on the structure of typhoon and its evolution will be discussed.

II. THE EEFECTS OF BETA TERM ON TYPHOON STRUCTURE

1. *Formula Derivation and Its Qualitative Analysis*

Take the quasi–geostrophic barotropic vorticity equation:

$$\frac{\partial}{\partial t}\nabla^2\psi + J(\psi,\nabla^2\psi) + \beta\frac{\partial\psi}{\partial x} = 0 \ , \tag{1}$$

where all symbols are conventional. Let L, V be characteristic horizontal scale and velocity respectively, and nondimensionize Eq.(1). Assume

$$\psi(x,y,t) = \overline{\psi} + \psi'(x,y,t) \ . \tag{2}$$

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The equation about the disturbed quantity $\psi'(x, y, t)$ is obtained by subsitituting Eq.(2) into the non–dimensional equation and assuming the basic flows everywhere to be zero. And then assuming the advection term to be zero, the linear equation about $\psi'(x, y, t)$ is obtained. (Primes are deleted in the following discussion):

$$\frac{\partial}{\partial t}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+\beta\frac{\partial\psi}{\partial x}=0 \ . \tag{3}$$

Then assume

$$\overline{\Psi}(k, l, t)=\sum_x\sum_y\psi(x, y, ,t)e^{i(kx+ly)}\Delta x\Delta y \ , \tag{4}$$

$$\overline{\Psi}(k, l, t)=\overline{\Psi}(k, l, 0)e^{i\omega t} \ , \tag{5}$$

where k and l are wavenumbers in the direction of x, y respectively. Thus we obtain

$$\psi(x, y, ,t)=\frac{1}{4\pi^2}\sum_k\sum_l\overline{\Psi}(k, l, 0)e^{-i(kx+ly)}e^{i\omega t}\Delta k\Delta l \ , \tag{6}$$

where $\overline{\Psi}(k, l, 0)$ can be obtained by substituting the initial stream function field $\psi(x, y, 0)$ into Eq.(4).

Substituting Eq.(6) into Eq.(3), the dispersion relation is obtained:

$$\omega=-\frac{\beta k}{k^2+l^2} \ . \tag{7}$$

From Eq.(7), we obtain the phase velocity formula in the east–west direction

$$C_x=-\frac{\beta}{k^2+l^2} \ , \tag{8}$$

and the group velocity formula in the same direction

$$C_{gx}=\frac{\beta(k^2-l^2)}{(k^2+l^2)^2} \ . \tag{9}$$

Substituting Eq.(7) into Eq.(6) , the analytic formula of the evolution of stream function field with time is obtained.

Chan and Williams (1987) discussed the effect of the Beta term on typhoon structure with the phase velocity formula, and pointed out that among the expansion components of the initial symmetrical typhoon vortex, the components standing for the outer circulation possess longer wavelength with smaller wavenumber k,l and the larger westward phase velocity of Rossby wave; and the components standing for the inner circulation possess shorter wavelength with larger wavenumbers and the smaller westward phase velocity. In consequence, the contours of the stream function in the eastern (western) part of typhoon become concentrated (dispersed), which is consistent with that of the observation. On the other hand, with the increase of time, the concentration of the contours and maximum wind velocity in the east will increase irrestrictively, which is inconsistent with the observation. The present study will show that the inconsistent phenomenon will not occur if we take into account both the effects of different phase velocities (which has been taken into account in the literature) and the process of typhoon energy dispersion (which has not taken into account). The following will be devoted to the discussion. First, make qualitative analysis of the group velocity formula (9).

From formula (9) we can see:

(1) When $k > l$ or $L_x < L_y$, then $C_{gx} > 0$, the energy of typhoon circulation disperses towards the eastside; when $k < l$ or $L_x > L_y$, then $C_{gx} < 0$, it disperses towards the westside. When $k = l$, i.e., $L_x = L_y$ then $C_{gx} = 0$, *it will not disperse outwardly. So, the system formed* due to energy dispersion may not show an approximate circle like typhoon, but an oval–shape. The system to the east of typhoon center becomes contracted along the east–west direction and stretched out in the north–south direction ($L_x < L_y$). The system to the west, streched out along the east–west direction and contracted along the north–south direction ($L_x > L_y$). We noticed that in the western part, different phase velocities would cause the outstrech of contour lines along the east–west direction as well. So it is quite difficult to identify *the west system while the east system is conspicuouse. The components which have the* equality $L_x = L_y$ will not disperse energy outward, thus making the circular typhoon vortex easy to maintain. Due to different phase velocities, the initial symmetrical typhoon vortex turned into asymmetrical one, which means that the expansion components with $L_x \neq L_y$ will increase. This provides a suitable background for the outward dispersion of typhoon energy.

(2) In the denominator of formula (9), there are quartic terms of wavenumbers k,l. Under *the condition of* $k \neq l$, *the larger (smaller) values* k,l *take, the shorter (longer) the wavelength* is, and the smaller (larger) the group velocity, the more difficult (the easier) for typhoon energy to be dispersed. Thus, the spatial scale of the system generated outside typhoon region should be larger or equal to that of the outer circulation.

(3) In Eq.(9), there is only the even power of l in the y direction. Therefore, the outside generating system takes the x axis as the symmetry axis. In the entire process of evolution, the symmetry will maintain.

2. *Comparison of the Results of Analytic Calculation and Numerical Experiment with Those of Qualitative Analysis*

The results of the above qualitative analysis have got proofs from analytic calculation and numerical experiment.

The initial relative vorticity fields of analytic calculation and numerical experiment are the same, in symmetrical distribution (Chan and Williams, 1987):

$$\zeta_0(r) = 2V_m / r_m \{1 - 0.5 \times (r / r_m)^b\} \exp\{(1/b)(1 - (r / r_m)^b)\} \ , \tag{10}$$

where $r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$, (x_0, y_0) is the coordinates of typhoon center at the initial time. V_m is the typhoon maximum wind velocity, and r_m the distance between the maximum wind velocity spot and typhoon center. Assume that $V_m = 20$ m / s, $r_m = 100$ km.

Analytic calculation: The initial stream function field $\psi(x, y, 0)$ is obtained from $\zeta(r)$, with the Poisson Iteration. Substituting $\psi(x, y, 0)$ into Eq.(4) yields $\overline{\Psi}(k, l, 0)$. And substituting $\overline{\Psi}(k, l, 0)$ and dispersion relation (7) into (6), we obtain the evolution of the stream function field with time. The calculating area is a square, with the side length being 3770 km, taking 77 × 77 grid points, $\Delta x = \Delta y = 49.6$ km. In 2–D Fourier expansion, $k_m = l_m = 38$, where $k_m = l_m$ are the maximum wavenumbers in the x, y directions respectively. That is, the stream function fields are expanded with and composed of 1521 modes. Assume in Eq.(6), $t = 12^h, 24^h, 36^h, \cdots, 336^h$, the stream function fields of the 12th, 24th, 36th, ···, 336th model hour have been calculated. Chan and Williams (1987) have only produced the results within 72 model hours. Now, the results within 336 model hours in this paper have been calculated so that a more complete understanding of the evolution of typhoon structure can be reached.

Numerical experiment: Using the model of the literature (Luo, 1991), we make two changes to the model so as to let it fit the result of the above. One is to assume all physical processes to be zero except the Beta term in the model equation. The other is to change the calculation region from 101 × 101 grid points into 77 × 77 grid points, with the grid spacing being 50 km and time 10 min. The initial relative vorticity field is derived from Eq.(10), and the model has been run for 8 model days (experiment 1).

In experiment 1, during the eight model days, the day– to– day evolution of the stream function is much similar to the corresponding analytic calculation result. One prominent characteristic on the stream function field at 168th model hour is that a low– high– low wavetrain occurs to the east of typhoon center (Fig. 1a) . This characteristic is also shown clearly in the corresponding figure of the analytic calculation (figure omitted). In Fig.1a, the system to the east of typhoon center is oval–shaped, with out– stretch in the north–south direction and contraction in the east–west direction; its spatial scale is larger than or equal to that of typhoon circulation; the system presents a symmetrical distribution in relation to the x axis. All these are consistent with the results of the above qualitative analysis. The results of the numerical experiment, analytic calculation and qualitative analysis are the basis for analyzing the process of typhoon energy dispersion outward from typhoon region.

3. *Variations of Wind Velocity outside and inside Typhoon Region with Time*

In the analytic calculation, 15 tangential wind velocity fields including 0^h, 24^h, $\cdots$, 336^h are given out (tangential wind velocity $V=\sqrt{u^2+v^2}$). In these tangential wind fields,

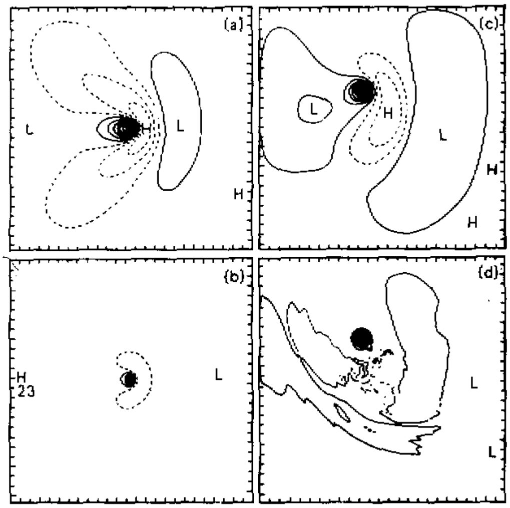

Fig. 1. The 168th model hour stream function fields and relative vorticity fields in experiments 1 and 2. Solid lines $\psi < 0, \zeta > 0$; dotted lines $\psi > 0, \zeta < 0$. (a) Stream function field in experiment 1, the interval of contours being $0.478 \times 10^6 m^2 s^{-1}$. (b) Relative vorticity field in experiment 1, the maximum value being $9.87 \times 10^{-4} s^{-1}$.(c) Stream function field in experiment 2, the interval of contours being $0.553 \times 10^6 m^2 s^{-1}$. (d) Relative vorticity field in experiment 2, the maximum value being $6.01 \times 10^{-4} s^{-1}$.

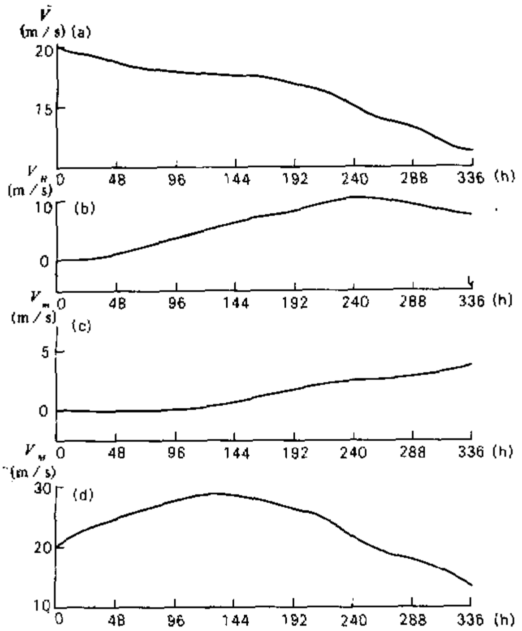

Fig. 2. Temporal variations of $\bar{V}$, V_H, V_m and V_M. (a) Wind velocity $\bar{V}$ inside typhoon region; (b) wind velocity V_H in the eastside of the high pressure system; (c) wind velocity V_m in the eastside of the low presuure system; and (d) Typhoon maximum wind velocity V_M.

draw a circle taking the typhoon center as its center, r_m (= 100 km) as its radius. On the circle, get the eight tangential wind velocity values in E, NE, N, NW, W, SW, S, SE directions. Then, get their mean value, marked as $\bar{V}$, which is taken to stand for the wind velocity of typhoon region. Result: $\bar{V}$ monotonously decreases with time (Fig. 2a), meaning that the kinetic energy in typhoon region decreases gradually. Because the control equation (3) is a conservation system, the total kinetic energy in the calculation region should be constant. The decrease of wind velocity and kinetic energy inside typhoon region calls for the increase of those outside typhoon region. We use wind velocity V_H east of the high pressure system to the east of typhoon center and V_m east of the low pressure system to reflect the wind velocity and kinetic energy outside typhoon region. The distance between the east side of the high (low) pressure system and the typhoon center is about 600 (1500) km. At the initial moment, the wind velocity $\bar{V}$ inside typhoon region is the largest, while the velocities $\bar{V}$, V_H and V_m beyond 600 km from the typhoon center are zero. During 0–96^h, $\bar{V}$ monotonously decreases, V_H monotonously increases, V_m remains zero, which indicates that the typhoon energy has not been dispersed to those regions beyond 1500 km. During 96^h–240^h, $\bar{V}$ keeps monotonously decreasing, while V_H and V_m increase simultaneously. During 240^h–336^h, both $\bar{V}$ and V_H decrease with time, while V_m monotonously increases, which indicates that the high pressure system downstream is weakening, and the typhoon energy begins to disperse outwardly (Figs. 2b and 2c). The evolution curves of $\bar{V}$, V_H and V_m with time are properly matched, reflecting the process of typhoon energy dispersion outwardly to the wide space under the effect of the Beta term. The maximum wind velocity in typhoon region appears in the east of typhoon. The variations of V_M with time (Fig. 2d) clearly show that V_M does not increase irrestrictively. During 0^h–120^h, V_M increases with time; but during 120^h–336^h, V_M decreases with time.

Obviously, the increase of V_M is attributed to the streamline distribution (west, dispersed; east, concentrated) caused by the different phase velocities of the harmonics of typhoon circulation. And the decrease of V_M is attributed to the outward dispersion of typhoon energy. Due to the dispersion, the maximum wind velocity in the east of typhoon cannot increase infinitely. It will become less strong when it attains a certain value (about 30 m / s at 120th model hour, see Fig. 2d). Chan and Williams (1987) have only presented the result of the first 72 model hours, i.e., the evolution before the turning points. It only considered the influence of different phase velocities and failed to show the entire process of the flow pattern under the Beta term effect. The present paper has offered a rather complete result.

III. JOINT EFFECTS OF THE BETA TERM AND NONLINEAR ADVECTION ON TYPHOON STRUCTURE

The primary constraint for typhoon movement is the absolute vorticity conservation, which needs taking into account the joint effects of the Beta term and nonlinear advection. Experiment 2 is designed for this purpose. The model used in experiment 2 is taken from Luo (1991). Experiment 2 takes into account the nonlinear advection with 101 × 101 grid points while experiment 1 does not. The other conditions are both the same. The 168th model hour stream function and relative vorticity fields of experiment 2 are plotted in Figs. 1c and 1d. From Fig. 1c, we can see that the low– high– low pressure system wavetrain still exists in the east of typhoon center, which is similar to the linear situation (Fig. 1a). The differences of the two situations are as follows: First, their wavetrain orientations are different, one is along the east–west direction, the other is along the northwest–southeast direction. Secondly, their distances between the typhoon center and the high pressure center are also different. The distance, after the nonlinear advection is introduced, has increased to about 4 times as long as the distance before. Thirdly, because of energy dispersion, the intensity of the system to the west of typhoon center is also different. Under the linear condition, there is no closed contour line. After introduction of nonlinear advection, there appear closed contour lines and the system is quasi– oval shape with concentration in the north–south direction, out– stretch in the east–west direction, in sharp contrast to the system east of typhoon center. This is consistent with the result of the above qualitative analysis. So we can see that under the joint effects of the Beta term and nonlinear advection, the function of the Beta term is recognized and that of nonlinear advection is also obvious.

There is a conspicuous difference between Figs.1b and 1d, namely their value difference of the relative vorticity in typhoon centers is rather sharp. In experiment 1, ζ_0 is 9.87×10^{-4} s^{-1}, and in experiment 2, ζ_0 is 6.01×10^{-4}s^{-1}. The relative vorticity fields of both initial time are completely the same, ζ_0 being 10.86×10^{-4}s^{-1}. Comparing with the initial fields, the value is reduced by 9. 0% and 44.6 % respectively. ζ_0's evolution with time is shown in Fig. 3: the rate of ζ_0's attenuation in experiment 1 is slower than that in experiment 2. Explanation: in the attenuation process of typhoon vortex intensity, the effect originated from nonlinear advection seems more important than that of the Beta term. McWilliams and Flierl (1979) obtained a similar conclusion after they studied the problem of the attenuation of ocean vortex amplitude. Besides, under the linear condition (experiment 1), the evolution of ζ_0 with time is monotonous and regular; after the introduction of nonlinear advection (experiment 2), the evolution becomes nonmonotonous and irregular.

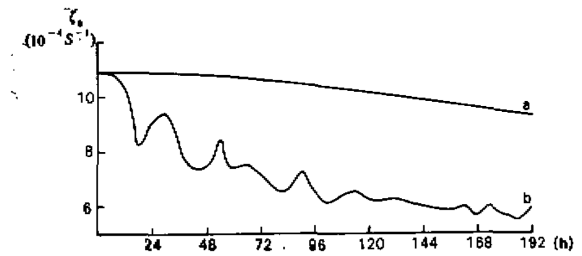

Fig. 3. Temporal evolution of ζ_0 (the value of relative vorticity of typhoon centers in experiments 1 and 2) for (a) experiment 1 and (b) experiment 2.

Experiment 3 is designed to see whether the high pressure system east of typhoon center has any influence on the typhoon vortex structure. In experiment 3, during each time step the intensity of the high pressure system is artificially half– reduced without changing the other conditions. If the intensity of the high pressure system is not related to typhoon structure, the results of experiments 1 and 2 should be the same. To make it clearer, three–point time smoothing has been conducted to the relative vorticity value sequence ζ_0 of typhoon center, the smoothed value marked as $\tilde{\zeta}_0$. The result indicates that the attenuation of $\tilde{\zeta}_0$ with time is much faster. The time, during which $\tilde{\zeta}_0$ decreases from the initial value $10.85 \times 10^{-4}s^{-1}$ to $5.80 \times 10^{-4}s^{-1}$, is 156 model hours in experiment 2, and however it is only 72 model hours in experiment 3. This seems to show that it is the Beta term that leads to typhoon energy dispersion and the formation of high pressure system. This dispersion process made the typhoon weakened, as is not favorable for the maintenance of the typhoon vortex; on the other hand, the existence of the high pressure system made typhoon vortex easy to maintain. This is an interesting phenomenon.

IV. RESULT AND DISCUSSION

The present paper has studied the effects of the Beta term on typhoon structure in terms

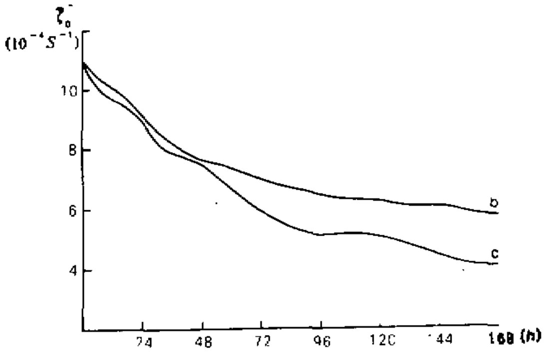

Fig. 4. Temporal evolution of the smoothed value $\tilde{\zeta}_0$ of the typhoon center relative vorticity in (a) experiments 2 and (b) experiment 3.

of an analytical method of 2– D Fourier representation and numerical experiments by a Beta– plane quasi– geostrophic barotropic model. The results show that the maximum wind velocity in the east of typhoon first increases reasonably with time, then decreasing gradually (Fig. 2d), as resulting from the joint effects of both the different Rossby phase velocities and the typhoon energy dispersion. Chan and Williams (1987) have only taken into account the effect of different phase velocities, and thus, the maximum wind velocity would increase irrestrictively. As compared to it, the present paper has obtained a more reasonable result.

As for the conservation system, the main factor for the attenuation of the intensity of large scale vorticity is the energy dispersion and nonlinear advection. The present paper shows that the effect of nonlinear advection is more important, which is consistent with the result obtained by McWilliams, et al. (1979) in the study of ocean vortex. Meanwhile, the function of the high pressure system formed by typhoon energy dispersion in the attenuation of typhoon intensity has been further discussed. And it is shown that the existence of the system will slow down the rate of attenuation caused mainly by nonlinear advection.

However, the present paper is unable to explain some facts for the effects of heating and dissipation were not taken into account. For instance, it can not explain the fact that both the maximum and mean velocities often increase together during the typhoon developing stage. Furthermore, it is reasonable that diabatic heating should be added into the process with a large time scale. All these need to be further improved in the future study.

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