

Application of a Shape-Preserving Advection Scheme to the Moisture Equation in an E-grid Regional Forecast Model

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ABSTRACT

This paper presents a methodology which is very useful to design shape-preserving advection finite difference scheme on general E-grid horizontal arrangement of variables through introducing a two-step shape-preserving positive definite advection scheme in the moisture equation of the LASG-REM (LASG regional E-grid eta-coordinate forecast model). By trial-forecasting six local heavy raincases, the efficiency of the shape-preserving advection scheme in practical application has been examined. The LASG-REM with the shape-preserving advection scheme has a good forecasting ability for local precipitation.

Key words: Eta-coordinate, E-grid, Shape-preserving advection scheme

I. INTRODUCTION

The arrangement of each dependent variable on the Arakawa E-grid presents a stagger form between neighbour mesh lines, which is different from other (A-D) grid arrangement. For each grid point of general E-grid, the closest neighbour mesh points, which arrange same variables as the grid point, are located in its diagonal directions but horizontal perpendicular mesh lines. Two-dimensional advection centre spatial finite difference scheme has not combined form and a simple generalization of one-dimensional results, and corresponding up-stream scheme is difficult to express. In finite difference models, all present positive definite advection schemes are almost based on upstream scheme, so it is also difficult to construct shape-preserving positive definite advection scheme in general E-grid arrangement. But it is often necessary to solve the advection equation for positive definite scalar functions in numerical modeling of atmospheric phenomena (moisture and pollutant transportation for example).

The LASG-REM solved the construction technique of the spherical-coordinate advection finite difference scheme, which suits to general rectangle E-grid, by using "half-space-increment" finite difference (Yu Rucong, 1989). Yu Rucong(1994) developed a two-step shape-preserving positive definite advection scheme(hereafter, TSPAS) which is simple reasonably and efficiently based on Arakawa C-grid. This paper presents a methodology to design shape-preserving positive definite advection scheme for general E-grid arrangement and examines the practical application efficiency of the TSPAS through introducing the TSPAS in the moisture equation of the LASG-REM.

Section 2 presents the LASG-REM governing differential equations and its advection scheme for non-quantitative variables in order to discuss it conveniently. In Section 3, an "upstream scheme" on E-grid is developed and the TSPAS is introduced to the moisture equation of LASG-REM. Section 4 gives trial-forecasting results of six local heavy rain

cases.

II. THE GOVERNING DIFFERENTIAL EQUATIONS OF LASG-REM

The mathematical model of LASG-REM is derived from the IAP-AGCM formulas. In order to consider steep mountains, the η -coordinate has been used as vertical coordinate. This has a very simple lower boundary condition similar to σ -coordinate, and avoids the excessive inclination of coordinate surfaces in the mountainous areas. The governing differential equations of LASG-REM are as follows:

$$\frac{\partial U}{\partial t} = - \sum_{m=1}^3 \mathcal{L}_m(U) - f^* V - P_x, \quad (1)$$

$$\frac{\partial V}{\partial t} = - \sum_{m=1}^3 \mathcal{L}_m(V) + f^* U - P_y, \quad (2)$$

$$\frac{\partial \Pi}{\partial t} = - \sum_{m=1}^3 \mathcal{L}_m(\Pi) + S(C_0 + \frac{R\Pi}{C_p P}) \left(\frac{1}{P\eta} \Omega^{(1)} + \Omega^{(2)} \right), \quad (3)$$

$$\frac{\partial P^2}{\partial t} = - \frac{1}{\eta_s} \int_0^{\eta_s} D_{xy} d\eta; \int_{\eta_2}^{\eta_1} \left(\frac{\partial P^2}{\partial t} + D_{xy} \right) d\eta = P^2 (\dot{\eta}_2 - \dot{\eta}_1), \quad (4)$$

$$\frac{\partial \Phi}{\partial \eta} = - C_0 \frac{S\Pi}{P\eta}, \quad (5)$$

$$\dot{\eta}|_{\eta=0; \eta_s} = 0, \quad (6)$$

where

$$\eta = \sigma \cdot \eta_s, \quad \eta_s = \frac{p_T(z_s) - p_t}{p_T(z_b) - p_t}, \quad \sigma = \frac{p - p_t}{p_s - p_t}, \quad (7)$$

$$P = \sqrt{p_{es}}, \quad p_{es} = (p_s - p_t) / \eta_s, \quad S = \frac{p - p_t}{p} = \eta / (\eta + p_t / P^2), \quad (8)$$

$$U = Pu, \quad V = Pv, \quad \Pi = \frac{RPT}{C_0}, \quad (9)$$

$$\mathcal{L}_1(F) = \frac{1}{2a \sin \theta} \left(2 \frac{\partial F u}{\partial \lambda} - F \frac{\partial u}{\partial \lambda} \right), \quad (10)$$

$$\mathcal{L}_2(F) = \frac{1}{2a \sin \theta} \left(2 \frac{\partial F v \sin \theta}{\partial \theta} - F \frac{\partial v \sin \theta}{\partial \theta} \right), \quad (11)$$

$$\mathcal{L}_3(F) = \frac{1}{2} \left(2 \frac{\partial F \dot{\eta}}{\partial \eta} - \frac{\partial \dot{\eta}}{\partial \eta} \right), \quad (12)$$

$$P_x = P \frac{\partial \Phi}{a \sin \theta \partial \lambda} + C_0 S \Pi \frac{\partial \ln P^2}{a \sin \theta \partial \lambda}, \quad P_y = P \frac{\partial \Phi}{a \partial \theta} + C_0 S \Pi \frac{\partial \ln P^2}{a \partial \theta}, \quad (13)$$

$$\Omega^{(1)} = - \int_0^{\eta_s} D_{xy} d\eta, \quad \Omega^{(2)} = V \frac{\partial \ln P^2}{a \partial \theta} + U \frac{\partial \ln P^2}{a \sin \theta \partial \lambda}, \quad (14)$$

$$D_{xy} = \frac{1}{a \sin \theta} \left(\frac{\partial P V \sin \theta}{\partial \theta} + \frac{\partial P U}{\partial \lambda} \right), \quad (15)$$

$$\dot{\eta} = \frac{d\eta}{dt} = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{a \sin \theta \partial \lambda} + v \frac{\partial \eta}{a \partial \theta}, \quad (16)$$

$$f^* = 2\omega \cos \theta + (ctg \theta / a) u, \quad (17)$$

and C_0 is the constant gravity wave speed of the standard atmosphere; a , R , C_p and ω are the radius of the Earth, gas constant, specific heat at constant pressure and angular velocity of the Earth's rotation respectively; u , v , T and Φ are the zonal wind, meridional wind and departures of temperature and geopotential from their "standards" respectively; θ and λ are the

common surface spherical coordinates; η and t are the vertical and time coordinates; z_s is the topographic height which is permitted to take a discrete set of values, chosen so that mountains are defined according to the three-dimensional grid box element of the model; z_b is a base elevation defined so as to include gentle slopes of the surface topography; $P_{ref}(z)$ is a reference pressure as a function of z ; p_s and p_t are the pressure, surface pressure and pressure of the model top, respectively.

The arrangement of the dependent variables on the grids of the model can refer to Fig.1 of Yu(1989). E-grid is used for the horizontal arrangement. In vertical, the geopotential is defined at the interface of the layers as well as vertical velocity so as to preserve hydrostatic consistency.

The horizontal advection finite difference scheme of LASG-REM has been developed by using "half-space-increment" difference scheme, the formulas of the newest model version are as follows:

$$\mathcal{E}_1(F) = \frac{1}{a \sin \theta \Delta \lambda} \left(2 \delta_{\frac{1}{2} \Delta \lambda}^1 u F - F \delta_{\frac{1}{2} \Delta \lambda}^1 u \right), \quad (18)$$

$$\mathcal{E}_2(F) = \frac{1}{a \sin \theta \Delta \theta} \left(2 \delta_{\frac{1}{2} \Delta \theta}^1 v \sin \theta F - F \delta_{\frac{1}{2} \Delta \theta}^1 v \sin \theta \right). \quad (19)$$

Let

$$V d_{i+\frac{1}{4}, j+\frac{1}{4}} = \frac{1}{a \sin \theta_j} \left(\frac{u_{i+\frac{1}{4}, j+\frac{1}{4}}}{\Delta \lambda} + \frac{v_{i+\frac{1}{4}, j+\frac{1}{4}} \sin \theta_{j+\frac{1}{4}}}{\Delta \theta} \right), \quad (20)$$

$$V d_{i-\frac{1}{4}, j-\frac{1}{4}} = \frac{1}{a \sin \theta_j} \left(\frac{u_{i-\frac{1}{4}, j-\frac{1}{4}}}{\Delta \lambda} + \frac{v_{i-\frac{1}{4}, j-\frac{1}{4}} \sin \theta_{j-\frac{1}{4}}}{\Delta \theta} \right), \quad (21)$$

$$V d_{i+\frac{1}{4}, j-\frac{1}{4}} = \frac{1}{a \sin \theta_j} \left(\frac{u_{i+\frac{1}{4}, j-\frac{1}{4}}}{\Delta \lambda} - \frac{v_{i+\frac{1}{4}, j-\frac{1}{4}} \sin \theta_{j-\frac{1}{4}}}{\Delta \theta} \right), \quad (22)$$

$$V d_{i-\frac{1}{4}, j+\frac{1}{4}} = \frac{1}{a \sin \theta_j} \left(\frac{u_{i-\frac{1}{4}, j+\frac{1}{4}}}{\Delta \lambda} - \frac{v_{i-\frac{1}{4}, j+\frac{1}{4}} \sin \theta_{j+\frac{1}{4}}}{\Delta \theta} \right), \quad (23)$$

then

$$\begin{aligned} [\mathcal{E}_1(F) + \mathcal{E}_2(F)]_{i, j, k} = & \frac{1}{2} [V d_{i+\frac{1}{4}, j+\frac{1}{4}} F_{i+\frac{1}{2}, j+\frac{1}{2}} - V d_{i-\frac{1}{4}, j-\frac{1}{4}} F_{i-\frac{1}{2}, j-\frac{1}{2}} \\ & + V d_{i+\frac{1}{4}, j-\frac{1}{4}} F_{i+\frac{1}{2}, j-\frac{1}{2}} - V d_{i-\frac{1}{4}, j+\frac{1}{4}} F_{i-\frac{1}{2}, j+\frac{1}{2}}]_{k}, \quad (24) \end{aligned}$$

where, if F represents U or V , then:

$$u_{i+\frac{1}{4}j+\frac{1}{4}} = 0.5(u_{ij} + u_{i+\frac{1}{2}j+\frac{1}{2}}) ,$$

if F represents Π , then:

$$u_{i+\frac{1}{4}j+\frac{1}{4}} = 0.5(u_{i+\frac{1}{2}j} + u_{ij+\frac{1}{2}}) .$$

The spatial finite difference schemes of other terms of the model equations are almost based on full-space-increment centre difference and can refer to Yu(1989).

III. APPLICATION OF THE TSPAS TO THE MOISTURE EQUATION IN LASG-REM

In order to have an efficient and reasonable mass conservative positive definite moisture advection scheme we chose the following flux-type moisture advection equation:

$$\frac{\partial Q}{\partial t} + \frac{1}{a \sin \theta} \left[\frac{\partial Q u}{\partial \lambda} + \frac{\partial Q v \sin \theta}{\partial \theta} \right] + \frac{\partial Q \dot{\eta}}{\partial \eta} = 0 , \quad (25)$$

where $Q = p_{es} q$.

Let

$$HA(Q) = \frac{1}{a \sin \theta} \left[\frac{\partial Q u}{\partial \lambda} + \frac{\partial Q v \sin \theta}{\partial \theta} \right] ; \quad VA(Q) = \frac{\partial Q \dot{\eta}}{\partial \eta} . \quad (26)$$

The horizontal advection spatial centre finite difference scheme can be written as the following by using "half-space-increment" finite difference method.

$$\begin{aligned} HA(Q)_{i,j,k}^c &= \frac{1}{a \sin \theta_j} \left(\frac{2}{\Delta \lambda} \frac{\delta_{\frac{1}{2} \Delta \lambda}^{1 \times \times \times}}{2^{\frac{1}{2} \Delta \theta}} u Q + \frac{2}{\Delta \theta} \frac{\delta_{\frac{1}{2} \Delta \theta}^{1 \times \times \times}}{2^{\frac{1}{2} \Delta \lambda}} v \sin \theta Q \right)_{i,j,k} \\ &= \frac{1}{2} [Vd_{i+\frac{1}{4}, j+\frac{1}{4}} (Q_{i,j} + Q_{i+\frac{1}{2}, j+\frac{1}{2}}) \\ &\quad - Vd_{i-\frac{1}{4}, j-\frac{1}{4}} (Q_{i,j} + Q_{i-\frac{1}{2}, j-\frac{1}{2}}) \\ &\quad + Vd_{i+\frac{1}{4}, j-\frac{1}{4}} (Q_{i,j} + Q_{i+\frac{1}{2}, j-\frac{1}{2}}) \\ &\quad - Vd_{i-\frac{1}{4}, j+\frac{1}{4}} (Q_{i,j} + Q_{i-\frac{1}{2}, j+\frac{1}{2}})]_{i,j,k} . \end{aligned} \quad (27)$$

The vertical advection spatial centre finite difference scheme can be shown as:

$$\begin{aligned} VA(Q)_{i,j,k}^c &= \frac{1}{\Delta \eta_k} [\delta_{\eta} (\dot{\eta} \bar{Q})]_{i,j,k} \\ &= \frac{1}{2 \Delta \eta_k} [\dot{\eta}_{k+\frac{1}{2}} (Q_k + Q_{k+1}) - \dot{\eta}_{k-\frac{1}{2}} (Q_k + Q_{k-1})]_{i,j} . \end{aligned} \quad (28)$$

Based on (27) and (28), an E-grid "upstream scheme" can be defined as follows:

$$\begin{aligned} HA(Q)_{i,j,k}^{us} &= \frac{1}{2} [(Vd + |Vd|)_{i+\frac{1}{4}, j+\frac{1}{4}} Q_{i,j} + (Vd - |Vd|)_{i+\frac{1}{4}, j+\frac{1}{4}} Q_{i+\frac{1}{2}, j+\frac{1}{2}} \\ &\quad - (Vd - |Vd|)_{i-\frac{1}{4}, j-\frac{1}{4}} Q_{i,j} - (Vd + |Vd|)_{i-\frac{1}{4}, j-\frac{1}{4}} Q_{i-\frac{1}{2}, j-\frac{1}{2}}] \end{aligned}$$

$$\begin{aligned}
& + (Vd + |Vd|)_{i+\frac{1}{4}, j-\frac{1}{4}} Q_{i, j} + (Vd - |Vd|)_{i+\frac{1}{4}, j-\frac{1}{4}} Q_{i+\frac{1}{2}, j-\frac{1}{2}} \\
& - (Vd - |Vd|)_{i-\frac{1}{4}, j+\frac{1}{4}} Q_{i, j} - (Vd + |Vd|)_{i-\frac{1}{4}, j+\frac{1}{4}} Q_{i-\frac{1}{2}, j+\frac{1}{2}}]_k \quad , \quad (29) \\
VA(Q)_{i, j, k}^{**} & = \frac{1}{2\Delta\eta_k} [(\dot{\eta} + |\dot{\eta}|)_{k+\frac{1}{2}} Q_k + (\dot{\eta} - |\dot{\eta}|)_{k+\frac{1}{2}} Q_{k+1} \\
& - (\dot{\eta} - |\dot{\eta}|)_{k-\frac{1}{2}} Q_k - (\dot{\eta} + |\dot{\eta}|)_{k-\frac{1}{2}} Q_{k-1}]_{ij} \quad . \quad (30)
\end{aligned}$$

It can be proved that the upstream moisture advection finite difference scheme

$$Q_{i, j, k}^{n+1} = Q_{i, j, k}^n - \Delta t (HA(Q)_{ij,k}^{**} + VA(Q)_{ij,k}^{**})^n \quad (31)$$

is of positivity, mass conservation and conditional stability similar to general upstream scheme in A or C grid.

Furthermore, based on the upstream scheme, the new positive definite advection scheme of Yu (1994) can be applied as follows:

$$Q_{i, j, k}^* = Q_{i, j, k}^n - \beta \Delta t [HA(Q)_{ij,k}^* + VA(Q)_{ij,k}^*]^n \quad , \quad (32)$$

$$Q_{i, j, k}^{n+1} = Q_{i, j, k}^n - \Delta t [HA(Q)_{ij,k}^{**} + VA(Q)_{ij,k}^{**}]^n \quad , \quad (33)$$

where $1 \leq \beta \leq 2$ (refer to Yu, 1994) and

$$\begin{aligned}
HA(Q)_{i, j, k}^* & = \frac{1}{2} [(Vd + |Ve|)_{i+\frac{1}{4}, j+\frac{1}{4}} Q_{i, j} + (Vd - |Ve|)_{i+\frac{1}{4}, j+\frac{1}{4}} Q_{i+\frac{1}{2}, j+\frac{1}{2}} \\
& - (Vd - |Ve|)_{i-\frac{1}{4}, j-\frac{1}{4}} Q_{i, j} - (Vd + |Ve|)_{i-\frac{1}{4}, j-\frac{1}{4}} Q_{i-\frac{1}{2}, j-\frac{1}{2}} \\
& + (Vd + |Ve|)_{i+\frac{1}{4}, j-\frac{1}{4}} Q_{i, j} + (Vd - |Ve|)_{i+\frac{1}{4}, j-\frac{1}{4}} Q_{i+\frac{1}{2}, j-\frac{1}{2}} \\
& - (Vd - |Ve|)_{i-\frac{1}{4}, j+\frac{1}{4}} Q_{i, j} - (Vd + |Ve|)_{i-\frac{1}{4}, j+\frac{1}{4}} Q_{i-\frac{1}{2}, j+\frac{1}{2}}]_k \quad , \quad (34)
\end{aligned}$$

$$\begin{aligned}
VA(Q)_{i, j, k}^* & = \frac{1}{2\Delta\eta_k} [(\dot{\eta} + |\dot{\eta}e|)_{k+\frac{1}{2}} Q_k + (\dot{\eta} - |\dot{\eta}e|)_{k+\frac{1}{2}} Q_{k+1} \\
& - (\dot{\eta} - |\dot{\eta}e|)_{k-\frac{1}{2}} Q_k - (\dot{\eta} + |\dot{\eta}e|)_{k-\frac{1}{2}} Q_{k-1}]_{ij} \quad , \quad (35)
\end{aligned}$$

$$\begin{aligned}
HA(Q)_{i, j, k}^{new} & = \frac{1}{2} [(Vd + |\tilde{V}e|)_{i+\frac{1}{4}, j+\frac{1}{4}} Q_{i, j} + (Vd - |\tilde{V}e|)_{i+\frac{1}{4}, j+\frac{1}{4}} Q_{i+\frac{1}{2}, j+\frac{1}{2}} \\
& - (Vd - |\tilde{V}e|)_{i-\frac{1}{4}, j-\frac{1}{4}} Q_{i, j} - (Vd + |\tilde{V}e|)_{i-\frac{1}{4}, j-\frac{1}{4}} Q_{i-\frac{1}{2}, j-\frac{1}{2}} \\
& + (Vd + |\tilde{V}e|)_{i+\frac{1}{4}, j-\frac{1}{4}} Q_{i, j} + (Vd - |\tilde{V}e|)_{i+\frac{1}{4}, j-\frac{1}{4}} Q_{i+\frac{1}{2}, j-\frac{1}{2}} \\
& - (Vd - |\tilde{V}e|)_{i-\frac{1}{4}, j+\frac{1}{4}} Q_{i, j} - (Vd + |\tilde{V}e|)_{i-\frac{1}{4}, j+\frac{1}{4}} Q_{i-\frac{1}{2}, j+\frac{1}{2}}]_k \quad , \quad (36)
\end{aligned}$$

$$VA(Q)_{i, j, k}^{new} = \frac{1}{2\Delta\eta_k} [(\dot{\eta} + |\tilde{\eta}e|)_{k+\frac{1}{2}} Q_k + (\dot{\eta} - |\tilde{\eta}e|)_{k+\frac{1}{2}} Q_{k+1}$$

$$-(\dot{\eta} - |\tilde{\eta}e|)_{k-\frac{1}{2}} Q_k - (\dot{\eta} + |\tilde{\eta}e|)_{k-\frac{1}{2}} Q_{k-1} l_{tj} , \quad (37)$$

$$|Ve| = \Delta t |Vd| |Vd|; \quad |\dot{\eta}e| = \Delta t |\dot{\eta}| |\dot{\eta}| / \Delta \eta , \quad (38)$$

$$|\tilde{V}e| = \alpha_1 |Vd|; \quad \alpha_1 = 1 \text{ or } \Delta t |Vd|; \quad (\text{refer to Yu, 1994}) , \quad (39)$$

$$|\tilde{\eta}e| = \alpha_2 |\dot{\eta}|; \quad \alpha_2 = 1 \text{ or } \Delta t |\dot{\eta}| / \Delta \eta; \quad (\text{refer to Yu, 1994}) . \quad (40)$$

The consistency, conservation, positivity and stability of the scheme(33) can be proved as Yu (1994).

IV. THE TRIAL-FORECASTING RESULTS

In this section, six local heavy rain cases were trial-forecasted based on LASG-REM with the TSPAS. The model dynamical framework can refer to Section 2 of this paper and Yu (1989). The physical processes of the model consist of: (1) large-scale precipitation; (2) modified Betts(1986) convective adjustment scheme; (3) a simple Bulk Aerodynamic PBL parameterization scheme. The model covers twenty five degrees longitude by twenty degrees latitude. The horizontal resolution of the model is a superposition of two C-grid with horizontal mesh widths of $1^\circ \times 1^\circ$ longitude latitude for each. In the vertical, the model is divided equally into 8 layers with $\Delta \eta = 0.125$, and $\eta = 0$ at $p_t = 100$ hPa. For all of the following integrations the models were run to produce 24-hour forecasts using almost some model parameters.

Six local heavy rain cases were chosen to examine the efficiency of the moisture advection scheme by using TSPAS to predict precipitation. First case is the world well-known 75.8 Henan China extraordinary heavy rain, the maximum 24-hour rainfall amount for the period 8/7/00Z through 8/8/00Z, 1975 has reached above 1000 millimeters. The second case is also an extraordinary local heavy rain which occurred near the outlet of Changjiang-Sanxia of China, the maximum 24-hour rainfall amount for the period 8/14/00Z through 8/15/00Z, 1990 is about 420 millimeters. The other four heavy rain cases took place in 1992 and located in the east periphery of the Tibetan Plateau (west Sichuan of China) where local heavy rains occur frequently.

Fig.1 shows the 24-hour observed(dashed line) and trial-forecasted (solid line) rainfall amount. The longitude and latitude of point(1,1) in each sub-graph have been shown in the lower left of the sub-graph, and the initial date of each sub-graph has been shown in the lower right. The smallest isoline is 25 mm and other isolines are 50 mm, 100 mm and 200 mm. The numerals rotated clockwise 45 degrees are the observed precipitation centre values and the not rotated numerals are the forecasted centre values. The LASG-REM with the TSPAS in moisture equation forecasted well the local characteristics and precipitation intensity, in the sense of mesh average rainfall amount, of these heavy rain cases. For all of these integrations never have negative moisture values occurred. Before being introduced the TSPAS in moisture equation, the LASG-REM used explicit diffusion to prevent computing instability of its explicit advection scheme. The integration results are sensitive to the explicit diffusion coefficients and it is difficult to have good results for all of cases in same model parameters. It is more important that non-positive definite advection scheme suffers from negative values, which is unreasonable in physics and unfavourable to solve same material transportation problem.

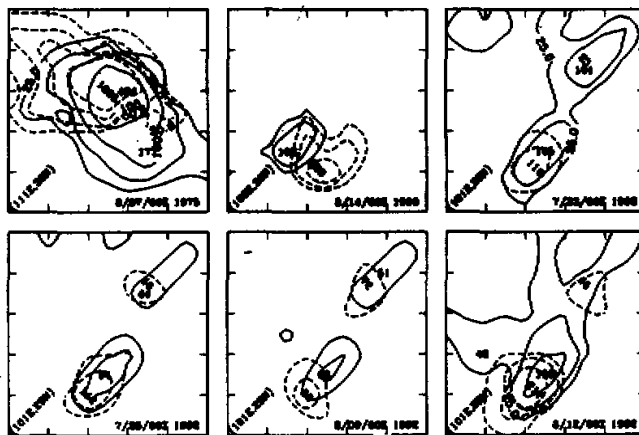


Fig. 1. 24-hour observed(dashed line) and trial-forecasted (solid line) rainfall amount, the smallest isoline is 25 mm, the rotated numerals are the observed precipitation centre values and the not rotated numerals are the forecasted centre values.

REFERENCES

- Yu Rucong (1989), Design of the limited area numerical weather prediction model with steep mountains, *Chinese Journal of Atmospheric Sciences*, 13(2): 145-158.
- Yu Rucong (1994), A New Two-Step Shape-Preserving Advection Scheme, *Advances in Atmospheric Sciences*, 11(4): 479-490.