Multivariate Objective Analysis of Wind and Height Fields in the Tropics

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ABSTRACT

The commonly used objective analysis scheme (Scheme—A) for the analysis of wind and geopotential height smoothen the divergent component of the wind which is rather important in the tropics, specifically over convective regions. To overcome this deficiency, a new analysis scheme in which divergent component is included in the statistical model of the wind forecast errors, has been proposed by Daley (1985). Following this scheme, a new set of correlation functions of forecast errors for the Indian region during monsoon season which are suitable for analysing the tropical wind are obtained. This analysis scheme (Scheme—B) as well as Scheme—A were used to make analyses for the period from 4 July to 8 July 1979 (12 GMT) at 850, 700 and 200 hPa levels over an area bounded by 1.875°N to 39.375°N and 41.250°E to 108.750°E and subsequently divergent component, velocity potential are computed for both schemes. Results from both these schemes show that in the monsoon depression region the velocity potential and divergence have increased in the later case (Scheme—B). This suggests that the divergent component has been enhanced in Scheme—B and that the objective of this study is realized to some extent.

Key words: Multivariate optimum interpolation scheme, Divergent part of the wind, Tropics, Objective analysis

I. INTRODUCTION

The objective analysis of wind and height fields in the tropics is more complex than at middle and higher latitudes. There are mainly two reasons (i) scarcity of observational data and (ii) the dynamical characteristics of the tropical atmosphere. In tropics, the convective process plays an important role in the development and evolution of the tropical systems. In most of the operational numerical weather prediction models, the amount of precipitation and consequently the latent heat released are underestimated during the first few hours of the forecast runs, particularly in tropics. According to Kasahara et al. (1988), this is because of the inaccuracies in the initial specification of the divergence and moisture fields. In order to define the convective regions of the atmosphere as closely to reality as possible, more attention should be paid to the divergent component of the wind.

In tropics the cumulus convection and the resulting diabatic heating are very important. Consequently the divergent part of the wind is comparable to the rotational part and is very important. Also, the important available potential energy—kinetic energy conversion term is dependent on a good estimate of the divergent wind. Tropical analysis with the nondivergent analysis scheme (Scheme—A) could be unrealistic since divergent wind can hardly be realized after the analysis. Julian (1984) on examination of FGGE analyses brought out the deficiencies in the analysis of tropical wind data related to the divergent part and the diabatic heating. Hence special efforts have to be made in the analysis scheme in order to retain the divergent part in the wind analysis, otherwise divergent part is smoothened out during analysis procedure.

Although, wind observations in tropics are more accurate, containing the divergent part, the present analysis scheme (Scheme–A) is not able to retain satisfactorily the divergent part. It is the shortcoming of the analysis scheme which smoothens the divergent part.

It is relevant to point out here that the geostrophic relation between wind and height fields is not valid specifically near the equator. This makes the multivariate objective analysis to degenerate into univariate analysis and thus the balance between height and wind fields is not ensured.

The cross–correlations between u– and v– components of wind based on a nondivergent statistical model of the wind forecast errors render the analysis increments (departure of the analysed field from the first guess field) nondivergent. Daley (1983, 1985) proposed a covariance model for prediction errors in terms of geopotential, stream function and velocity potential. Daley investigated the effects on two–dimensional analyses of cross–correlations between geopotential, stream function and velocity potential and the effects of varying the distribution of the wind prediction error between stream function and velocity potential error. Unden (1989) made analysis on operational basis at the ECMRWF using divergent structure function. He took into consideration three different vertical correlation functions for the velocity potential. In the present work we will discuss the scheme which would retain the divergent component of the wind in the tropics.

II. FORMULATION OF ANALYSIS SCHEME

We will first describe Gandin's (1963) theory of optimum interpolation. The analysis of geopotential height z, u– and v– components of wind at the grid point "gp" (z^a_{gp}, u^a_{gp}, v^a_{gp}) at a particular level are computed as the sum of linear combination of the weighted corrections (z^c_i, u^c_i, v^c_i) added to the first guess values (z^g_{gp}, u^g_{gp}, v^g_{gp}) at the grid point.

$$\begin{aligned} z^a_{gp} &= z^g_{gp} + \sum_{i=1}^{I} a^z_i z^c_i + \sum_{j=1}^{J} b^z_j u^c_j + \sum_{k=1}^{K} c^z_k v^c_k \ , \\ u^a_{gp} &= u^g_{gp} + \sum_{i=1}^{I} a^u_i z^c_i + \sum_{j=1}^{J} b^u_j u^c_j + \sum_{k=1}^{K} c^u_k v^c_k \ , \\ v^a_{gp} &= v^g_{gp} + \sum_{i=1}^{I} a^v_i z^c_i + \sum_{j=1}^{J} b^v_j u^c_j + \sum_{k=1}^{K} c^v_k v^c_k \end{aligned} \tag{1}$$

where $a^z, b^z, c^z, a^u, b^u, c^u, a^v, b^v, c^v$ are weights and z^c, u^c, v^c are the deviations of the observed values from the initial guess values at the observing locations.

In order to solve Eq.1 for obtaining the weights, one has to derive an expression for the mean square interpolation error (E) from it and then to impose the condition that the quantity E, is minimum.

$$E = \overline{\left[z^a_g - z^g_{gp} - \sum_{i=1}^{I} a^z_i z^c_i - \sum_{j=1}^{J} b^z_j u^c_j - \sum_{k=1}^{K} c^z_k v^c_k \right]^2} \ , \tag{2}$$

where the overbar denotes an ensemble average. This leads to a system of linear equations.

$$\begin{aligned} &\sum_{i=1}^{I} \overline{z_{l'} z_i}\, a^z_i + \sum_{j=1}^{J} \overline{z_{l'} u_j}\, b^z_j + \sum_{k=1}^{K} \overline{z_{l'} v_k}\, c^z_k = \overline{z_{l'} z_{gp}}, \qquad l' = 1, \cdots, I \\ &\sum_{i=1}^{I} \overline{u_{m'} z_i}\, a^z_i + \sum_{j=1}^{J} \overline{u_{m'} u_j}\, b^z_j + \sum_{k=1}^{K} \overline{u_{m'} v_k}\, c^z_k = \overline{u_{m'} z_{gp}}, \qquad m' = 1, \cdots, J \end{aligned} \tag{3}$$

$$\sum_{i=1}^{I}\overline{v_{n'}z_i}a_i^z+\sum_{j=1}^{J}\overline{v_{n'}u_j}b_j^z+\sum_{k=1}^{K}\overline{v_{n'}v_k}c_k^z=\overline{v_{n'}z_{gp}},\qquad n'=1,\cdots,K$$

and similar system for u_{gp} and v_{gp}. Upto this point our derivation has basically followed that of Schlatter (1975), Bergman (1979) and Sinha et al. (1992). The traditional method of deriving correlation functions between different variables on the basis of height–height correlation would not retain the divergent part of wind which is important for the wind field analysis in the tropics. Daley (1985) introduced correlation functions between three scalar quantities: height (z), Velocity potential (χ) and the stream function (ψ). Assuming homogeneity and isotropy of the scalar covariances, let us define the following nine covariance functions of z, χ, ψ.

$$\begin{aligned}
&\langle\psi_i\psi_j\rangle=E_\psi^2F(r_{ij})\ ,\\
&\langle\chi_i\chi_j\rangle=E_\chi^2G(r_{ij})\ ,\\
&\langle\psi_i\chi_j\rangle=\langle\chi_i\psi_j\rangle=E_\chi E_\psi H(r_{ij})\ ,\\
&\langle z_i\psi_j\rangle=\langle\psi_i z_j\rangle=E_Z E_\psi I(r_{ij})\ ,\\
&\langle z_i\chi_j\rangle=\langle\chi_i z_j\rangle=E_Z E_\chi J(r_{ij})\ ,\\
&\langle z_i Z_j\rangle=E_Z^2K(r_{ij})\ ,
\end{aligned}$$

where E_ψ, E_χ and E_Z are the errors of the stream function, velocity potential and height. F, G and H are the corresponding correlation functions and r_{ij} is the scalar distance between two arbitrary points i and j.

$$r_{ij}^2=(x_i-x_j)^2+(y_i-y_j)^2\ ,$$

where x_i, x_j and y_i, y_j are the cartesian coordinates of the points. On the basis of Helmholtz's theorem

$$u=-\frac{\partial\psi}{\partial y}+\frac{\partial\chi}{\partial x}\ ,\quad v=\frac{\partial\psi}{\partial x}+\frac{\partial\chi}{\partial y} \tag{5}$$

the prediction error correlations can be written as

$$\begin{aligned}
&E_v^2\langle u_iu_j\rangle^*=E_\psi^2\Gamma[F(r_{ij})]+E_\chi^2\Delta[G(r_{ij})]+2E_\chi E_\psi\Theta[H(r_{ij})]\ ,\\
&E_v^2\langle v_iv_j\rangle^*=E_\psi^2\Delta[F(r_{ij})]+E_\chi^2\Gamma[G(r_{ij})]-2E_\chi E_\psi\Theta[H(r_{ij})]\ ,\\
&E_v^2\langle v_iu_j\rangle^*=E_v^2\langle u_iv_j\rangle^*\\
&\qquad=E_\psi^2\Theta[F(r_{ij})]-E_\chi^2\Theta[G(r_{ij})]+E_\chi E_\psi\Lambda[H(r_{ij})]\ ,\\
&E_vE_Z\langle z_iu_j\rangle^*=-E_vE_z\langle u_iz_j\rangle^*=E_zE_\psi\Xi[I(r_{ij})]-E_zE_\chi\Pi[J(r_{ij})]\ ,\\
&E_vE_z\langle z_iv_j\rangle^*=-E_vE_z\langle v_iz_j\rangle^*=-E_zE_\psi\Pi[I(r_{ij})]-E_zE_\chi\Xi[J(r_{ij})]\ ,\\
&E_z^2\langle z_iz_j\rangle^*=E_z^2K(r_{ij})\ ,
\end{aligned} \tag{6}$$

where

$$\Gamma=-\left[\frac{1}{r}\frac{\partial}{\partial r}+(y_i-y_j)^2\frac{1}{r}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\right],$$

$$\Delta=-\left[\frac{1}{r}\frac{\partial}{\partial r}+(x_i-x_j)^2\frac{1}{r}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\right],$$

$$\Theta = \left[(x_i - x_j)(y_i - y_j) \frac{1}{r}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} \right] , \qquad (7)$$

$$\Lambda = \left[(y_i - y_j)^2 - (x_i - x_j)^2 \right] \frac{1}{r}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} ,$$

$$\Xi = \left[\frac{(y_i - y_j)}{r}\frac{\partial}{\partial r} \right] ,$$

$$\Pi = \left[\frac{(x_i - x_j)}{r}\frac{\partial}{\partial r} \right] .$$

We will assume that the autocorrelation function of the three quantities ψ, χ and z are identical and have the form of Gaussian distribution.

$$G(r_{ij}) = F(r_{ij}) = K(r_{ij}) = a \times \exp(-b \times r_{ij}^2) . \qquad (8)$$

Since there exists a close correlation between the height and stream function even at low latitude for the slow motion of the atmosphere (Matsuno, 1966), we can assume that

$$I = \mu F , \qquad (9)$$

where μ is a measure of geostrophic approximation and which relates the stream function–height correlation to the autocorrelation of stream function. We will further assume that H and J are related to F as follows:

$$H = \lambda F ,$$
$$J = \lambda^* F , \qquad (10)$$

where λ and λ^* are constans and relate the velocity potential–stream function and velocity potential–height correlations with the stream function autocorrelation. Thus we can write the prediction error correlations as a function of F and G

$$\begin{aligned}
\langle u_i u_j \rangle^* &= (1-v)\Gamma(F) + v\Delta(G) + 2(v - v^2)^{1/2}\lambda\Theta(F) , \\
\langle v_i v_j \rangle^* &= (1-v)\Delta(F) + v\Gamma(G) - 2(v - v^2)^{1/2}\lambda\Theta(F) , \\
\langle u_i v_j \rangle^* &= \langle v_i u_j \rangle^* = (1-v)\Theta(F) - v\Theta(G) + (v - v^2)^{1/2}\lambda\Lambda(F) , \qquad 2(11) \\
\langle z_i u_j \rangle^* &= -\langle u_i z_j \rangle^* = (1-v)^{1/2}\mu\Xi(F) - \sqrt{v}\lambda^*\Pi(F) , \\
\langle z_i v_j \rangle^* &= -\langle v_i z_j \rangle^* = -(1-v)^{1/2}\mu\Pi(F) - \sqrt{v}\lambda^*\Xi(F) , \\
\langle z_i z_j \rangle^* &= F ,
\end{aligned}$$

where the parameter v defined as

$$v = E_{v_\chi}^2 / E_v^2 \qquad (12)$$

$$= \frac{\text{Prediction error variance in the divergent wind component}}{\text{Prediction error variance of the total wind}} ,$$

$$(1 - v) = E_{v_\psi}^2 / E_v^2 \qquad (13)$$

The asterisk shows the correlation function which is different from the above defined covariance function. Seaman and Gauntlett (1980) found that λ and λ^* were very small. Following Daley (1985) we too decided to set λ and λ^* to zero.

With the above assumptions we can finally write the nine prediction error correlations.

$$
\begin{aligned}
\langle u_i u_j \rangle^* &= \{(1-\nu)[2b-(y_i-y_j)^2 4b^2] + \nu[2b-(x_i-x_j)^2 4b^2]\} a e^{-b r_{ij}^2} , \\
\langle v_i v_j \rangle^* &= \{(1-\nu)[2b-(x_i-x_j)^2 4b^2] + \nu[2b-(y_i-y_j)^2 4b^2]\} a e^{-b r_{ij}^2} , \\
\langle u_i v_j \rangle^* &= \langle v_i u_j \rangle^* = (1-2\nu) 4b^2 (y_i-y_j)(x_i-x_j) a e^{-b r_{ij}^2} , \\
\langle z_i u_j \rangle^* &= -\langle u_i z_j \rangle^* = -\sqrt{(1-\nu)}\, \mu 2b (y_i-y_j) a e^{-b r_{ij}^2} , \\
\langle z_i v_j \rangle^* &= -\langle v_i z_j \rangle^* = \sqrt{(1-\nu)}\, \mu 2b (x_i-x_j) a e^{-b r_{ij}^2} , \\
\langle z_i z_j \rangle^* &= a e^{-b r_{ij}^2} ,
\end{aligned}
\tag{14}
$$

when $\nu = 0$, $\mu = 0.1$, Eq.14 degenerates into the quasigeostrophic and nondivergent situation. If we take $\nu = 0$, $\mu = 0$ there is no relation between wind and height and the multivariate analysis will be degenerated into univariate analysis. The parameter ν (divergence parameter) describes how the wind components u– and v– are related and μ (geostrophic parameter) describes how z is to be related to u and v.

III. ANALYSIS EXPERIMENTS

1. *Synoptic Situation*

A real data experiment was carried out over India and adjoining region (region bounded by 1.875°N to 39.375°N and 41.250°E to 108.750°E) with a grid resolution of 1.875°. In order to see how the new scheme (Scheme–B) performs under realistic condition, we have made analysis for the period 4 to 8 July 1979, 12 GMT for 850, 700 and 200 hPa levels. We have chosen this period because during this period, a low was formed over the Head of Bay of Bengal with its central region near 20°N and 90°E on 4 July 1979 and moved westnorthwestward. It moved slowly at the beginning and intensified into a depression on 7 July and crossed the Indian coast on 8 July.

2. *Discussions*

a. Costants μ and ν

The parameters μ and ν are multivariate coupling parameters. In order to obtain the proper value of divergence, following Daley (1985) we set $\mu = 0.1$ and $\nu = 0.1$. He found that for $\nu = 0.1$, the ratio of standard deviation of the divergent part to the total wind forecast error is 0.32 which agrees reasonably well with the result of Hollingsworth and Lonnberg (1986). Also the effect of the analyzed rotational wind is small since the standard deviation of the rotational part of the wind error is only reduced by 5% with this choice of ν.

b. Wind field

In this experiment we have made analysis with nondivergent structure function (Scheme–A) as well as with divergence structure function (Scheme–B) with $\nu = 0.1$ for the three levels and five days as mentioned earlier. From these analyzed wind field at the grid points, divergence and velocity potential are calculated. We will present here the results of only 7 July 1979 for 700 and 200 hPa levels. Fig. 1(a) and Fig. 1(b) show the wind analyses at 700 and 200 hPa levels using Scheme–B. The wind speeds and directions are indicated by the arrows. The magnitude of the wind is indicated in upper right hand corner which shows a wind arrow and the corresponding speed in m / s. It is found that the main synoptic features viz. centre of the depression (700 hPa) and strong westerly jet (200 hPa) are well depicted in the analyses.

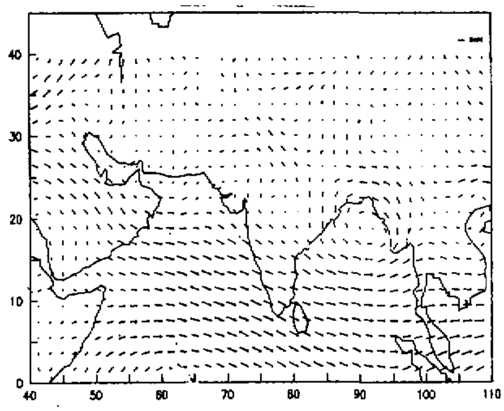

Fig. 1(a). Wind analysis of 700 hPa level using Scheme–B.

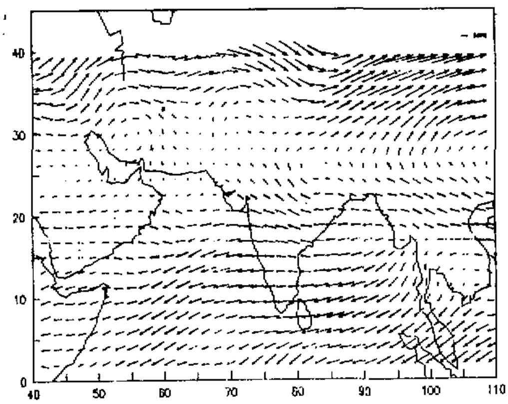

Fig. 1(b). Same as Fig. 1(a) but for 200 hPa level.

Fig. 2(a) and Fig. 2(b) show the differences between the two types of wind analyses (Scheme–B minus Scheme–A) for 700 and 200 hPa levels respectively. The wind difference shows clearly the impact of divergent analysis specially at 200 hPa level. The maximum wind speed difference was around 8 m / s at 200 hPa and is shown in Fig. 2(b). The largest wind speed difference at 700 hPa level is of the order of 6 m / s, Fig. 2(a). We have also made analyses with different values of v to test the sensitivity. Analysis with $v = 0.05$ and $v = 0.2$ at 200 hPa level produced features which are similar to Fig. 2(b). This is in aggrement with the theoretical computation made by Xue and Wang (1992). They found that the analysis is not so sensitive when v increases from 0.1 to 0.5.

c. Velocity Potential and Divergence

We have computed from the analyzed wind field using Scheme–A and Scheme–B the divergence and the velocity potential following the technique developed by Tandon (1992), in which truncated double Fourier series is used. Velocity potential gives the information about inflow / outflow and magnitude of divergence wind. Comparing Fig. 3(a) and Fig. 3(b) we found that the velocity potential has increased in the new scheme (Scheme–B). The maximum

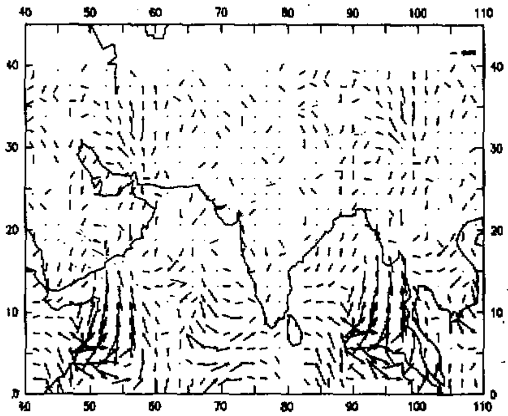

Fig. 2(a). Wind differences between two schemes (Scheme–B minus Scheme–A) at 700 hPa level.

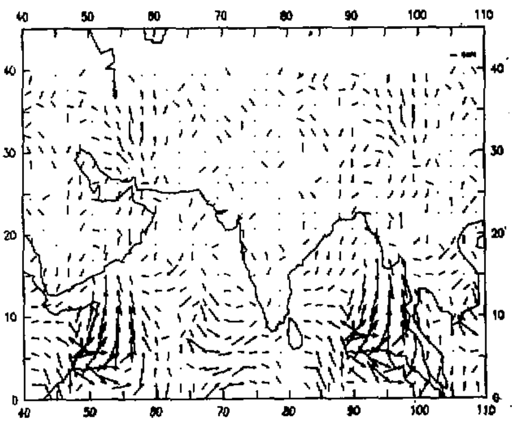

Fig. 2(b). Same as Fig. 2(a) but for 200 hPa level.

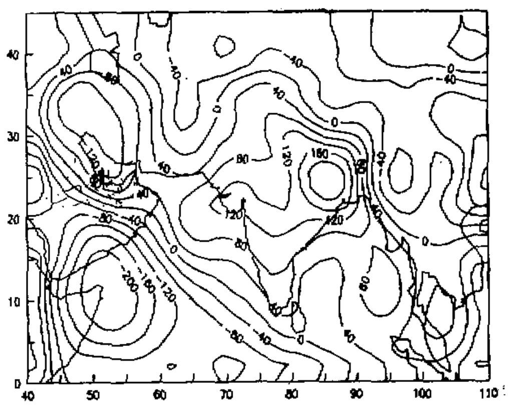

Fig. 3(a). Velocity potential at 700 hPa level using Scheme–B. Unit is $10^4 m^2 s^{-1}$.

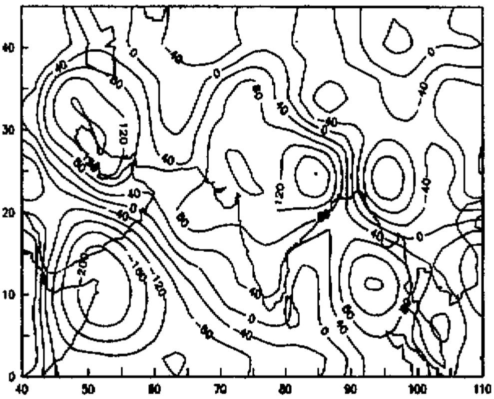

Fig. 3(b). Same as Fig. 3(a) but for Scheme–A.

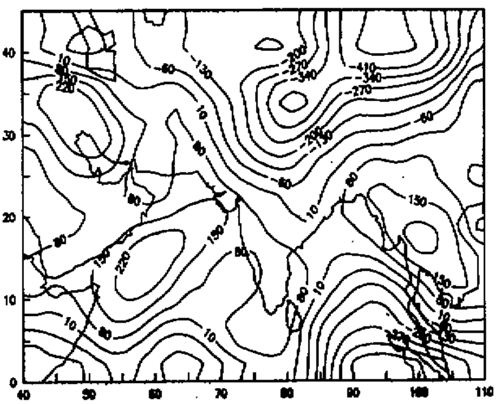

Fig. 4(a). Velocity potential at 200 hPa level using Scheme–B. Unit is $10^4m^2s^{-1}$.

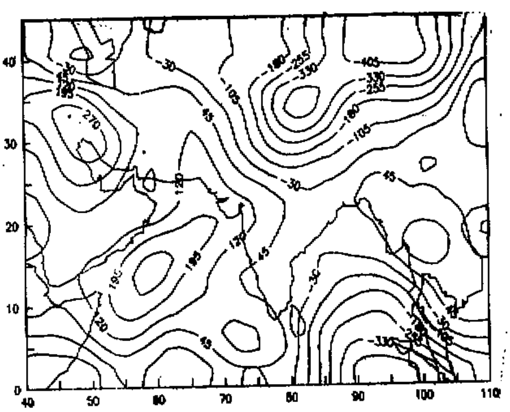

Fig. 4(b). Same as Fig. 4(a) but for Scheme–A.

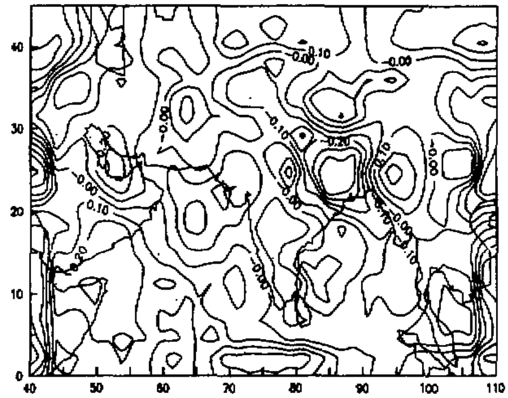

Fig. 5(a). Divergence at 700 hPa level for Scheme–B. Unit is $10^{-5}s^{-1}$.

Fig. 5(b). Same as Fig. 5(a) but for 200 hPa level.

value of velocity potential on 7 July at the grid point 24.375°N and 84.375°E at 700 hPa level with Scheme–B is $253\times10^4m^2s^{-1}$ whereas in the case of Scheme–A it is $214\times10^4m^2s^{-1}$. Similarly comparing Fig. 4(a) and Fig. 4(b) for 200 hPa level we found that there is no appreciable increase in the velocity potential from Scheme–A to Scheme–B. On examination of the divergence field in the case of Scheme–B (Fig. 5(a)) we found that value of divergence field at grid point 24.375°N and 84.375°E at 700 level is $-0.5\times10^{-5}s^{-1}$. Whereas at upper level (200 hPa) it is $0.9\times10^{-5}s^{-1}$. (Fig. 5(b)).

Let us examine the analysis increment of divergence (analyzed field minus first guess field) for the 200 hPa level for two schemes. From Fig. 6(a) and Fig. 6(b) we found that the values of divergence increment for the Scheme–B at the grid point 20.625°N and 84.375°E are $0.27\times10^{-5}s^{-1}$ whereas in the Scheme–A it is $0.13\times10^{-5}s^{-1}$. Thus on the basis of the above experiment we found that there is a positive impact in the analysis when divergent component is included in the statistical model of the wind forecast errors.

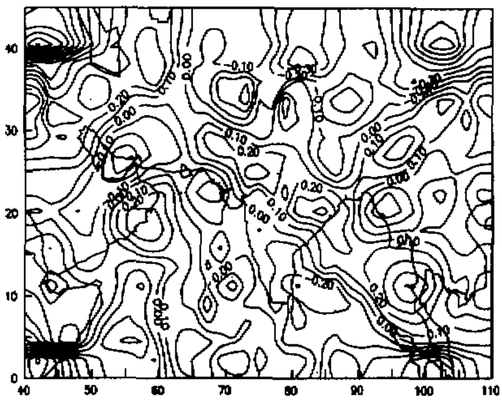

Fig. 6(a). Divergence increment at 200 hPa level for Scheme–B. Unit is $10^{-5}s^{-1}$.

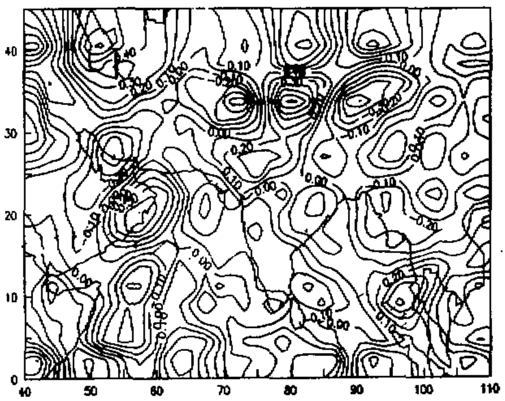

Fig. 6(b). Same as Fig. 6(a) but for Scheme–A.

Table 1. Root Mean Square Errors for u– and v– components of Wind in mps

		850 hPa		700 hPa		500 hPa	
		u	v	u	v	u	v
4.7.79	Scheme–A	4.1	3.6	3.3	4.3	6.6	6.4
	Scheme–B	2.3	2.5	2.6	2.8	6.8	5.5
5.7.79	Scheme–A	2.8	2.8	2.8	2.8	7.6	7.8
	Scheme–B	2.3	2.2	2.5	2.5	6.1	5.9
6.7.79	Scheme–A	3.2	2.9	2.8	3.0	6.3	5.4
	Scheme–B	2.0	1.9	2.6	2.5	4.9	5.0
7.7.79	Scheme–A	3.9	3.3	2.7	2.9	6.5	6.2
	Scheme–B	2.5	2.6	2.7	2.7	5.5	5.5
8.7.79	Scheme–A	3.4	3.8	4.1	4.5	6.7	6.6
	Scheme–B	2.9	3.0	2.9	3.5	5.8	5.3

Root mean square errors for both the schemes were computed by comparing the two sets of analyses with the FGGE analyses. From the table we found that RMS errors for Scheme–B at all the three levels on most of the days are less than the Scheme–A.

IV. CONCLUSIONS

We have investigated the analysis of divergent wind (which is very much important over the tropical region) based on Gandin's (1963) statistical interpolation procedure modified as per Daley's (1985) scheme of synoptic scale divergence. In this study we have not taken into account the vertical correlation problem and confined to only two–dimensional horizontal analysis problem.

The parameters μ and v are multivariate coupling parameters. μ describes how height field is related to wind field and v describes how wind components u and v are related.

Experiments conducted with this new scheme (Scheme–B) produce analyses with enhanced divergence and velocity potential as compared to Scheme–A. Improved analysis of divergence in tropical region is important as the inflow of low level moisture improves the simulation of cumulus convection.

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