Some Unique Characteristics of Atmospheric Interannual Variability in Rainfall Time Series over India and the United Kingdom

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ABSTRACT

Continuous periodogram analyses of two 50–years (1871–1920 and 1936–1985) of summer monsoon rainfall over the Indian region and one 84–years set (1893–1976) of winter half–year rainfall over England and Wales show that the power spectra of disparate rainfall regimes follow the universal and unique inverse power law form of the statistical normal distribution with the percentage contribution to total variance representating the eddy probability corresponding to the normalized standard deviation equal to [(log L / log T_{50})–1] where L is the period in years and T_{50} the period up to which the cumulative percentage contribution to total variance is equal to 50. The above results are consistent with a recently developed non–deterministic cell dynamical system model for atmospheric flows. The implications of the above results for prediction of interannual variability of rainfall is discussed.

Key words: Interannual variability in rainfall, Chaos, Fractals, Self–organized criticality

I. INTRODUCTION

The interannual variability of atmospheric flows as recorded in meteorological parameters such as windspeed, temperature and pressure at the earth's surface and in the atmospheric column extending up to the stratosphere has been investigated extensively and major quasiperiodic oscillations such as the QBO (quasi–biennial oscillation) and the 3–7 year ENSO (El Nino / Southern Oscillation) cycle have been identified (Lamb, 1972; Philander, 1990; Burroughs, 1992). Such dominant cycles are however superimposed on an appreciable "background noise" contributed by a continuum of eddies of all scales within the time, space scales investigated (Lorenz, 1990; Tsonis and Elsner, 1990). It is important therefore, to identify the physics of multiple scale interactions (Barnett, 1991) and quantify the total pattern of fluctuations of atmospheric flows for predictability studies. Long–range spatiotemporal correlations manifested as the selfsimilar fractal geometry to the global cloud cover pattern and the inverse power law form for atmospheric eddy energy spectrum documented by Lovejoy and Schertzer (1986) and Tessier et al. (1993) are signatures of selforganized criticality (Bak, Tang and Wiesenfeld, 1988) or deterministic chaos (Mary Selvam, 1990, 1993; Mary Selvam et al., 1992; Selvam and Radhamani, 1994; Selvam and Joshi, 1994) in atmospheric flows. The physics of self–organized criticality is not yet identified. In this paper a cell dynamical model for atmospheric flows (Mary Selvam, 1990; Mary Selvam et al., 1992) is summarized. The model predicts self–organized criticality as intrinsic to quantum–like mechanics governing atmospheric flows and, as a natural consequence leads to the result that the atmospheric eddy energy spectrum represents the statistical normal distribution. The model predictions are in agreement with continuous periodogram analyses of two sets of 50–years summer monsoon rainfall over the Indian region and one set of 84–years winter half–year rainfall over England and Wales. Such unique quantification, namely the inverse power law form of the statistical normal distribution for the atmospheric eddy energy spectrum implies predictability of the total pattern of atmospheric fluctuations. The applications of the above result for prediction of interannual variabilty of atmospheric flows is discussed.

II. CELL DYNAMICAL SYSTEM MODEL FOR ATMOSPHERIC FLOWS

In summary (Mary Selvam, 1990; Mary Selvam et al., 1992; Mary Selvam, 1993) the model is based on Townsend's concept (Townsend, 1956) that large eddies form as envelopes of enclosed turbulent eddies in fluid flows. The root mean square (r. m. s) speeds W, w and corresponding radii R, r of large and turbulent eddy respectively are related as

$$W^2 = \frac{2}{\pi}\frac{r}{R}w^2 \quad . \tag{1}$$

Eq.(1) is a statement of law of conservation of energy for large eddy growth from turbulent fluctuations with ordered energy flow between the larger and smaller scales.

(1) Since the large eddy is but the integrated mean of enclosed turbulent eddies the eddy energy (kinetic) distribution follows statistical normal distribution according to Central Limit Theorem.

The square of the eddy amplitude or variance is proportional to kinetic energy and represents the probability. Such a result that the additive amplitude of eddies, when squared, represents probability densities is found in the subatomic dynamics of quantum systems such as the electron or photon. Atmospheric flows therefore follow quantum-like mechanical laws.

(2) Atmospheric flows consist of an overall logarithmic spiral with the quasiperiodic Penrose tiling pattern for the internal structure.

Conventional power spectral analysis will resolve such spiral flow structure as a continuum of eddies with progressive increase in phase angle. The eddy continuum will have embedded dominant wavebands, the bandwidth increasing with period length. The dominant peak periodicities P_n are given as

$$P_n = \tau^n (2+\tau)T \ , \tag{2}$$

where τ is the golden mean equal to $(1+\sqrt{5})/2 = 1.618$, T is the primary perturbation time period equal to the annual (summer to winter) cycle of solar heating in the present study and n is an integer ranging from negative to positive values including zero.

(3) The power spectrum will follow the universal inverse power law form of the statistical normal distribution with the percentage probability corresponding to normalized standard deviation t gives as

$$t = (\log L / \log T_{50}) - 1 \ ,$$

where L is the period in years and T_{50} the period upto which the cumulative percentage contribution to total variance is equal to 50.

III. DATA AND ANALYSIS

The following rainfall time series were used in the study.

(1) Indian region summer monsoon (June–September) rainfall for 29 meteorological subdivisions for two sets of 50–years periods (1871–1920, 1936–1985) (Parthasarathy et al., 1987).

(2) England and Wales winter half–year rainfall (as % 1941–1970 annual average rain-

fall (AAR)=912 mm) for the period 1893/94 to 1975/76 (Jenkinson, 1977). The broadband power spectrum of the rainfall time series can be computed accurately by an elementary but very powerful method of analysis developed by Jankinson (1977) which provides a quasicontinuous form of the classical periodogram allowing systematic allocation of the total variance and degrees of freedom of the data series to logarithmically spaced elements of the frequency range (0.5, 0). The periodogram is constructed for a fixed set of 10000 (m) periodicities which increase geometrically as $L_m = 2\ \exp\ (Cm)$ where $C=0.001$ and $m=0, 1, 2 \cdots$ m. The data series y_t for the N data points was used. The periodogram estimates the set of $A_m \cos(2\pi v_m s - \varphi_m)$ where A_m, v_m and φ_m denote respectively the amplitude, frequency and phase angle for the mth periodicity and s is the time in years. The cumulative percentage contribution to total variance was computed starting from the high frequency side of the spectrum. The period T_{50} at which 50% contribution to total variance occurs is taken as reference and the normalized standard deviation t_m values are computed as

$$t_m = (\log L_m / \log T_{50}) - 1 \ .$$

The cumulative percentage contribution to total variance and the corresponding t–values are plotted as continuous lines in Figs. 1–3 for the Indian and U. K. rainfall respectively.

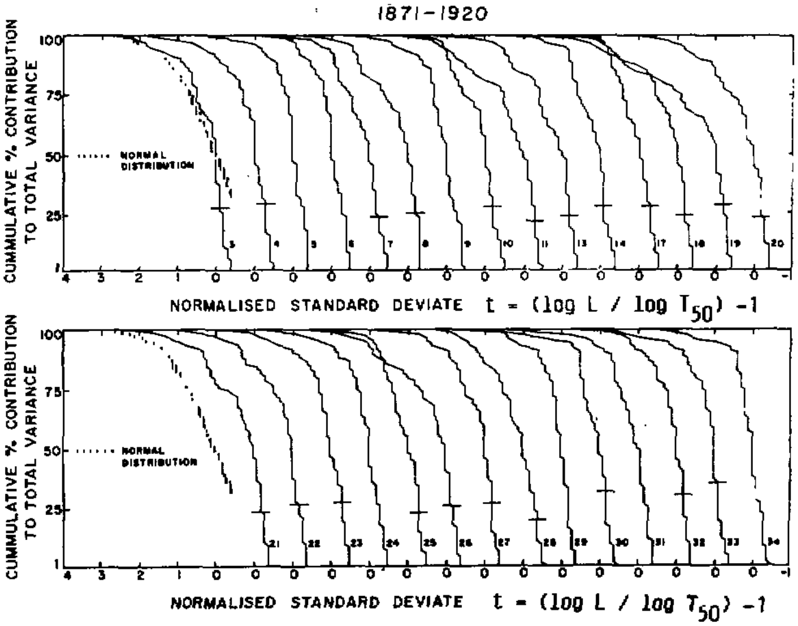

Fig. 1. The periodogram of 50 years (1871–1920) summer monsoon (June–September) rainfall for 29 meteorological sub–divisions (numbers ranging from 3 to 34) plotted with progressive shift in zero by one division on the t–axis. The meteorological sub–division numbers are indicated on the lower pert of the figure. The continuous line is the periodogram while the crosses refer to the corresponding cumulative normal probability density distributions. The horizontal lines on the lower part of the figure indicate the lower limit above which the periodogram is the same as the cumulative normal probability density distribution as determined by the chi–square test at 95% level significance. Absence of the horizontal line indicates that the fit is not significant.

Table 1: Periodogram Estimates for Indian Rainfall (1871–1920)

MET. SUB-DIV. No.	Mean mm	Standard deviation mm	T_{50} years	T_{75} years	T_{90} years	Periodicities (years) contributing to maximum normalised variance (H) in wave band with H ≥ 1
3	1460	200	4.00	7.77	13.89	2.9,3.3,3.6,3.9,4.6,6.4,7.7,9.5,12.7, 21.0,49.5
4	1469	184	3.86	6.03	13.59	2.1,2.3,2.5,2.7,2.9,3.1,3.3,3.6,4.0,6.1 9.8,13.9,24.8
5	2022	343	3.76	4.59	8.22	2.1,2.2,2.3,2.6,3.4,3.9,4.6,5.3,6.4,10.
6	1131	164	4.48	5.66	10.60	2.1,2.2,2.4,2.5,2.8,3.2,4.1,4.7,5.9, 7.8,9.8,16.4
7	1169	161	4.08	7.75	14.5	2.0,2.1,2.3,2.6,2.8,3.0,3.5,4.2,4.9, 5.6,7.4,13.5,25.9
8	1101	147	3.03	6.12	12.72	2.1,2.9,3.1,3.9,4.7,6.2,12.5,26.5
9	1060	217	3.94	6.12	9.72	2.1,2.3,2.8,3.0,3.4,3.9,4.9,6.3,9.5,26.
10	894	211	5.09	13.00	24.17	2.5,2.7,3.4,3.8,4.5,8.7,14.2,23.8
11*	760	200	3.79	7.43	35.52	2.0,2.2,2.5,2.7,2.9,3.1,3.8,4.6,5.2, 6.0,7.2,62.0
13	439	137	3.53	7.44	20.56	2.4,2.8,3.2,4.1,4.7,6.0,7.2,20.6
14*	469	174	3.24	6.51	8.09	2.0,2.3,2.5,2.8,3.2,3.5,4.1,5.9,7.0
17	253	112	4.44	7.81	13.04	2.0,2.2,2.4,2.7,3.2,3.4,4.1,4.7,7.3,12.
18	623	187	3.46	7.09	22.13	2.1,2.3,2.5,2.7,2.9,3.2,3.4,3.8,4.1,4.7 7.3,8.9,11.6,57.5
19	897	179	3.20	9.49	57.35	2.1,2.2,2.3,2.5,2.7,3.2,3.8,8.7,11.0,6[illegible]
20	1218	187	3.34	6.97	18.14	2.0,2.1,2.4,2.7,3.3,4.3,7.1,8.8,11.5,2[illegible]
21	860	275	3.44	8.16	16.81	2.1,2.5,2.8,3.5,4.1,6.1,19.1
22	437	195	3.23	5.76	14.70	2.0,2.2,2.8,3.4,4.0,10.2,20.1
23	2259	507	4.20	6.06	13.39	2.1,2.3,2.5,2.8,3.2,4.1,6.4,25.6
24	581	145	4.55	6.70	15.64	2.0,2.5,2.8,3.4,4.0,6.0
25	703	220	4.49	8.05	16.70	2.0,2.7,3.4,4.1,4.9,6.0,19.3
26	898	190	3.25	7.09	23.95	2.2,2.4,2.8,5.6,9.9,18.5
27	494	108.	3.43	6.19	10.49	2.4,2.7,4.4,20.7
28	692	182	4.78	10.71	26.98	2.7,3.4,4.0,6.3,18.7
29	428	134	3.47	4.57	7.25	2.1,2.8,3.3,4.5,6.0
30	319	77	3.79	6.37	7.72	2.1,2.5,4.8,5.9,11.1
31	2754	537	3.17	5.19	7.46	2.0,2.4,2.7,3.2,4.1,7.0,10.4
32	594	131	3.53	4.87	10.19	2.4,2.7,3.8,4.5,6.3,11.9,22.1
33	504	116	3.49	4.80	9.52	2.5,2.8,3.1,4.0,4.9,7.0,16.8
34	1910	367	3.81	5.13	6.49	2.0,2.2,3.3,6.8,9.9

* Indicates that the rainfall data does not follow normal distribution characteristics.

The cumulative normal probability density distribution corresponding to the normalized standard deviation *t* values are shown as crosses in Figs. 1–3. It is seen that the cumulative percentage contribution to total variance closely follows the cumulative normal probability density distribution. The "goodness of fit" was tested using the standard statistical chi–square test (Spiegel, 1961). The short horizontal lines in the lower pert of Figs. 1–3 indicate the lower limit above which the fit is good at 95% confidence level.

Table 2: Periodogram Estimates for Indian Rainfall (1936–1985)

MET. SUB-DIV. No.	Mean mm	Standard deviation mm	T_{50} years	T_{75} years	T_{90} years	Periodicities (years) contributing to maximum normalised variance (H) in wave band with $H \geq 1$
3	1436	135	3.54	10.28	19.17	2.1,2.7,3.4,4.3,9.7,19.6
4	1432	185	4.38	12.77	57.87	2.1,2.3,2.6,2.9,3.2,4.5,10.8,21.7,66.1
5	1941	256	4.97	8.92	15.41	2.1,2.4,2.8,3.2,4.1,5.1,6.3,9.3,16.5
6	1154	170	3.54	9.48	17.76	2.1,2.4,2.8,3.1,3.5,6.1,7.2,9.0,15.4,34
7	1159	155	4.12	23.04	64.66	2.2,2.7,3.2,4.1,14.7,23.7,66.8
8	1090	178	4.82	8.97	17.66	2.1,2.5,2.8,3.2,3.5,5.8,6.9,8.8,16.1, 29.8,73.4
9	1011	174	3.89	7.0	16.40	2.0,2.1,2.2,2.7,3.2,3.5,3.9,4.7,5.3,7.1 11.8,16.6,51.9
10	928	205	3.54	6.79	12.48	2.1,2.2,2.5,2.8,3.2,3.5,4.6,5.4,6.4,8.8 11.6,16.0,50.7
11	788	164	3.09	6.46	9.60	2.1,2.2,2.3,2.5,2.7,3.1,3.5,6.9,8.3,13.
13	477	131	3.89	10.82	46.07	2.4,2.4,2.7,3.0,3.8,5.8,6.9,8.4,10.5, 14.7,67.2
14	531	164	4.08	13.36	25.59	2.4,2.9,4.3,14.1,22.3,55.1
17	259	94	3.87	10.82	16.98	2.1,2.2,2.4,2.6,2.8,3.3,5.6,8.1,10.6, 15.6,26.0
18	651	155	4.26	13.90	16.98	2.1,2.4,2.6,2.8,3.1,3.5,4.1,8.3,15.0,25
19	948	156	3.34	12.15	15.20	2.1,2.2,2.4,2.7,3.2,4.9,7.3,14.2
20	1183	208	3.41	10.69	58.22	2.1,2.4,2.7,3.1,3.4,4.1,8.1,12.6,71.4
21	870	274	3.52	6.36	14.24	2.1,2.4,2.8,3.3,4.2,5.6,6.4,7.9,11.1,.6
22*	437	190	3.32	8.91	16.09	2.0,2.3,3.0,3.3,4.3,8.8,11.6,16.6,28.8
23	2515	438	4.88	9.60	28.11	2.0,2.3,2.5,2.7,3.1,4.0,5.0,6.7,9.4,27.
24	580	106	3.79	6.12	10.72	2.1,2.3,2.4,2.7,2.9,3.2,3.8,4.1,5.2,6.9 11.4,16.5
25	685	166	4.09	6.51	26.37	2.0,2.2,2.4,2.5,2.7,3.4,4.0,5.1,6.8,8.3 39.7
26	900	174	3.23	7.02	16.76	2.0,2.2,2.4,2.7,3.5,4.2,5.4,6.5,8.5,10. 18.8,80.7
27	521	119	3.01	9.64	26.39	2.1,2.2,2.4,2.7,2.8,3.3,4.1,5.4,27.3
28	745	158	2.89	6.73	20.11	2.1,2.2,2.4,2.5,2.7,4.0,5.5,7.2,11.2,28
29	425	112	3.70	5.60	9.84	2.1,2.3,2.4,2.8,3.2,3.7,4.1,4.4,5.5,6.3 8.9
30	311	64	2.84	8.82	12.72	2.0,2.1,2.3,2.4,2.8,3.1,5.9,11.4,78.2
31*	2939	509	3.87	6.90	13.23	2.1,2.3,2.5,2.7,3.1,3.5,4.1,7.0,12.2,26
32	620	112	4.42	8.59	30.85	2.2,2.5,2.7,3.7,4.0,4.4,5.3,6.2,7.1,11. 31.1
33	505	89	2.96	5.05	11.05	2.1,2.2,2.3,2.5,3.0,4.1,4.8,6.5,14.8
34*	1927	332	3.61	6.94	11.16	2.0,2.2,3.5,4.0,6.7,10.7

* Indicates that the rainfall data does not follow normal distribution characteristics.

Tables 1–3 give the following periodogram estimates corresponding to Figs. 1–3 respectively: (1) The mean and standard deviation of the data series. (2) The atmospheric eddies of periodicities up to T_{50}, T_{75} and T_{90} which contribute respectively to 50, 75 and 90

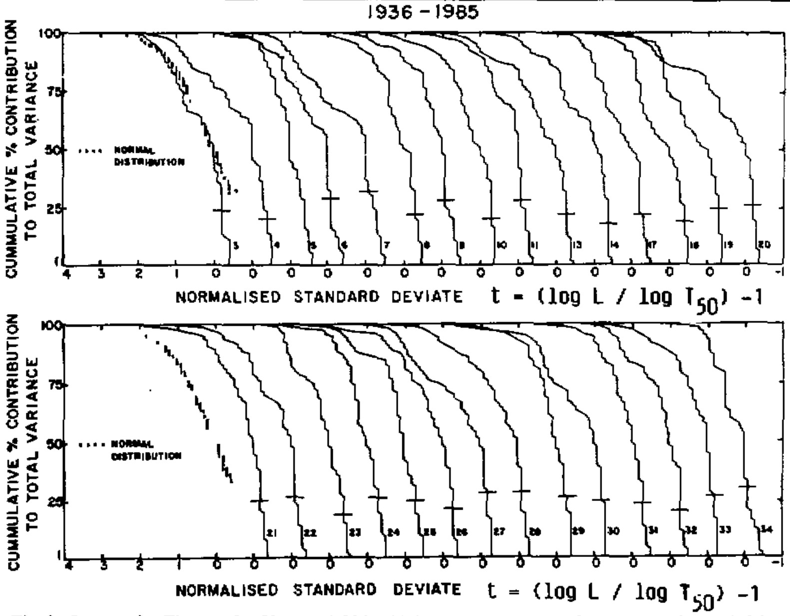

Fig. 2. Same as for Figure 1 for 50 years (1936–1985) summer monsoon (June–September) rainfall.

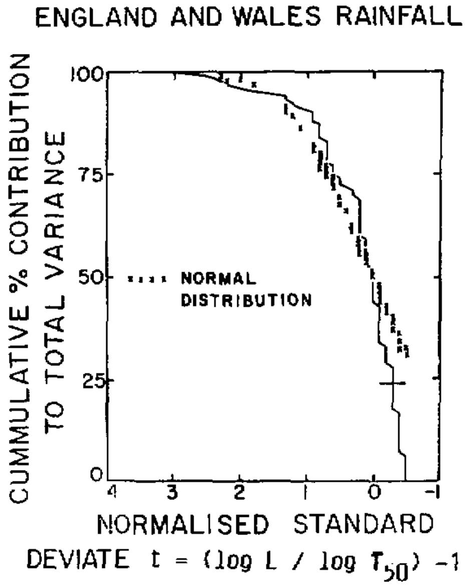

Fig. 3. The periodogram of England and Wales winter half–year rainfall for 84 years (1893–1976). The continuous line is the periodogram while the crosses refer to the corresponding cumulative normal probability density distribution. The horizontal line on the lower pert of the figure indicates the lower limit above which the periodogram is the same as the cumulative normal probability density distribution as determined by the chi–square test at 95% level significance.

Table 3. Periodogram Estimates for England and Wales winter Rainfall (1893–1976)

Mean % of AAR	Standard deviation	T_{50} years	T_{75} years	T_{90} years	Periodicities (years) contributing to maximum normalized variance (H) in wave band with $H \geqslant 1$
54.6	10.1	4.04	8.93	14.66	2.1, 2.2, 2.5, 2.8, 3.0, 3.4, 3.9 5.0, 11.0, 24.1, 90.8

AAR: Annual Average Rainfall

percent of the total variance. (3) The periodicities contributing maximum normalized variance in wavebands with normalized variance equal to or exceeding 1. Almost all the rainfall time series used in this study are found to exhibit normal distribution characteristics except for a few marked by asterisks in Tables 1–3.

IV. IMPLICATIONS FOR PREDICTION OF RAINFALL PATTERN

It is seen from Figs. 1–3 and Tables 1–3 that the atmospheric eddy energy spectrum derived from rainfall time series for two disparate climatic regimes, namely India and U. K., exhibits universal characteristics of the statistical normal distribution. Meteorologically, inverse power law form of normal distribution for power spectrum represents the correlation (or the structure function) relating the intensities of fluctuations of different scales. Tessier et al. (1993) have documented and discussed extensive observational evidence for inverse power law form for power spectra of temporal fluctuations in meteorological parameters. Such inverse power law spectra imply scale invariance, namely small–and large–scale (statistical) properties are related by a scale changing operation involving only the scale ratio. There is no characteristic size. Scale invariance is basic symmetry of the atmosphere and is independent of the exact details of the dynamics of the fluctuations.

Short term periodicities in rainfall ranging up to 5 years contribute to as much as 50% of the total variance. Future rainfall patterns may therefore be determined by dominant periodicities such as the QBO (quasi–biennial oscillation) in atmospheric flows.

The peak periodicities in the wave–bands with normalized variance equal to or more than 1 (Tables 1–3) correspond to the time periods of internal circulations of successively larger radii which follow the Fibonacci mathematical number series and therefore may be expressed as (Eq.2) $T(2+\tau)\tau^{-1} = 2.2,\ T(2+\tau)\tau^{0} = 3.6T,\ T\tau(2+\tau) = 5.8T,\ T\tau^{2}(2+\tau) = 9.5T, \cdots$ where T, the primary perturbation time period is the annual (summer–winter) cycle of solar heating. The QBO (quasi–biennial oscillation) is present in all the time series and is in agreement with earlier studies (Burroughs, 1992). The dominant periodicities in the rainfall time series may therefore be expressed as functions of the golden mean.

V. DISCUSSION AND CONCLUSION

The cell dynamical system model developed by one of the authors (Mary Selvam, 1990; Mary Selvam et al., 1992) enables to predict the signatures of quantum–like mechanics and deterministic chaos in atmospheric flows. It is shown that the atmospheric eddy energy spectrum is the same as the normal probability density distribution. The normal probability distribution follows the inverse power form t^{-B} which is identified as the temporal signature of deterministic chaos or self–organized criticality in atmospheric flows (Mary Selvam, 1990; Mary Selvam et al., 1992). Therefore quantum–like mechanics and the inverse power law for the atmospheric eddy energy structure are intrinsic to real world atmospheric flows. The model predictions are confirmed by determining the atmospheric eddy energy spectrum using continuous periodogram analysis technique for two sets of 50–years (1871–1920, 1936–1985) summer monsoon (June–September) rianfall time series for 29 meteorological sub–divisions over the Indian region and one set of 84–years (1893–1975) winter half–year rainfall time series for England and Wales. The important results of the present study are as follows: (1) Temporal (years) fluctuations in rainfall contribute to form a self–organized unique pattern, namely, that of the statistical normal distribution with the square of the eddy amplitude representing the normal probability density corresponding to the normalized standard deviation equal to $[(\log L / \log T_{50})] - 1$ where L is the period length in years and T_{50} the period up to which the cumulative % contribution to total variance is equal to 50 and $t = 0$. (2) periodicities up to 5 years contribute to as much as 50% of the total variance. Quantification of the non–linear variability of atmospheric flows in terms of the unique and universal characteristics of the statistical normal distribution implies predictability based on a knowledge of dominant periodicities. Further, since short term periodicities up to 5 years contribute to as much as 50% of total variance, short period (up to 50–years used in this study) time series data are sufficient for predictability studies. Universal spectrum for interannual variability rules out linear secular trend in rainfall. Energy input into the atmospheric eddy continuum (Eq.1) due to man–made greenhouse gas related atmospheric warming may result in propagation of energy to all scales of weather systems and manifested immediately as intensification of high frequency fluctuations such as the QBO and ENSO.

The important result of the present study is that the interannual variability of rainfall in two disparate climate regimes such as India and United Kingdom exhibits scale invariance and can be quantified in terms of the universal characteristics of the statistical normal distribution. The interannual variability in rainfall is therefore independent of the exact details of dynamical mechanisms in different climate regimes. These results are consistent with scale invariance in temporal fluctuations of meteorological parameters identified by Tessier et al. (1993).

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