Propagation of Envelope Solitons in Baroclinic Atmosphere①

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ABSTRACT

The propagation of finite amplitude baroclinic wave packets in the two-layer model is investigated by using the multiple-scale method. It is shown that the propagation of the wave packets can be described by the so-called unstable nonlinear Schrodinger equation which possesses envelope soliton solutions. The speeds of the solitons are independent of their amplitudes, while the width of the solitons is directly proportional to their speeds but inversely proportional to their amplitudes.

Key words: Envelope solition, Baroclinic wave packet, Baroclinic instability

I. INTRODUCTION

The important role played by baroclinic waves in the large scale dynamics of both the atmosphere and the ocean has been widely accepted since the early investigations by Charney (1947) and Eady (1949) of the linear stability of baroclinic flows. In the last two decades or more considerable efforts have been made to obtain a theoretical understanding of the behavior of finite amplitude of baroclinic wave train and wave packets. Pedlosky (1970, 1971, 1972a, 1979), Drazin (1970, 1972), Pedlosky and Frenzen (1980), Brindley and Moroz (1980) and Moroz and Brindley (1984) studied the time evolution of a single baroclinic wave train. They found that the amplitude oscillates periodically in time in the inviscid limit and approaches a steady solution in the strong viscous limit. In the intermediate case the behavior of the amplitude may be steady, oscillatory or chaotic.

The time and space evolution of a baroclinic wave packet was studied for the first time by Pedlosky (1972b) by use of a two-layer model and later by Moroz and Brindley (1981) by use of a continuously stratified model. They obtained the AB equations (The meanings of the symbles A and B are given in next section.). Gibbon et al.(1979) showed that the AB equations can be transformed either into the sine-Gordon equation or the so-called self-induced transparency equations which can be solved by the inverse scattering method and possess a variety of soliton or envelope soliton solutions. Later Moroz and Brindley (1984) extended their 1981's work further by including dissipation and topographic forcing. They obtained variant amplitude evolution equations for different parameter ranges but did not discuss the solutions of these equations.

The goal of the present paper is to study by analytical means the baroclinic wave packets

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in the two–layer model further. Indeed our work is an extension of Pedlosky's (1972b). In the work of Pedlosky the theory was developed for a flow whose vertical shear is restricted to the minimal critical shear. This restriction will be relaxed here, that is, the vertical shear will be allowed to exceed the minimal critical shear arbitrarily. The next section will show, in this case, that the marginal baroclinic wave packets are governed by the so–called unstable nonlinear Schrodinger equation, not the AB equations. Like the AB equations the unstable nonlinear Schrodinger equation can be solved by the inverse scattering method and possesses envelope soliton solutions. The envelope soliton solutions and their properties are found and discussed in Section 3. Finally the difference of properties between the envelope solitons for the unstable nonlinear Schrodiger equation and the solitons or envelope solitons for the AB equation are discussed. At the same time the results of our paper are compared with the properties of the envelope solitons in the observations and numerical models obtained by Lee and Held (1993).

II. DERIVATION OF THE EVOLUTION EQUATIONS

The model used is the conventional two–layer model (e.g. see Pedlosky, 1987). The model consists of two layers of incompressible, homogeneous fluids of slightly different densities, in a gravitationally stable configuration (light fluid on top), rotating with angular speed Ω about the vertical axis. The fluid is contained in a channel of width L. The nondimensional quasi–geostrophic governing equations without dissipation and topographic forcing are

$$\left(\frac{\partial}{\partial t}+U_n\frac{\partial}{\partial x}\right)[\nabla^2\varphi_n+F(\varphi_1-\varphi_2)(-1)^n]+[\beta-F(U_1-U_2)(-1)^n]\frac{\partial\varphi_n}{\partial x}$$
$$=-J[\varphi_n,\nabla^2\varphi_n+F(\varphi_1-\varphi_2)(-1)^n]\ ,\qquad n=1,2. \tag{2.1}$$

The basic shear flow is U_n, the parameter F is the square of the ratio of the channel width L to the deformation radius, $\sqrt{(g'D)}/f$, where D is the undisturbed depth of each layer. The parameter β is $\beta_* L^2/U$, where β_* is dimensional planetary vorticity gradient and U is the scale for the horizontal velocity field.

For the linear problem,instability occurs when

$$\beta<F(U_1-U_2)=\beta_M\ , \tag{2.2}$$

the critical curve is given by

$$\beta_l=\frac{U_1-U_2}{2F}K^2(4F^2-K^4)^{1/2}\ . \tag{2.3}$$

Where K^2 is the square of the total wavenumber, i.e. k^2+l^2, k and l are the zonal and meridional wave numbers, respectively. The critical curve is shown in Fig.1. In this figure the domain above the critical curve is stable, while the domain under the critical curve is unstable. When $\beta=\beta_M$, all disturbances are stable except one with wavenumber $K=K_0=\sqrt{2F}$ which is marginally stable. When $\beta=\beta_M-\Delta$ and $|\Delta|\ll\beta_M$, all wavenumbers in a band of width $|\Delta|^{1/2}$ around the wavenumber K_0 are destabilized. Pedlosky (1972b) studied the finite amplitude dynamics of this group of waves. He obtained the packet propagation equations

$$\left(\frac{\partial}{\partial T}+C_{g1}\frac{\partial}{\partial X}\right)\left(\frac{\partial}{\partial T}+C_{g2}\frac{\partial}{\partial X}\right)A=\sigma A-NAB \tag{2.4a}$$

$$\left(\frac{\partial}{\partial T}+C_{g2}\frac{\partial}{\partial X}\right)B=\left(\frac{\partial}{\partial T}+C_{g1}\frac{\partial}{\partial X}\right)|A|^2 \tag{2.4b}$$

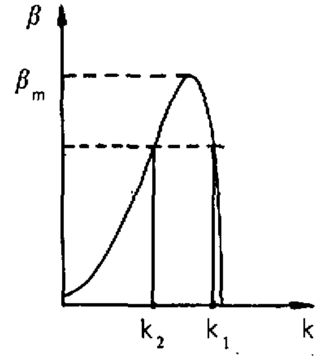

Fig.1. A schematic rendering of the two–layer marginal stability curve in the β, k plane. β is the planetary vorticity, k is the total wave number.

Here A is the amplitude of the packet, B is a measure of the modification of the mean flow induced by the wave packet, $T=|\Delta|^{1/2}t$, $X=|\Delta|^{1/2}x$ are the slow time and space coordinates, C_{g1}, C_{g2}, σ, and N are real constants. Equations (2.4a,b) are now usually called the AB equations. The AB equations have soliton or envelope soliton solutions, the properties of these solutions will be discussed in Section 4.

When $\beta<\beta_M$, wave numbers in a band of width from K_1 to K_2 are unstable. Here K_1 and K_2 are given by

$$K_1^2=\sqrt{2}F\{1+[1-\beta^2/(F^2(U_1-U_2)^2)]^{\frac{1}{2}}\}^{1/2} \quad , \tag{2.5}$$

$$K_2^2=\sqrt{2}F\{1-[1-\beta^2/(F^2(U_1-U_2)^2)]^{\frac{1}{2}}\}^{1/2} \quad , \tag{2.6}$$

which are margionally stable. The phase speeds of these two marginal waves are given by

$$C=\frac{U_1+U_2}{2}-\frac{\beta(K^2+F)}{K^2(K^2+2F)} \quad . \tag{2.7}$$

It can be shown that the long wave, K_2, always travels with a phase speed less than U_2 while the short wave, K_1, always travels with a speed greater than U_2.

In the present paper we shall examine the finite amplitude dynamics of the packets whose central wavenumber is given by (2.5) or (2.6) when β is slightly less than the critical value β_I, i.e.

$$\beta=\beta_I-\Delta \ , \qquad |\Delta|\ll\beta_I \ . \tag{2.8}$$

To obtain the packet propagation equation, the multiple–scale method will be used. Besides the fast variables x, y and t, the following slow time and space variables are introduced:

$$T_1=|\Delta|^{\frac{1}{2}}t, \qquad T_2=|\Delta|t, \qquad X_1=|\Delta|^{\frac{1}{2}}x, \qquad X_2=|\Delta|x \ . \tag{2.9}$$

Thus, in (2.1),

$$\frac{\partial}{\partial t}\to\frac{\partial}{\partial t}+|\Delta|^{\frac{1}{2}}\frac{\partial}{\partial T_1}+|\Delta|\frac{\partial}{\partial T_2} \ , \qquad \frac{\partial}{\partial x}\to\frac{\partial}{\partial x}+|\Delta|^{\frac{1}{2}}\frac{\partial}{\partial X_1}+|\Delta|\frac{\partial}{\partial X_2} \ . \tag{2.10}$$

The solution to (2.1) is sought in the form of an asymptotic series

$$\varphi_n = |\Delta|^{\frac{1}{2}} \varphi_n^{(1)} + |\Delta|^{\frac{3}{2}} \varphi_n^{(2)} + \cdots . \tag{2.11}$$

The insertion of (2.10) and (2.11) into (2.1) yields a sequence of linear problems for the $\varphi_n^{(j)}$ after terms of like order in $|\Delta|^{1/2}$ are balanced.

The lowest order solution, at $O(|\Delta|^{1/2})$ yields

$$\varphi_1^{(1)} = A(X_1, X_2; T_1, T_2)e^{i\theta}\sin ly + * , \tag{2.12a}$$

$$\varphi_2^{(1)} = \gamma A(X_1, X_2; T_1, T_2)e^{i\theta}\sin ly + * , \tag{2.12b}$$

where

$$\theta = k(x - ct) ,$$

$$\gamma = \frac{K^2 + F}{F} - \frac{\beta_I + F(U_1 - U_2)}{F(U_1 - c)} . \tag{2.13}$$

where β_I and c are given by (2.3) and (2.7), respectively, γ is the ratio between the amplitudes in the lower and upper layers. The asterisks in (2.13) denote the complex conjugate of the proceeding term. The amplitudes A are functions of (X_1, X_2, T_1, T_2) and still undetermined.

The $O(|\Delta|)$ problem for $\varphi_n^{(2)}$ is

$$\left(\frac{\partial}{\partial t} + U_1\frac{\partial}{\partial x}\right)[\nabla^2\varphi_1^{(2)} - F(\varphi_1^{(2)} - \varphi_2^{(2)})] + \frac{\partial q_1}{\partial y}\frac{\partial \varphi_1^{(2)}}{\partial x}$$

$$= \frac{\partial q_1 / \partial y}{U_1 - c}\left\{\frac{\partial}{\partial T_1} + \left[U_1 - \frac{\partial q_1 / \partial y - 2k^2(U_1 - c)}{(\partial q_1 / \partial y)/(U_1 - c)}\right]\frac{\partial}{\partial X_1}\right\}Ae^{i\theta}\sin ly + * , \tag{2.14a}$$

$$\left(\frac{\partial}{\partial t} + U_2\frac{\partial}{\partial x}\right)[\nabla^2\varphi_2^{(2)} - F(\varphi_2^{(2)} - \varphi_1^{(2)})] + \frac{\partial q_2}{\partial y}\frac{\partial \varphi_2^{(2)}}{\partial x}$$

$$= \frac{(\partial q_1 / \partial y)\gamma}{U_2 - c}\left\{\frac{\partial}{\partial T_1} + \left[U_2 - \frac{\partial q_2 / \partial y - 2k^2(U_2 - c)}{(\partial q_2 / \partial y)/(U_2 - c)}\right]\frac{\partial}{\partial X_1}\right\}Ae^{i\theta}\sin ly + * , \tag{2.14b}$$

where

$$\frac{\partial q_n}{\partial y} = \beta_I - (-1)^n F(U_1 - U_2) . \tag{2.15}$$

The inhomogeneities in Eqs.(2.14a,b) are proportional to $\exp(ik_n x - i\omega_n t)$ sinly which is a free solution of the linear operator on the left–hand side. They thus appear to force a resonance, but this, as we shall see, is illusory. To investigate this problem further we attempt to find the particular solutions to (2.14a,b) in the form

$$\varphi_{1p}^{(2)} = A^{(2)}e^{i\theta}\sin ly + * , \tag{2.16a}$$

$$\varphi_{2p}^{(2)} = B^{(2)}e^{i\theta}\sin ly + * , \tag{2.16b}$$

Substituting (2.16a,b) into (2.14a,b) yields,after a moderate algebra,

$$-\gamma A^{(2)} + B^{(2)} = \frac{\partial q_1 / \partial y}{iFK(U_1 - c)^2}\left\{\frac{\partial}{\partial T_1} + \left[c + \frac{2k^2(U_1 - c)^2}{\partial q_1 / \partial y}\right]\frac{\partial}{\partial X_1}\right\} A , \tag{2.17a}$$

$$-\gamma A^{(2)}+B^{(2)}=-\frac{(\partial q_2/\partial y)\gamma^2}{iFK(U_2-c)^2}\left\{\frac{\partial}{\partial T_1}+\left[c+\frac{2k^2(U_2-c)^2}{\partial q_2/\partial y}\right]\frac{\partial}{\partial X_1}\right\}A\ . \tag{2.17b}$$

The left hand sides of (2.17a,b) are identically the same, so the right hand sides must equal to each other. Thus we have

$$\frac{2k^2}{iFK}(1+\gamma^2)\frac{\partial A}{\partial X_1}=0\ . \tag{2.18}$$

To obtain (2.17),we have used the relation

$$\frac{\partial q_1/\partial y}{(U_1-c)^2}+\gamma^2\frac{\partial q_2/\partial y}{(U_2-c)^2}=0\ . \tag{2.19}$$

As γ is real and k is not equal to zero, (2.18) requires that

$$\frac{\partial A}{\partial X_1}=0\ . \tag{2.20}$$

This condition means that the amplitude A is independent of X_1. Eqs.(2.16a,b) are hence redundant, and solving for $B_n^{(2)}$ yields

$$B_2^{(2)}=\gamma A^{(2)}+\frac{\partial q_1/\partial y}{iFK(U_1-c)^2}\frac{\partial A}{\partial T_1}\ . \tag{2.21}$$

Without a loss in generality, we may set $A^{(2)}$ equal to zero. This is equivalent to a normalization condition stating that all of the structures of the neutral waves is present in the $O(|\Delta|^{1/2})$ mode. All (2.21) yields is the phase difference between the waves in the two layers. That is, to this order, the wave packet has the form

$$\varphi_1=|\Delta|^{1/2}Ae^{ik(x-ct)}\sin ly+\ *\ , \tag{2.22a}$$

$$\varphi_2=|\Delta|^{1/2}Ae^{ik(x-ct)}\sin ly(\gamma+\frac{|\Delta|^{1/2}}{ikF}\frac{\partial q_1}{\partial y}\frac{1}{(U_1-c)^2}\frac{1}{A}\frac{\partial A}{\partial T_1})+\ *\ . \tag{2.22b}$$

There will be a phase difference between the wave packets in the upper and lower layers if $(1/A)(\partial A/\partial T_1)$ has a real part different from zero.

To the particular solution (2.16a,b) we may add the almost trivial (but as it turns out to be essential) homogeneous solution of (2.14a,b), i.e.,

$$\varphi_n^{(2)}=\Phi_n(y,T_1,T_2;X_2)\ . \tag{2.23}$$

This represents a zonal flow correction of $O(|\Delta|)$ to the origional mean flow. It is a function of y, T_1, T_2 and X_2. We can add such a homogeneous solution at each and every stage of the expansion but, as we shall see, this zonal flow is directly forced by the nonlinear self–interaction of the basic wave field. At this order, the zonal flow correction Φ_n and the amplitude A are still undetermined. In order to determine them, we press on to the $O(|\Delta|^{3/2})$ problem for $\varphi_n^{(3)}$:

$$\left(\frac{\partial}{\partial t}+U_1\frac{\partial}{\partial x}\right)[\nabla^2\varphi_1^{(3)}+F(\varphi_2^{(3)}-\varphi_1^{(3)})]+\frac{\partial q_1}{\partial y}\frac{\partial\varphi_1^{(3)}}{\partial x}$$

$$= -\left(\frac{\partial}{\partial T_2} + U_1\frac{\partial}{\partial X_2}\right)[\nabla^2\varphi_1^{(1)} + F(\varphi_2^{(1)} - \varphi_1^{(1)})] - 2\left(\frac{\partial}{\partial t} + U_1\frac{\partial}{\partial x}\right)\frac{\partial^2\varphi_1^{(1)}}{\partial x\partial X_2}$$

$$\frac{\partial q_1}{\partial y}\frac{\partial\varphi_1^{(1)}}{\partial X_2} + \frac{\Delta}{|\Delta|}\frac{\partial\varphi_1^{(1)}}{\partial x} - \frac{\partial}{\partial T_1}[\nabla^2\varphi_1^{(2)} + F(\varphi_2^{(2)} - \varphi_1^{(2)})]$$

$$-J[\varphi_1^{(1)}, \nabla^2\varphi_1^{(2)} + F(\varphi_2^{(2)} - \varphi_1^{(2)})] - J[\varphi_1^{(2)}, \nabla^2\varphi_1^{(1)} + F(\varphi_2^{(1)} - \varphi_1^{(1)})] \ , \tag{2.24a}$$

$$\left(\frac{\partial}{\partial t} + U_2\frac{\partial}{\partial x}\right)[\nabla^2\varphi_2^{(3)} + F(\varphi_1^{(3)} - \varphi_2^{(3)})] + \frac{\partial q_2}{\partial y}\frac{\partial\varphi_2^{(3)}}{\partial x}$$

$$= -\left(\frac{\partial}{\partial T_2} + U_2\frac{\partial}{\partial X_2}\right)[\nabla^2\varphi_2^{(1)} + F(\varphi_1^{(1)} - \varphi_2^{(1)})] - 2\left(\frac{\partial}{\partial t} + U_2\frac{\partial}{\partial x}\right)\frac{\partial^2\varphi_2^{(1)}}{\partial x\partial X_2}$$

$$\frac{\partial q_2}{\partial y}\frac{\partial\varphi_2^{(1)}}{\partial X_2} + \frac{\Delta}{|\Delta|}\frac{\partial\varphi_2^{(1)}}{\partial x} - \frac{\partial}{\partial T_1}[\nabla^2\varphi_2^{(2)} + F(\varphi_1^{(2)} - \varphi_2^{(2)})]$$

$$-J[\varphi_2^{(1)}, \nabla^2\varphi_2^{(2)} + F(\varphi_1^{(2)} - \varphi_2^{(2)})] - J[\varphi_2^{(2)}, \nabla^2\varphi_2^{(1)} + F(\varphi_1^{(1)} - \varphi_2^{(1)})] \ . \tag{2.24b}$$

Using the results of the $O(|\Delta|^{1/2})$ and $O(|\Delta|)$ problem, we can obtain inhomogeneities in (2.24a,b). They are of two types. One part is independent of x and t. Considering the form of the linear operators on the left hand sides of (2.22a,b),it is clear that these inhomogeneities are secular terms and must be removed, leading to conditions which relate the $(|\Delta|)$ corrections to the mean flow to the wave amplitudes, i.e.

$$\frac{\partial}{\partial T_1}\left[\frac{\partial^2\Phi_1}{\partial y^2} + F(\Phi_2 - \Phi_1)\right] = \frac{l\partial q_1/\partial y}{(U_1 - c)^2}\frac{\partial}{\partial T_1}|A|^2\sin 2ly \ , \tag{2.25a}$$

$$\frac{\partial}{\partial T_1}\left[\frac{\partial^2\Phi_1}{\partial y^2} + F(\Phi_1 - \Phi_2)\right] = -\frac{l\partial q_1/\partial y}{(U_1 - c)^2}\frac{\partial}{\partial T_1}|A|^2\sin 2ly \ . \tag{2.25a}$$

with

$$\frac{\partial\Phi_n}{\partial y} = 0 \qquad \text{at} \qquad y = o,\pi \ . \tag{2.26}$$

The solutions of (2.25a,b) which satisfy (2.26) are

$$\Phi_1 = -\Phi_2$$

$$= -\frac{1}{4l^2 + 2F}\frac{l\partial q_1/\partial y}{(U_1 - c)^2}|A|^2\left(\sin 2ly - \frac{\sqrt{2}\,l\sinh\sqrt{2F}(y - \pi/2)}{\sqrt{F}\cosh\sqrt{F/2}\,\pi}\right) . \tag{2.27}$$

Another part of the inhomogeneous terms in (2.24a,b) is proportional to $\exp(i\theta)\sin ly$. In order to obtain nonzero solutions, as doing in the $O(|\Delta|)$ problem, the following condition must be satisfied, i.e.

$$i\chi\frac{\partial A}{\partial X_2} + \alpha\frac{\partial^2 A}{\partial T_1^2} + \lambda|A|^2 A + \mu A = 0 \ , \tag{2.28}$$

where χ, α, β, λ and σ are constants which are given by

$$\chi = -\frac{2k(1 + \gamma^2)}{F} \ , \tag{2.29a}$$

$$\alpha=\frac{\partial q_1/\partial y(U_2-U_1)K^4}{2F^3k^2(U_1-c)^3(U_2-c)}\ , \tag{2.29b}$$

$$\lambda=\frac{l^2\partial q_1/\partial y}{F}\frac{1}{(U_1-c)^2}\left[(\frac{1}{U_1-c}-\frac{\gamma^2}{U_2-c})-\frac{1}{4l^2+2F}\right.$$
$$\left.\times\left(\frac{\partial q_1/\partial/y}{(U_1-c)^2}-\frac{(\partial q_2/\partial y)\gamma^2}{(U_2-c)^2}\right)\left(1+\frac{8l^2\tanh\sqrt{F/2}\pi}{\sqrt{F/2}(4l^2+2F)}\right)\right]\ , \tag{2.29c}$$

$$\mu=\frac{1}{F}\left[\frac{1}{U_1-c}+\frac{\gamma^2}{U_2-c}\right]\frac{\Delta}{|\Delta|}\ . \tag{2.29d}$$

Equation (2.28) is a fundamental equation governing the finite amplitude baroclinic wave packets whose central wave number is given by (2.5) or (2.6). In the following section we shall find its solutions.

3. *Envelope Soliton Solution of the Unstable Nonlinear Schrodinger Equation*

The packet equation (2.28) is usually called the unstable nonlinear Schrodinger equation, UNS equation in short, because it appears frequently in some unstable physical systems (e.g. see Yajima and Tanaka, 1988). The UNS equation is different in form from the conventional nonlinear Schrodinger equation and becomes the conventional nonlinear Schrodinger equation under interchange of X_2 and T_1. If we define

$$A(X_2,T_1)=E(X_2,T_1)\exp[i\frac{\mu}{\chi}X_2]\ , \tag{3.1}$$

we can rewrite the UNS equation in the following standard form

$$i\chi\frac{\partial E}{\partial X_2}+\alpha\frac{\partial^2 E}{\partial T_1^2}+\lambda|E|^2E=0\ . \tag{3.2}$$

In (3.2) the coefficients α and λ are called the dispersion coefficient and the Landau constant, respectively.

Yajima and Wadati (1990) shown that like the conventional nonlinear Schrodinger equation, the UNS equation can also be solved by the inverse scattering method, i.e., it is integrable. Its solution consists of radiation plus a number of envelope solitons. Here we do not go into the details of the inverse scattering method (The interesting reader is referred to their original work for details.), we shall find an isolated single envelope soliton solution to the UNS equation by the travelling wave method. We allow

$$E(X_2,T_1)=e^{i(rX_2-sT_1)}\psi(\theta)\ ,\qquad \theta=X_2-VT_1\ . \tag{3.3}$$

Where r and s are constants. On substitution, the ordinary differential equation for ψ is

$$\alpha V^2\psi''+i(\chi+2\alpha sV)\psi'-(r\chi+\alpha s^2)\psi+\lambda\psi^3=0\ . \tag{3.4}$$

We now choose

$$s=-\frac{\chi}{2\alpha V}\ ,\qquad r=\frac{\alpha}{\chi}(\eta^2-s^2)\ , \tag{3.5}$$

the first being the important one to eliminate the term ψ'. Then ψ may be taken to be real and

$$\psi'' - \frac{\eta^2}{V^2}\psi + \frac{\lambda}{\alpha V^2}\psi^3 = 0 \ . \tag{3.6}$$

It may be integrated once to

$$\psi' = \left[\frac{\lambda}{2\alpha V^2}\psi^2\left(\frac{2\alpha}{\lambda}\eta^2 - \psi^2\right)\right]^{1/2} \ , \tag{3.7}$$

which can be solved in elliptic functions. The limiting case of the solitary wave is possible when $\alpha\lambda > 0$. The solution is

$$\psi = \sqrt{\frac{2\alpha}{\lambda}}\eta \mathrm{sech}\left[-\frac{\eta}{V}(X_2 - VT_1)\right] \ . \tag{3.8}$$

Where the parameters η and V are independent parameters and determined by the initial state of A. This means that the speed of the soliton is independent of its amplitude, which is very different from the property of the solitons for the KdV equation, the speed of the solitons for the KdV equation is amplitude–dependent.

It is also clear from (3.8) that the width of the soliton for the UNS equation is directly proportional to its velocity, inversely proportional to its amplitude. This means that for a soliton with a fixed amplitude, the faster (slower) the speed is, the wider (narrower) the width is. While for a soliton with a fixed speed, the larger (smaller) the amplitude is, the narrower (wider) the width is.

As the soliton solution (3.8) represents the envelope for E, also for A and $\varphi_n^{(1)}$, so it is usually called envelope solitons. They can be produced only when the condition $\alpha\lambda > 0$ is satisfied. For the atmospherical parameters we have verified that this condition is satisfied. So it is possible for the baroclinic atmosphere to produce the envelope solitons.

4. *Concluding Remarks and Discussions*

It is shown in this paper that the propagation of the finite–amplitude marginal baroclinic wave packets in the two–layer model can be described by the so–called unstable nonlinear Schrodinger equation when the basic vertical shear exceeds the minimal critical shear arbitrarily. Like the conventional nonlinear Schrodinger equation, the UNS equation can also be solved by the inverse scattering method and posseses envelope soliton solutions. It is shown that the speeds of the solitons are amplitude–independent, while the widths of the solitons are directly proportional to their velocities and inversely proportional to their amplitudes.

In Section 2 we pointed out that Pedlosky (1972b) shown that when the vertical shear is restricted to the minimal critical shear, that is, $\beta = \beta_M - \Delta$ (where $|\Delta| << \beta_M$), the marginal wave packets in the two–layer model is governed by the AB equations, not the UNS equation. The form of the AB equations is different from the form of the UNS equation. The most significant difference is that the AB equations are second order in space, while the UNS equation is first order in space. This difference may be a reflection of the fact that the carrier wave of the marginal wave packet corresponds to two wave modes which coalesce when the vertical shear is near the critical minimal shear, while the carrier wave of the marginal wave packet corresponds to only one wave mode when the vertical shear is above the minimal critical shear arbitrarily. This difference leads to some differences in properties between the envelope soliton solutions for the UNS equation and the soliton or envelope soliton solutions for the AB equations. (The reader is referred to the papers by Pedlosky (1972b) and by Gibbon, et al. (1979) for the soliton or the envelope soliton solutions for the AB equations.) One of the most significant difference is that the speeds of the solitons or envelope solitons for the AB equation are amplitude–dependent, while the speeds of the envelope solitons for the UNS equation are not, as pointed out in the previous section.

It is very interesting to note here that envelope solitons exist not only in baroclinic atmosphere but also in barotropic atmosphere. Yamagata (1980), Boyd (1983) and Luo and Ji (1989) showed that the envelope solitons in barotropic atmosphere can be described by the famous conventional nonlinear Schrodinger equation which becomes the unstable nonlinear Schrodinger equation obtained in our paper under the interchange of the time and space coordinates. Most of the properties of the envelope soliton solution to the conventional nonlinear Shrodinger equation are similar to those of the envelope soliton solution to the unstable nonlinear Shrodinger equation. The significant difference is that the width of the envelope soliton of the conventional nonlinear Schrodinger equation is velocity–independent, while the width of the envelope soliton of the unstable nonlinear Schrodinger equation is velocity–dependent.

It is also interesting to compare here the envelope Rossby solitons studied in our paper with the Rossby solitons studied by Redekopp (1977) and others. It is clear that the envelope Rossby solitons are a special form of nonlinear Rossby wave packets whose envelopes behave like solitons, while the Rossby solitons are a special form of nonlinear Rossby waves whose zonal length scale is much larger than as meridional length scale. Usually the Rossby solitons satisfy the famous KdV equation or the mKdV equation. The significant characteristic of the Rossby solitons is that their speeds are amplitude–dependent.

Very recently, the existence of the envelope solitons in the real atmosphere and in the numerical models is confirmed by Lee and Held (1993). In their paper, Lee and Held analyzed the propagation of baroclinic wave packets in the observations and numerical models. They found that the envelopes of the wave packets behave like solitons in the sense that the envelopes remain their shapes unchanged during their propagating process. They also found that the speeds of the wave packets are amplitude–independent, which is in good agreement with the result of our paper and suggests that the UNS equation may some of the dynamical balance maintaining the wave packets.

In our present paper we have studied only one single baroclinic wave packet, we have ignored the interactions between the wave packets. We have also ignored the effects of topography and friction on the wave packets. These will be reported in other papers.

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