CISK–related Rossby Waves in the Tropical Atmosphere

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ABSTRACT

In terms of a baroclinic quasi–geostrophic wave–filtering technique in connection with a dimensionless parameter, $\eta(z)$, of condensation–released latent heat that indicates the CISK mechanism, a model is established for describing tropical atmosphere CISK–Rossby waves alongside its analytical solution. Theoretical study shows that there exists pronounced difference between Rossby waves, CISK–involving and classic, and the former can be used to interpret some aspects of the low–frequency oscillation in the tropical atmosphere.

Key words: Low–frequency oscillation, Cumulus convection feedback, CISK–Rossby wave

I. INTRODUCTION

Low–frequency oscillation (LFO) is a phenomenon of much importance in the tropical atmosphere. Madden and Julian (1971) reported first the 30–50–day oscillation in zonal wind over the tropical Pacific and revealed later that such oscillation occurs at tropical latitudes on a global basis (Madden and Julian, 1972). Subsequent data analysis and theoretical research (Krishnamurti, et al., 1982; Murakami, et al., 1984; Lau, et al., 1986) show that LFO is ubiquitous in the tropical atmosphere. Also, data analysis indicates that such tropical LFO travels slowly (10 m / s) eastward alongside the equator as wavenumber 1 but it may propagate westward although the eastward motion is predominant (Li, 1993).

Feedback from convective latent heat release represents the key factor for genesis and maintenance of a tropical system. Theoretical and numerical studies (Yeh, et al., 1991) show such feedback to be one of the causes of the LFO occurrence. Li (1985) proposed a hypothesis of 30–50–day oscillation in mobile CISK wave–driven monsoon trough / ridge. Lau and Peng (1987) reported the mechanism for 30–50–day oscillation excited by mobile wave–CISK. All these findings reveal the important role of the feedback in the tropical LFO.

Recent studies (Li, 1990; Liu, et al., 1992) claim that equatorial LFO can be assumed to be related to the combined effects of CISK–associated Kelvin and Rossby waves. And these studies adopted a semi–geostrophic model with wave–CISK mechanism incorporated. In view of the fact that tropical LFO is displayed zonally as long waves (wavenumbers 1–6 with wavelength $L \geqslant 6 \times 10^6$m) and the atmospheric motion at tropics, particularly in the range of 5–15°N / S (Wu, 1990), is featured by semi–geostrophicity, the rationality of the model is obvious, with which CISK–Kelvin and Rossby waves can also be produced (Liu, et al., 1992). However, careful inspection of the CISK–Rossby mode solution in the semi–geostrophic framework indicates that the solution with no cumulus convection feedback involved does not agree entirely with the tropical free Rossby wave solution of classical nature, i.e., the solution is slightly deformed, which is evidently relevant to semi–geostrophic model is applied in this work to explore atmospheric CISK–Rossby mode at tropics by introducing CISK mechanism. As pointed out by Li (1985), tropical ultralong waves are similar in properties to midlatitude long waves, suggestive of quasi–geostrophicity, leading to the fact that quasi–geostrophic approximation is of use to the study of planetary motion in the tropical atmosphere. Following the conclusion (Liu, 1990), low–latitude Rossby waves are undistorted based on the filterings of quasi–geostrophic model results and so the filter will favor the study in dealing with CISK–Rossby mode. The present work shows that CISK–Rossby waves from the quasi–geostrophic model are really equally good in explaining some aspects of the tropical LFO and even better than those from a semi–geostrophic model.

II. CISK–ROSSBY WAVE FROM THE QUASI–GEOSTROPHIC MODEL

For a baroclinic model incorporating wave–CISK mechanism in the condition of static equilibrium under equatorial beta–plane approximation and Boussinesq approximation, the linear equations take the form

$$\begin{cases} \frac{\partial u}{\partial t} - \beta y v = -\frac{\partial \varphi}{\partial x}, \\ \frac{\partial v}{\partial t} + \beta y u = -\frac{\partial \varphi}{\partial y}, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \\ \frac{\partial}{\partial t}(\frac{\partial \varphi}{\partial z}) + N^2 w = N^2 \cdot \eta w_B, \end{cases} \tag{1}$$

where t denotes the time; x, y and z the coordinates, directed east–, north– and upward, respectively, in a positive sense; u, v and w the velocity components, in order, for the directions; N the Brunt–Vaisala frequency; β(=constant) the Rossby parameter; $\varphi = p' / \rho_0$ where p' is the pressure deviation from the value in a stationary atmosphere and ρ_0 the density of the atmosphere; w_B the vertical velocity at the boundary–layer top; η the dimensionless parameter of latent heating which is generally set to be the function of z in such a way that it is nonzero for $w_B > 0$. $N^2 \eta w_B$ in the last equation of (1) represents the latent heating through condensation of the CISK mechanism.

Filtering made of baroclinic quasi–geostrophic model results leads to basic equations of a quasi–geostrophic model involving wave–CISK mechanism for the CISK–Rossby wave. They are in the form

$$\begin{cases} \frac{\partial}{\partial t}(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}) + \beta \frac{\partial \varphi}{\partial x} - \beta^2 y^2 \frac{\partial w}{\partial z} = 0, \\ \frac{\partial}{\partial t}(\frac{\partial \varphi}{\partial z}) + N^2 w = N^2 \eta w_B, \end{cases} \tag{2}$$

which, if φ is eliminated therefrom, will give the expression containing W only, viz.,

$$\frac{\partial}{\partial t}(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\beta^2 y^2}{N^2} + \frac{\partial^2 w}{\partial y^2}) + \beta \frac{\partial w}{\partial x} = \eta \frac{\partial^3 w_B}{\partial t \partial x^2} + \eta \frac{\partial^3 w_B}{\partial t \partial y^2} + \beta \eta \frac{\partial w_B}{\partial x} \quad , \tag{3}$$

which is the very basic equation for the CISK–Rossby wave in question.

III. ANALYTICAL SOLUTION TO CISK–ROSSBY MODE

1. *Solution at $\eta = 0$*

The used CISK–Rossby wave model contains only CISK mechanism without any other types of forcing considered so that it can be inferred that the solution at $\eta=0$ should be a free baroclinic Rossby mode subject to no forcing at all.

Set $\eta=0$ in (3) and we get

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\beta^2 y^2}{N^2}\cdot\frac{\partial^2 w}{\partial z^2}\right)+\beta\frac{\partial w}{\partial x}=0\ . \tag{4}$$

Evidently, (4) is the standard basic equation for free Rossby waves of linear baroclinic features at tropical latitudes (Liu et al., 1990).

Then the orthogonal mode scheme is utilized. Let

$$w=W(y)e^{i(kx+nz-\sigma t)}, \tag{5}$$

which is put into (4), resulting in the expression of circular frequency of tropical baroclinic Rossby waves

$$\sigma=-\frac{\beta k}{k^2+(2m+1)\frac{\beta}{C_1}}\ ,\qquad (m=0,1,2,\cdots) \tag{6}$$

with

$$C_1=\frac{N}{n}=\frac{NH}{\pi}, \tag{7}$$

where H refers to the mean free–surface height, $n(=\frac{\pi}{H})$ to the wavenumber in the z direction, k to zonal wavenumber and σ to the circular frequency.

From (6) comes the zonal velocity of Rossby wave

$$C_m=-\frac{\beta}{k^2+(2m+1)\beta/C_1}\ ,\qquad (m=0,1,2,\cdots) \tag{8}$$

and the oscillation period

$$T_m=\frac{(2m+1)L}{C_1}+\frac{4\pi^2}{\beta L}\ ,\qquad (m=0,1,2,\cdots) \tag{9}$$

where $L(=\frac{2\pi}{k})$ is zonal wavelength.

And the eigenfunction $W(y)$ has the form

$$W(y)=A_m e^{-\frac{\beta y^2}{2C_1}}H_m\left(\left(\frac{\beta}{C_1}\right)^{1/2}y\right), \tag{10}$$

where H_m stands for m–order Hermite polynomial and A_m is the constant.

The reader is referred to Refs. (Liu et al., 1990) and (Gill, 1988) for the wavenumber k–varying curve of the circular frequency of the classical Rossby wave in the tropical atmosphere and the variation of σ as a function of wavelength L, and the $W(y)$–given meridional structure of the wave will be presented later.

One can see from (8) that in the absence of latent heating tropical atmosphere free Rossby waves travel westward all the time, which is obviously unable to explain the eastward propagation of 30–50–day oscillation of the waves. Judged only from the zonal shift, the free Rossby waves with no latent heating involved fail to interpret the tropical LFO.

2. *Analytical Solution at $\eta \neq 0$*

$\eta \neq 0$ means the occurrence of latent heating and the three terms on the rhs of (3) represent the heating effect. For simplicity, the third term is first considered and (3) is reduced to

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\beta^2 y^2}{N^2}\frac{\partial^2 w}{\partial z^2}\right)+\beta\frac{\partial w}{\partial x}=\beta\eta\frac{\partial w_B}{\partial x} . \tag{11}$$

With the orthogonal mode scheme used, we assume

$$\begin{cases} w = W(y,z)e^{i(kx-\sigma t)}, \\ w_B = W_B e^{i(kx-\sigma t)}, \end{cases} \tag{12}$$

which is put into (11), yielding

$$\sigma\left(\frac{\beta^2 \cdot y^2}{N^2}\frac{\partial^2 W}{\partial z^2}+\frac{\partial^2 W}{\partial y^2}-k^2 W\right)-\beta k W=\beta\eta k W_B, \tag{13}$$

which is the partial differential equation of W with respect to y and z. For the sake of easy solution, a simple two–level model(Li, 1990) is adopted whose schematic view is presented in Fig.1, where Δz denotes the interval between the equispaced levels in the z direction. Based on the boundary conditions $W_0 = W_2 = 0$, (13) is written on the mid level and the derivative with respect to z is replaced by difference coefficient, namely,

$$\left(\frac{\partial^2 W}{\partial z^2}\right)_1=\frac{W_0-2W_1+W_2}{(\Delta z)^2} , \tag{14}$$

so that

$$\sigma\left(\frac{d^2 W_1}{dy^2}+\frac{\beta^2 y^2}{N^2}\cdot\frac{-2W_1}{(\Delta z)^2}-k^2 W_1\right)-\beta k W_1=-\beta k\eta_1 W_B . \tag{15}$$

Calculation based on measurements shows that the low–level vertical velocity in the tropical troposphere is normally marked by nearly linear distribution so that it is possible to simply set $W_B = bW_1$ (Li, 1993; Li, 1990) with $b \leqslant 1$. Then (15) is rewritten as

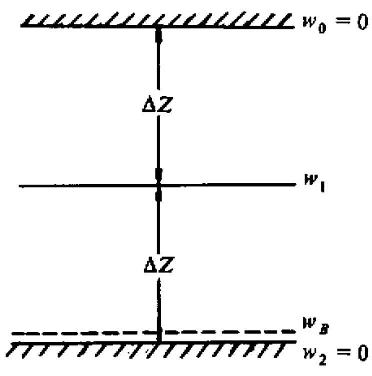

Fig.1. Schematic of the two–level model in use.

$$\frac{d^2 W_1}{dy^2} + [-\frac{\beta k}{\sigma}(1-b\eta_1) - k^2 - \frac{2\beta^2}{N^2 \Delta z^2} y^2] W_1 = 0, \qquad (16)$$

where $N = 1 \times 10^{-2} s^{-2}$ (Li, 1993) is generally taken and $\Delta z = 7$ km for the two-level model if

$$C_1 = \frac{N \cdot \Delta z}{\sqrt{2}} \qquad (17)$$

is set and in this case $C_1 = 49.5 \text{ m s}^{-1}$.

Using (17), (16) is changed into

$$\frac{d^2 W_1}{dy^2} + [-\frac{\beta k}{\sigma}(1-b\eta_1) - k^2 - (\frac{\beta}{C_1})^2 y^2] W_1 = 0 \qquad (18)$$

with

$$-\frac{\beta k}{\sigma}(1-b\eta_1) - k^2 = (2m+1)\frac{\beta}{C_1} \quad (m = 0,1,2,\cdots). \qquad (19)$$

W_1 has a bounded solution of the form

$$W_1^{(m)} = A_m e^{-\frac{\beta y^2}{2C_1}} \cdot H_m((\frac{\beta}{C_1})^{1/2} y), \qquad (20)$$

where A_m and H_m have the same meaning as in (10). From (19) we get the circular frequency of CISK–Rossby waves in the form

$$\sigma = -\frac{\beta k(1-b\eta_1)}{k^2 + (2m+1)\beta / C_1}, \quad (m = 0,1,2,\cdots). \qquad (21)$$

by which we find the expression of zonal velocity component

$$C_m = \frac{\sigma}{k} = -\frac{\beta(1-b\eta_1)}{k^2 + (2m+1)\frac{\beta}{C_1}} \quad (m = 0,1,2,\cdots). \qquad (22)$$

and the formula for oscillation period

$$T_m = \frac{2\pi}{|\sigma|} = \frac{1}{|1-b\eta_1|}[(2m+1)\frac{L}{C_1} + \frac{4\pi^2}{\beta L}] \quad (m = 0,1,2,\cdots). \qquad (23)$$

If $\eta_1 = 0$ is assumed, implying no such heating involved in (21)–(23), then $1 - b\eta_1 = 1$ and (21)–(23) are changed, respectively, to (6), (8) and (9) for classical Rossby wave, results those are in entire agreement with what is expected.

For $\eta_1 \neq 0$, suggestive of latent heating through condensation available, it follows from (21) that CISK–Rossby waves are stable all the time, and then it is known from (22) that for weaker heating, i.e.,

$$0 < 1 - b\eta_1 < 1, \qquad (24)$$

the CISK–Rossby wave is marching westward but at a lower speed than the free Rossby analog; for $1 - b\eta_1 = 0$ the CISK–Rossby wave is stationary; for stronger heating ($\eta_1 > \frac{1}{b}$), meaning

$$1-b\eta_1<0, \tag{25}$$

the CISK–Rossby wave is going eastward. From the foregoing analysis we come to the conclusion that the Rossby waves, forced and free, differ strongly in propagation, with the former having its motion westward, stationary or eastward under the action of the CISK mechanism, depending on the intensity of condensation–released latent heating.

Now we examine the oscillation periods of the CISK–Rossby wave. Fig.2 portrays the curve of period T_m as a function of η_1 through (23) with $C_1=49.5$ m s^{-1}, $L=1\times10^7$m and $b=0.4$. It is apparent that with the heating available, the east–travelling wave has 30–50–day period when the heating density is appropriate, the ($m=0$) wave is marked by the same period with $\eta_1=2.72\sim2.86$, and so is the $m=1$ wave with $\eta_1=2.95\sim$ 3.25 and the $m=2$ mode at $\eta_1=3.19\sim$ 3.64.

Now the relationship between the velocity values and LFO is investigated. Multiplying (22) by (23) yields

$$T_m\cdot|C_m|=\frac{2\pi}{k}=L, \tag{26}$$

which also holds for classical free Rossby waves. Observations show the tropical LFO propagates slowly eastward preferentially as wavenumber 1 with $L=4\times10^7$m, for which $C_m=9\sim$ 15 m s^{-1} is found when $T_m=30$–50 days, an outcome that is in concert with the observational fact. As stated earlier, only the Rossby wave associated with CISK mechanism tends to move east and as long as the heating density is appropriate, the wave travelling at a rate, on average, of 10 m / s or so will occur. And the CISK–Rossby wave at $m=0$ ($m=1$) will gain $C_1=49.5$ m s^{-1} with $\eta_1=2.98\sim3.30$ (3.89$\sim$4.81).

From the above we reach the conclusion that this type of Rossby wave is of value to interpreting some aspects of the tropical LFO.

Comparison of (10) and (20) shows that the classical type is the same in meridional structure as the CISK–Rossby wave in the tropical atmosphere, suggesting that the introduction of

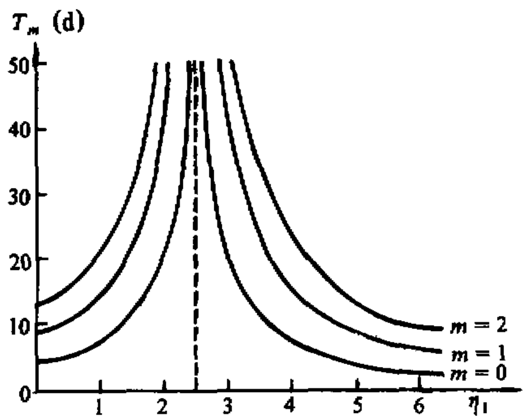

Fig.2. Variation in period T_m versus η_1 for the study wave with $L=1\times10^7$m and $b=0.4$ and $C_1=49.5$ m s^{-1} ($m=0,1,2$).

CISK mechanism does not alter the structure and such structures will be given below for the modes ($m=0,1,2$) of W_1, viz.,

$$W_1^{(0)}(y)=A_0 e^{-\frac{\beta \cdot y^2}{2C_1}},$$
$$W_1^{(1)}(y)=A_1 \cdot \left(\frac{\beta}{C}\right)^{1/2} y \cdot e^{-\frac{\beta \cdot y^2}{2C_1}}, \quad (27)$$

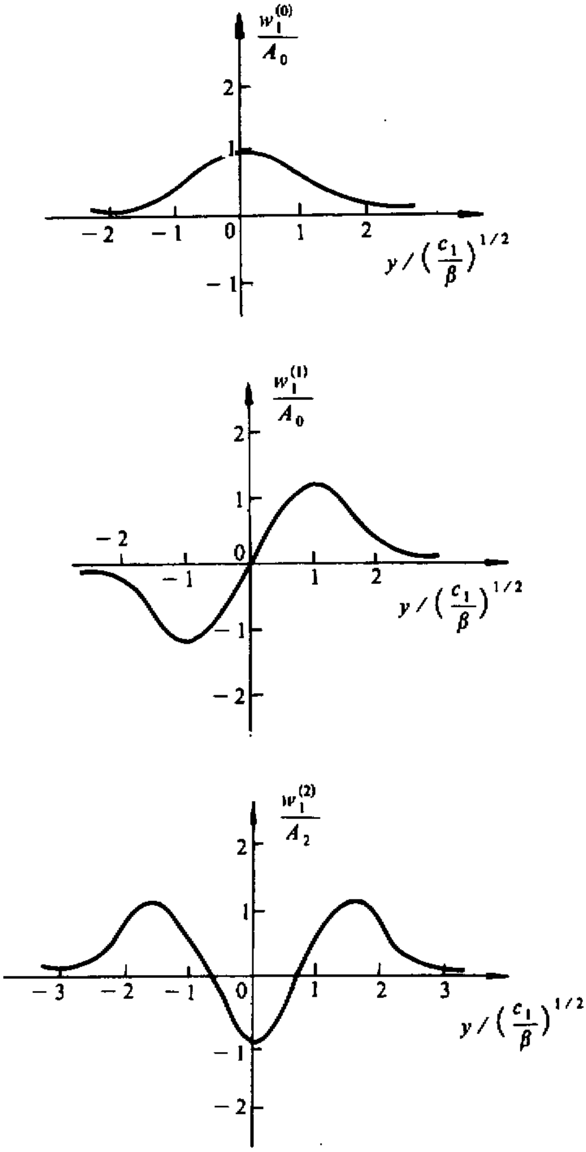

Fig.3. Meridional structures of W_1 at $m=0,1,2$.

$$W_1^{(2)}(y)=A_2\cdot(\frac{2\beta}{C_1}y^2-1)e^{-\frac{\beta\cdot y^2}{2C_1}},$$

and portrayed in Fig.3. One sees that in the case of $m=0$,
a maximum is at the equator ($y=0$), other values being symmetric about the equator; for $m=1$, a maximum occurs at $y=+(\frac{C_1}{\beta})^{1/2}$ (in the Northern Hemisphere) and a minimum at $y=-(\frac{C_1}{\beta})^{1/2}$ (in the Southern Hemisphere), antisymmetric about the equator; for $m=2$, a minimum shows up at $y=0$ and maxima at $\frac{\sqrt{5}}{2}(\frac{C_1}{\beta})^{1/2}$ from the equator symmetric about the equator.

Now, we compare the CISK Rossby wave from the quasi–geostrophic model to that from the semigeostrophic model (Liu, et al., 1992). Following Liu (1990), the tropical semi–geostrophic model yields Rossby mode at zonal wavenumber $k\to 0$ only while quasi–geostrophic model–produced CISK Rossby wave bears a relation to zonal wavenumber (tropical LFO belongs to a category of ultralong waves. Therefore, only the CISK–Rossby wave from the quasi–geostrophic model under longwave approximation is comparable to the analog from a semi–geostrophic model. Based on (22) and (23), the velocity C_m and period T_m of the waves in the framework of longwave approximation are

$$C_m=-\frac{(1-b\eta_1)C_1}{(2m+1)},\tag{28}$$

$$T_m=\frac{(2m+1)L}{|1-b\eta_1|C_1},\tag{29}$$

which, compared to the semigeostrophic model results, show the similarity in propagation feature and period change to each other except the difference in magnitude but both model generated zonal structures of $W^{(1)}(y)$ and $W^{(2)}(y)$ differ completely from each other. Also, it can be seen that the wave from the quasi–geostrophic model without CISK mechanism incorporated ($\eta=0$) is the same in all these aspects as the classical counterpart, which is, however, different from the semi–geostrophic model wave to considerable extent, which is obviously related to semi–geostrophic approximation considered, indicating that the involved semi–geostrophic approximation is responsible for the distortion of the wave. As such, the CISK–Rossby mode from the quasi–geostrophic model is superior to that from the semi–geostrophic model.

IV. CONCLUDING REMARKS

It is known that the dimensionless parameter, η, of latent heating is normally adopted to characterize CISK mechanism in the wave – CISK theory. The CISK–Rossby wave solution is obtained in this work in terms of the same scheme for a quasi–geostrophic model. Results show that, being always a west–travelling feature, the free Rossby wave in a baroclinic atmosphere at low latitudes with no CISK mechanism involved fails to account for the 30–50–day oscillation; when the mechanism ($\eta\neq0$) is included in such a way that the heating density is stronger, the CISK –Rossby wave will interpret quite well the direction, velocity and period of the tropical LFO, indicating that the wave–CISK mechanism is really responsible for the oscillation.

Also, the present study shows that the quasi–geostrophic CISK–Rossby model gives rise to no deformed wave of the type and contains the solution in the framework of logwave approximation so that it is superior to that from the semi–geostrophic model as far as Rossby wave related to CISK mechanism is concerned.

It is worth noting that all aspects of the LFO can not be explained satisfactorily by the CISK–Rossby waves, which are useful mainly to the LFO moving east slowly with more vigorous heating available but can do nothing about the case of feeble heating density that is interpreted only with the aid of CISK–Kelvin wave (Liu, et al., 1992). Both the types, taken together, indicate that the tropical LFO should be the result of their joint action, an inference that coincides with that of Miyahara (1987), who claimed that the LFO is just composed of Kelvin– and Rossby–type responses, as viewed from the 30–50–day oscillation structure.

The model used for CISK–Rossby wave in this work, though it is able to explain some of the LFO's aspects, particularly as regards horizontal propagation, fails to do so about the difference in travelling between upper and lower levels of the troposphere because of its undue simplicity. This problem awaits future research.

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