Estimation of Hourly and Daily Global Solar Radiation at Clear Days Using an Approach Based on Modified Version of Gaussian Distribution

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ABSTRACT

The performance of two models, Jain and Baig, based on the modified version of Gaussian distribution function in estimating the daily total of global solar radiation and its distribution through the hours of the day from sunrise to sunset at any clear day is evaluated with our own measured data in the period from June 1992 to May 1993 in Qena / Egypt. The results show a high relative deviation of calculated values from measured ones, especially for Jain model, in the most hours of the day, except for those near to local noon. This misfit behavior is quite obvious in the early morning and late afternoon. A new approach has been proposed in this paper to estimate the daily and hourly global solar radiation. This model performs with very high accuracy on the recorded data in our region. The validity of this approach was verified with new measurements in some clear days in June and August 1994. The resultant very low relative deviation of the calculated values of global solar radiation from the measured ones confirms the high performance of the approach proposed in this work.

Key words: Global solar radiation, Measurements, Models, Gaussian distribution function, Statistical treatment

I. INTRODUCTION

The interest in application of solar energy has largely increased in the last years particularly in the regions where sunshine is available in abundance (Hoyt, 1978). This requires the knowledge of the amount and variability of solar insolation at the earth's surface, which is necessary for design and optimization of all solar systems (Robinson and Hill, 1987; Rahmen and Chowdhury, 1988; Kuye and Jagtap, 1992). Obviously the "best" data to be used are radiation observations at the site of the proposed solar energy system. However, these observations are not always feasible. It is therefore necessary to be able to estimate reliably the radiation incident on any particular location using number of simple models especially in locations which do not have solar data stations.

Qena, upper Egypt (26°17N, 32°43E, Height 78 m asl), a city of abundant solar radiation is one of many locations in Egypt, in which the solar radiation data are not available. Because of this lack of data, we attempt in this study to develop, through a combination of existing empirical methods, an approach to estimate the daily global solar radiation and its distribution during the hours of any clear day of the year from sunrise to sunset. The problem of estimating hourly global solar radiation incoming on a horizontal surface has been analyzed by many authors (Amato et al., 1988).

In such an attempt we follow Jain (Jain, 1984) and Baig (Baig et al., 1991) in using Gaussian distribution function to fit the experimental data. This model is characterized with simplicity and high accuracy in estimating the daily total of global solar radiation and its hourly distribution in most hours of the day.

II. EXPERIMENTAL DATA

1. *Measurements*

The hourly global solar radiation data, which is used in this study were recorded through a program for measuring the solar radiation components, supervised by the author, over Qena / Egypt. Precision pyranometer (Kipp and Zonen Model GM 6 B) was used in conjunction with a two channel solar integrator (Kipp and Zonen CC 12) to record the values of the hourly global solar radiation G_h from sunrise to sunset as well as its daily total G_d in the period from June 1992 to May 1993. More datails about the experimental techniques and the exposure of the instruments were given in a previous contribution (El−shazly, 1994).

2. *Analysis of the Data*

The measured data have been analyzed in view of two models according to Jain (1984) and Baig (1991) who proposed a Gaussian function to fit the experimental ratio of hourly to daily solar radiation data (RT) using the following formulae.
For Jain

$$RT = (1 / \sigma\sqrt{2\pi})\mathrm{Exp}(-(t-12)^2 / 2\sigma^2) \ . \quad (1)$$

For Baig

$$RT = (1 / 2\sigma\sqrt{2\pi})\{\mathrm{Exp}(-(t-12)^2 / 2\sigma^2) + \cos(180^\circ(t-12)/(S_0 - 1))\} \ . \quad (2)$$

RT is the ratio of hourly G_h to daily G_d global radiation and calculated as follows

$$RT = G_h / G_d \ . \quad (3)$$

t is the local time in hours, σ is the standard deviation obtained by matching the experimental recorded value of RT at local noon time in Eq.1 or 2 as

$$\sigma = 1 / (RT)_{12}\sqrt{2\pi} \quad (4)$$

and S_0 is the day length in hours obtained from the following equations

$$S_0 = (2/15)\cos^{-1}(-\tan\varphi\tan\delta) \ , \quad (5)$$

$$\delta = 23.45\sin(360^\circ(n+284)/365) \ , \quad (6)$$

where δ represents the declination angle, n is the day number of the year ($n = 1$ for 1 January) and φ is latitude of the measurement site ($=26.17^\circ$).

Once the value of σ in Eq.1 or the values of σ and S_0 in Eq.2 are known, the theoretical values of RT along the day length from sunrise to sunset will be available. Consequently, the values of G_h through the day time and their daily total G_d can be calculated using a measured value of G_h in any hour of the day.

III. RESULTS AND DISCUSSION

1. *Determination of σ*

As we mentioned above that the determination of σ is essential for application of equations 1 and 2 to calculate RT and then G_d and G_h along the hours of the day. For this purpose we make use of the great linear correlation, which is found between σ and S_0 in clear days for the most data points. This relation is reported earlier by Jain (1984) and Baig (1991)

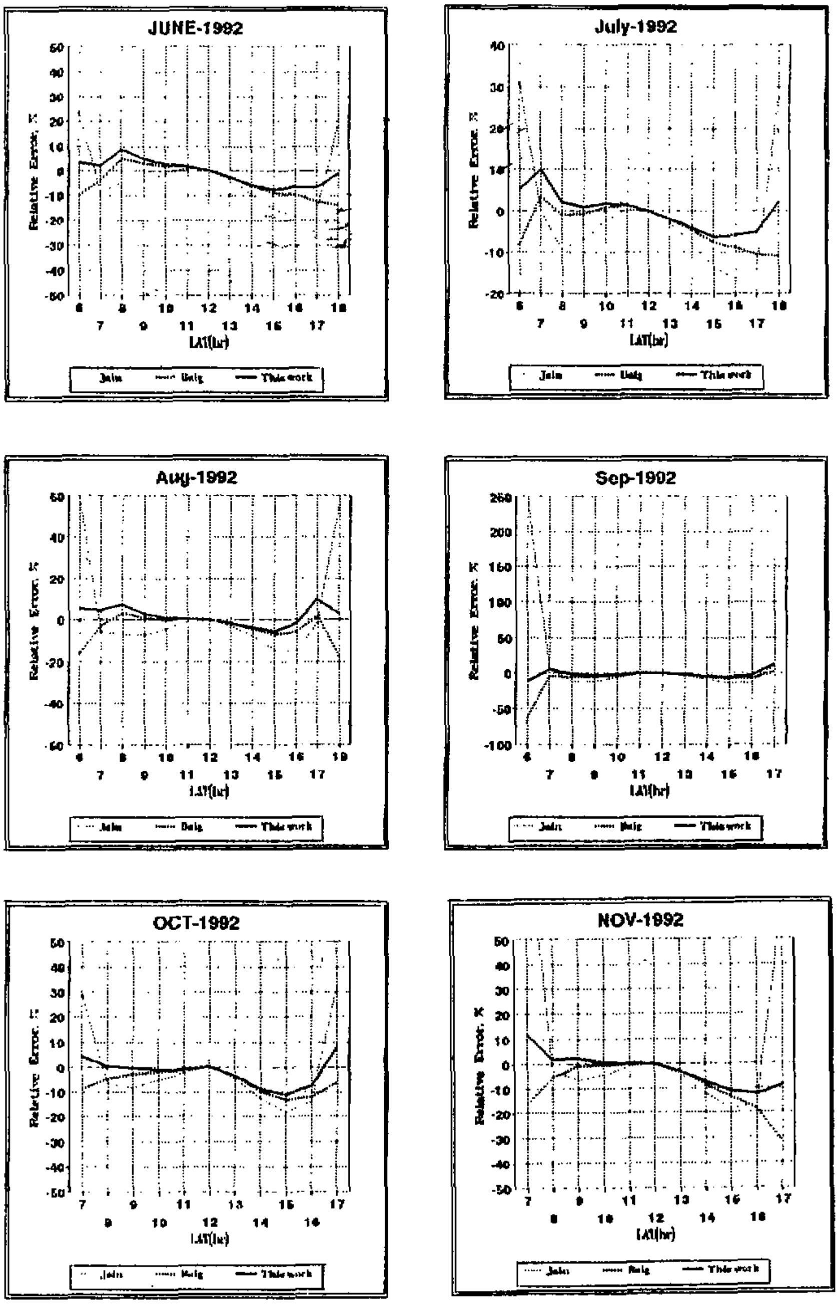

Fig. 1. Plot of the relative errors of the calculated values of *RT* with respect to the experimental ones against the local time in the period from June to November 1992.

for data in Triesta in Italy and Islamabad in Pakistan. It is supported here for our data series at Qena / Egypt in the following form:

$$\sigma = 0.174S_0 + 0.768 \ , \tag{7}$$

with a correlation coefficient equals 0.8. As long as the data points obey the linear equation 7 (i.e. the calculated values of σ approach to their practical ones), the calculated RT will give extremely good fit to the practical data at different local time leading to estimation of daily global solar radiation in a good accuracy. Otherwise a good fit may not be achieved.

2. *Evaluation of Global Solar Radiation Calculated Using Jain and Baig Models*

Figs.1 and 2 summarize the results of the relative errors of RT calculated by Eqs. 1 and 2 through the hours of clear days with respect to the experimental ones in the 15 th days of the measurement months. The global solar radiation values in these days are representative to very high extend for the monthly mean values. In some months this day was found to be cloudy, so the nearest day to it has been taken. The incomplete hours at sunrise and sunset were not considered in our treatment. From the both figures, one can conclude the following:

i) In the months from June to December 1992, RT results, which calculated with both equations show a good fit to the experimental data, with an obvious better fit for those calculated using the Baig equation, in the most hours of the day, except near sunrise and sunset hours. In these hours, both models, especially of Jain, give rather poor fitting. Relative errors of RT values calculated with Jain model during the time from 7 to 14 hrs were found to be 10–12% in the months from June to September and 8% during the time from 8–13 hrs in the October–December period. This corresponds to relative error of values $<5\%$ for Baig model at the same periods of time and months. With respect to the RT values calculated at early morning and late afternoon hours, relative error up to $>200\%$ was obtained in some months for Jain model compared to $\approx 62\%$ for Baig model.

ii) In the months from January to May 1993, neither Baig nor Jain model is suitable enough for application in many hours in the day (see Fig.2). the time, in which both equations estimate RT values with least error ranges from 5 hours for Jain model to 6 hours for Baig model.

The above given results show clearly that the measured data RT, Exp is not in a good agreement with the calculated ones RT, Jain and RT, Baig in certain hours in some months. The misfit behavior begins to be obvious in October, November, and December and quite large in the months from January to May 1993, especially in January, April and May, in which the fitting is rather poor in major parts of the day except those very near to the noon time. This is owing to the different variations characteristics of the atmospheric constituents, namely gases, liquid and aerosol particles, at Qena / Egypt in comparison with the sites, for which the two models were derived. So care should be exercised in applying them beyond their domains. Qena / Egypt is not an industrial district and characterized with a high content of aerosol dust particles, dispersed from near eastern desert and various man activities (El–shazly, 1989). Accordingly the lack of agreement between the models and observed data in the above mentioned months may be mainly explained in view of the asymmetry of those data due to the random changes of aerosol dust particles in these months. This explanation is confirmed by the corresponding fluctuations of the turbidity factor T_L (m), which is a very good representation for the aerosol contents in the atmosphere, at the same periods

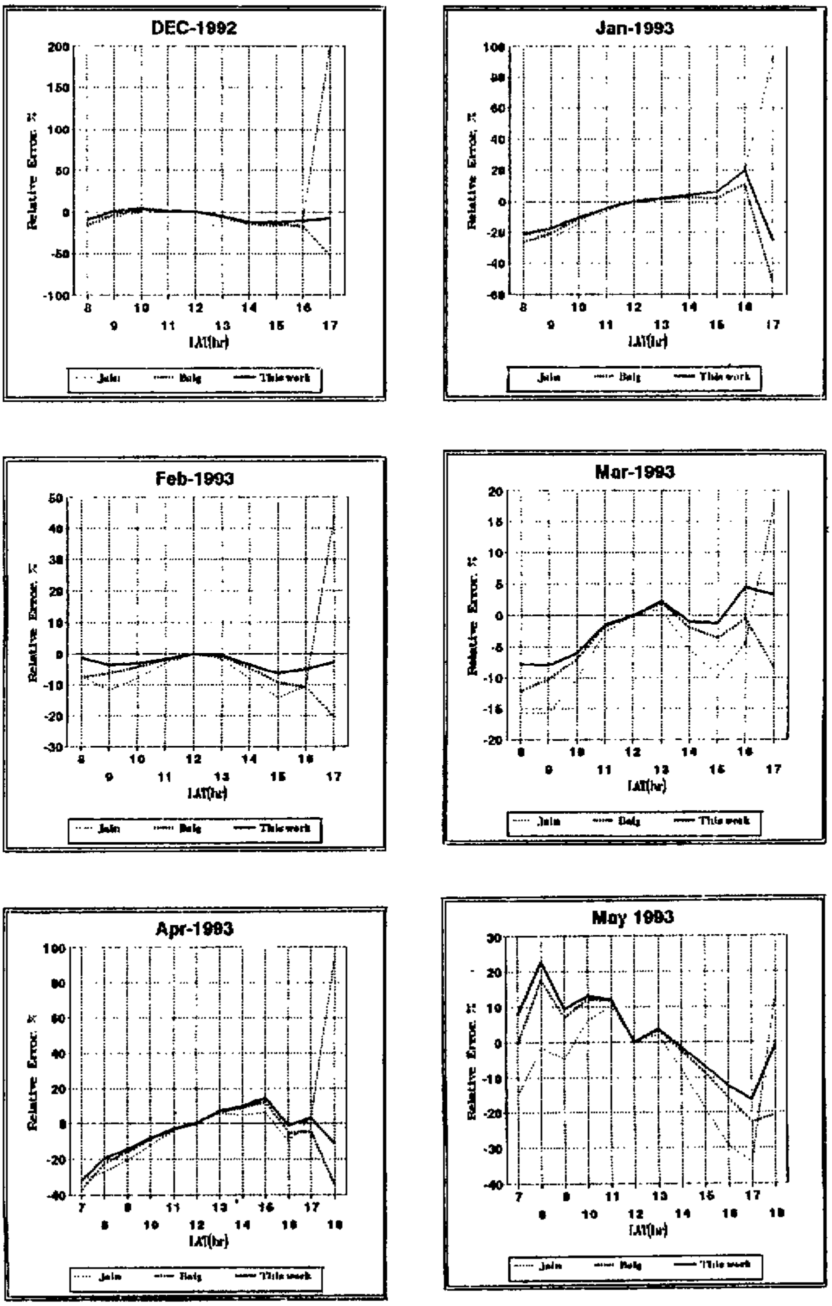

Fig. 2. Plot of the relative errors of the calculated values of RT with respect to the experimental ones against the local time in the period from December 1992 to May 1993.

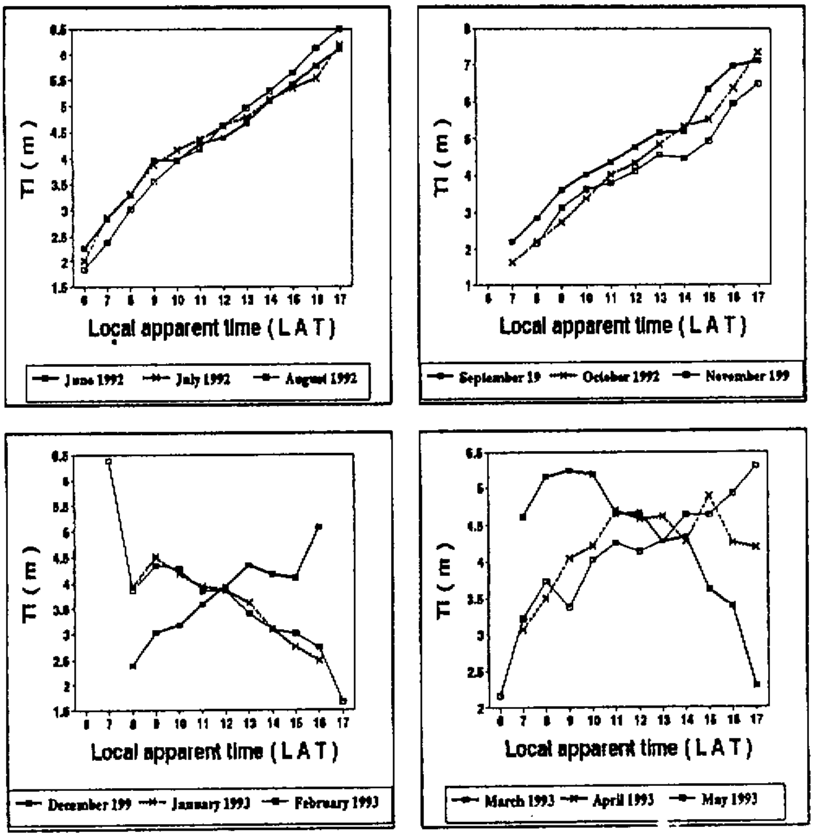

Fig. 3. Diurnal variation of TL (m) at clear days in the period from June 1992 to May 1993.

[see Fig.3]. This variability is quite large in April and May owing to the "Khamaseen winds" which blow in these months. Considering the above evaluation of Jain and Baig models in describing the measured global solar radiation in Qena / Egypt, it seems clearly the need to make some modifications in them to be more usable in the estimation of the global radiation in our region with better accuracy. These corrections may also change from place to another reflecting their local characteristics.

3. *Estimation of Clear Day Global Solar Radiation (Our Proposed Model)*

In this work, a new empirical equation is suggested to be used to estimate the hourly and daily global solar radiation at clear days at Qena / Egypt, with a very good accuracy. The new equation reads as

$$RT,\ \text{This work} = (1/2.2\sigma\sqrt{2}\pi)\{\text{Exp}(-(t-12)^2/2\sigma^2) + 1.2\cos(180^\circ(t-12)/(S_0-0.65))\} \quad (8)$$

Table 1. Values of Measured (G, Exp) and Calculated (G, This work) Hourly global Solar Radiation (Wh / m²) in 1992 / 1993

Date / hours		6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18	daily total
15 / 06 / 92	G,Exp	340	510	716	895	1015	1075	1061	975	812	594	373	152	8557
	G,This work	384	553	752	921	1036	1077	1036	921	752	553	348	150	8465
	RE Error	2.287	8.558	5.021	2.926	2.095	0.204	−2.331	−5.500	−7.372	−6.830	−6.637	−1.125	−1.08
14 / 07 / 92	G,Exp	310	533	733	892	1005	1060	1039	944	788	577	359	142	8466
	G,This work	341	544	739	906	1019	1059	1019	906	739	544	341	145	8281
	RE Error	9.85	2.11	0.89	1.56	1.38	−0.09	−1.93	−4.03	−6.18	−5.83	−5.20	2.14	−2.19
15 / 08 / 92	G,Exp	249	418	615	787	898	943	919	821	668	457	237	82	7075
	G,This work	261	448	632	791	902	941	902	791	632	448	261	84	7198
	RE Error	4.68	7.14	2.79	0.55	0.36	−0.19	−1.90	−3.65	−5.40	−2.05	9.80	2.79	1.73
15 / 09 / 92	G,Exp	205	421	636	808	911	955	927	830	650	430	193		6994
	G,This work	217	418	618	793	915	959	915	793	618	418	217		6971
	RE Error	5.65	−0.74	−2.87	−1.80	0.47	0.39	−1.22	−4.44	−4.92	−2.75	12.28		−0.33
15 / 10 / 92	G,Exp	135	332	530	707	830	865	851	768	592	360	131		6056
	G,This work	141	333	528	701	822	866	822	701	528	333	141		6092
	RE Error	5	0	−0	−1	−1	0	−3	−9	−11	−7	8		0.59
14 / 11 / 92	G,Exp	57	232	406	576	696	741	719	625	465	269	70		4808
	G,This work	64	236	415	579	697	741	697	579	415	236	64		4907
	RE Error	12	2	2	1	0	0	−3	−7	−11	−12	−9		2.07
14 / 12 / 92	G,Exp		200	339	469	591	635	629	559	389	203	31		4060
	G,This work		181	342	489	596	635	596	489	342	181	29		4097
	RE Error		−9	1	4	1	0	−5	−12	−12	−11	−8		0.91
15 / 01 / 93	G,Exp		264	467	609	692	702	646	523	362	174	55		4529
	G,This work		208	384	545	661	703	661	545	384	208	41		4558
	RE Error		−21.07	−17.82	−10.5	−4.59	0.177	2.2965	4.0962	5.9137	19.956	−25.17		0.64
15 / 02 / 93	G,Exp		297	511	695	816	850	806	698	527	308	102		5680
	G,This work		293	493	673	802	849	802	673	493	293	99		5743
	RE Error		−1.39	−3.49	−3.15	−1.75	−0.11	−0.52	−3.54	−6.50	−4.97	−2.81		1.11
14 / 03 / 93	G,Exp		374	581	747	832	861	801	709	541	330	152		6089
	G,This work		345	534	702	819	862	819	702	534	345	157		6063
	RE Error		−7.85	−8.02	−6.00	−1.48	0.04	2.24	−0.96	−1.27	4.62	3.21		−0.43
16 / 04 / 93	G,Exp	384	584	782	921	991	1007	901	770	584	475	254	75	7695
	G,This work	262	468	671	845	964	1007	964	845	671	468	262	67	9970
	RE Error	−31.8	−19.77	−14.27	−8.25	−2.71	−0.02	6.98	9.68	14.75	−1.33	3.10	−11.29	29.56
19 / 05 / 93	G,Exp	285	425	672	816	943	1104	1018	940	788	595	368	110	7947
	G,This work	308	521	735	924	1057	1105	1057	924	735	521	308	109	8118
	RE Error	8	23	9	13	12	0	4	−2	−7	−12	−16	−1	2.15

Table 2. Values of Measured (G,Exp) and Calculated (G.This work) Hourly Global Solar Radiation (Mj / m^2) at Selective Days in June 1994

Date / hours		6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18	daily total
1 / 06 / 94	G,Exp	1.12	1.941	2.682	3.272	3.71	3.884	3.778	3.432	2.874	2.167	1.367	0.476	31.10
	G,This work	1.22	1.95	2.65	3.24	3.64	3.79	3.64	3.24	2.65	1.95	1.22	0.519	30.39
	RE Error	8.91	0.31	−1.32	−0.95	−1.78	−2.49	−3.55	−5.56	−7.91	−10.15	−10.77	9.16	−2.28
3 / 06 / 94	G, Exp	1.1	1.75	2.58	3.19	3.68	3.86	3.76	3.42	2.90	2.18	1.35	0.538	30.70
	G,This work	1.23	1.97	2.69	3.30	3.71	3.86	3.71	3.30	2.69	1.97	1.23	0.519	30.20
	RE Error	11.81	12.77	4.20	3.40	0.76	0.01	−1.38	−3.73	−7.20	−9.48	−9.02	−3.46	−1.63
6 / 06 / 94	G,Exp	1.08	1.90	2.64	3.23	3.63	3.83	3.74	3.39	2.89	2.12	1.31	0.48	30.60
	G,This work	1.24	1.97	2.68	3.28	3.69	3.83	3.69	3.28	2.68	1.97	1.24	0.53	30.06
	RE Error	13.92	3.82	1.50	1.61	1.65	0.01	−1.37	−3.19	−7.15	−6.91	−5.03	10.59	−1.76
7 / 06 / 94	G,Exp	1.12	1.927	2.657	3.202	3.618	3.844	3.750	3.430	2.838	2.033	1.219	0.466	30.50
	G,This work	1.23	1.97	2.68	3.29	3.70	3.84	3.70	3.29	2.68	1.97	1.23	0.526	30.15
	RE Error	9.62	2.14	0.83	2.60	2.20	0.01	−1.40	−4.22	−5.60	−3.18	1.08	12.90	−1.15
8 / 06 / 94	G,Exp	1.047	1.812	2.537	3.132	3.502	3.637	3.657	3.343	2.797	2.077	1.297	0.508	29.71
	G,This work	1.20	1.90	2.56	3.12	3.50	3.64	3.50	3.12	2.56	1.90	1.20	0.531	28.57
	RE Error	15.00	4.80	1.03	−0.25	0.02	−0.01	−4.22	−6.55	−8.36	−8.57	−7.17	4.55	−3.84
12 / 06 / 94	G,Exp	1.16	1.94	2.65	3.22	3.61	3.81	3.72	3.42	2.88	2.17	1.37	0.54	30.88
	G,this work	1.20	1.98	2.68	3.27	3.67	3.81	3.67	3.27	2.68	1.98	1.26	0.555	29.98
	RE Error	8.17	2.07	1.22	1.67	1.64	0.01	−1.53	−4.34	−6.84	−8.52	−8.52	2.20	−2.91
16 / 06 / 94	G, Exp	1.184	1.948	2.637	3.156	3.59	3.801	3.652	3.348	2.811	2.125	1.224	0.457	30.36
	G,This work	1.24	1.96	2.66	3.25	3.66	3.80	3.66	3.25	2.66	1.96	1.24	0.540	29.91
	RE Error	4.49	0.71	0.88	3.12	1.89	0.01	0.16	−2.80	−5.37	−7.68	1.07	18.33	−1.48

Table 3. Values of Measured (G, Exp) and Calculated (G,This work) Hourly Global Solar Radiation (Wh / m^2) at Selective Days in August 1994

Date / hours		6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16	16–17	17–18	daily total
16 / 08 / 94	G,Exp	240.20	458.60	647.90	808.10	933.50	988.40	972.80	870.50	688.10	473.10	271.20	71.00	7470.00
	G,This work	267.32	466.06	661.29	830.37	946.75	988.41	946.75	830.37	661.29	466.06	267.32	79.72	7465.00
	RE Error	11.29	1.63	2.07	2.76	1.42	0.00	−2.68	−4.61	−3.90	−1.49	−1.43	12.29	−0.07
17 / 08 / 94	G, Exp	230.80	442.80	631.20	781.50	898.00	959.30	947.70	856.80	688.70	475.00	275.90	75.40	7307.00
	G, This work	260.90	454.20	643.50	806.95	919.16	959.28	919.16	806.95	643.50	454.20	260.90	78.04	7235.00
	RE Error	13.04	2.57	1.95	3.26	2.36	−0.00	−3.01	−5.82	−6.56	−4.38	−5.44	3.50	−0.99
18 / 08 / 94	G, Exp	237.90	457.10	644.70	794.70	919.40	975.40	970.10	871.50	729.50	524.50	290.70	80.20	7529.00
	G,This work	267.76	464.92	657.10	822.22	935.16	975.45	935.16	822.22	657.10	464.92	267.76	80.60	7345.00
	RE Error	12.55	1.71	1.92	3.46	2.69	0.01	−3.60	−5.65	−9.92	−11.36	−7.89	0.50	−2.44
19 / 08 / 94	G, Exp	242.8	457.8	644	794.4	907.2	975.1	970	878.5	716.4	511.1	295	84.2	7524.00
	G,This work	266.57	463.97	656.36	821.67	934.73	975.07	934.73	821.67	656.36	463.97	266.57	79.22	7332.00
	Re Error	9.79	1.35	1.92	3.43	3.04	−0.00	−3.64	−6.47	−8.38	−9.22	−9.64	−5.91	−2.55

σ, S_0 have the same definitions as in Eqs.1 and 2. The results of the relative errors of RT values calculated according to the new approach in Eq.8 with respect to the experimental ones are represented also in Figs. 1 and 2. From both figures one can see clearly that using the equation suggested in this work leads in general to theoretical RT values agree to high extend with the experimental ones during the day from sunrise to sunset in comparison to Jain and Baig equations. A slight misfit was observed in very little points, especially in January, April and May, for the same reasons discussed in Section 3.2, but it still quite smaller than that obtained applying Eqs.1 and 2.

Now the hourly and daily totals global solar radiation can be calculated at any clear day with the aid of estimated values of RT, provide that we know the value of G_h at any hour in the day. The measured value of G_h at noon time, in which $(RT)_{th}$ agree very well with $(RT)_{exp}$, was used in this work. The results of estimation process are summarized in Table 1 for the same days as in Figs.1 and 2. As expected, our model estimates daily and hourly global solar radiation with least errors. Furthermore the validity of this model was verified with new measurements in selective clear days in June and August 1994 as illustrated in Tables 2 and 3. The very low relative errors, which given in this tables indicate the high performance of our equation in estimating the daily and hourly global solar radiation in Qena / Egypt.

IV. CONCLUSION

Hourly global solar radiation data measured at clear days in Qena / Upper Egypt have been analyzed in view of models based on modified version of Gaussian distribution function, namely Jain model, Baig model and a new model, which proposed in this work. The following conclusions are deduced:

1. In general the model suggested in this work and Baig model provide the best hourly and daily estimates of global solar radiation, with our model usually best in the most hours of the day.

2. The very good agreement between our model values and new measurements in selective clear days, which were not used in its deduction, confirms its performance in our region.

3. Jain model did not perform as good as the other two models, except very near to local noon, although it is similar in principle to them.

4. The performance of the better global irradiation models is probably limited by the random variability of the atmospheric turbidity caused by the changes in the content of the absorbing and scattering gases and molecules in the atmosphere.

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