Study on Instability in Baroclinic Vortex Symmetric Disturbance under Effect of Nonuniform Environmental Parameters①

Fei Jianfang (费建芳) and *Lu Hancheng* (陆汉城)
Air Force Institute of Meteorology, Nanjing 211101
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ABSTRACT

With the aid of the baroclinicity parameter M^2, inertial instability parameter F^2 and the stratification instability paramter N^2 as the slowly varying function both spatially and temporally, an energetic equation is derived of symmetric perturbation waves in baroclinic vortices in the framework of progressively changing wavetrain theory, or WKB, alongside the examination of effects of these parameters upon the vortex disturbance.

Key words: Baroclinic vortex, Symmetric disturbance, Energetic equation, Disturbance development

I. INTRODUCTION

It is known that such entities as typhoons and cyclones are common in the atmosphere and often exhibit characteristic features of wave, the examples being the rainbands related to a typhoon spiral cloud and a frontal cyclone's warm sector. The mechanisms for the typhoon spiral structure genesis have been intensively investigated (Oyoama, 1996; Liu and Yang, 1980; Huang and Cao, 1980; Fei and Lu, 1996). As shown in Fei and Lu, the criteria for symmetric instability of a baroclinically circular vortex are gained in terms of the constant values of M^2, F^2 and N^2, and possible causes of the formation / development of the spiral cloud and rainbands in a typhoon are interpreted in a more reasonable fashion by virtue of symmetric instability concept. The three parameters are in reality space / time-dependent so that are bound to exert significant influence on the shift / evolution of the disturbance in a baroclinic vortex. For this purpose, the WKB technique is adopted to investigate the impact on the perturbation of the spatially / temporally varying instability paramters M^2, F^2 and N^2 of the ambient field.

II. BASIC EQUATIONS

Under the Boussinesq approximation, with an axially-symmetric vortex in the background field, the two-dimensional linearized disturbance equations in a column coordinate system take the form (Fei and Lu, 1996)

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$$\begin{cases} \dfrac{\partial u'}{\partial t} - f_1 v' + \dfrac{1}{\bar{\rho}} \dfrac{\partial p'}{\partial r} = 0 \ , \\ \dfrac{\partial v'}{\partial t} + f_2 u' + \dfrac{\partial \Omega r}{\partial z} w' = 0 \ , \\ \dfrac{\partial w'}{\partial t} + \dfrac{1}{\bar{\rho}} \dfrac{\partial p'}{\partial z} - g \dfrac{\theta'}{\bar{\theta}} = 0 \ , \\ \dfrac{\partial \theta'}{\partial t} + \dfrac{\partial \bar{\theta}}{\partial r} u' + \dfrac{\partial \bar{\theta}}{\partial z} w' = 0 \ , \\ \dfrac{1}{r} \dfrac{\partial r u'}{\partial r} + \dfrac{\partial w'}{\partial z} = 0 \ , \end{cases} \tag{1}$$

which can be reduced, after manipulation, to

$$\begin{cases} \dfrac{\partial u'}{\partial t} - f_1 v' + \dfrac{1}{\bar{\rho}} \dfrac{\partial p'}{\partial r} = 0 \ , \\ f_1 \dfrac{\partial v'}{\partial t} + F^2 u' + M^2 w' = 0 \ , \\ \dfrac{\partial w'}{\partial t} + \dfrac{1}{\bar{\rho}} \dfrac{\partial p'}{\partial z} - g \dfrac{\theta'}{\bar{\theta}} = 0 \ , \\ \dfrac{g}{\bar{\theta}} \dfrac{\partial \theta'}{\partial t} + M^2 u' + N^2 w' = 0 \ , \\ \dfrac{1}{r} \dfrac{\partial r u'}{\partial r} + \dfrac{\partial w'}{\partial z} = 0 \ , \end{cases} \tag{2}$$

where $M^2 [= f_1 \dfrac{\partial \Omega r}{\partial Z} = \dfrac{g}{\bar{\theta}} \dfrac{\partial \bar{\theta}}{\partial r}]$ denotes the baroclinicity parameter; $F^2 [= f_1 f_2 = (2\Omega + f)(2\Omega + f + r\dfrac{\partial \Omega}{\partial r})]$ the inertial instability parameter; $N^2 [= \dfrac{g}{\bar{\theta}} \dfrac{\partial \bar{\theta}}{\partial z}]$ the stratification instability parameter, i, e., the Brunt–Väisälä frequency.

III. WAVE FREQUENCY EQUATION

Introduce the slowly–changing coordinates

$$R = \varepsilon r, \quad Z = \varepsilon z, \quad T = \varepsilon t \ , \tag{3}$$

where $\varepsilon (\ll 1)$ is the small parameter. Assume the solution to (2) to be

$$\psi = \Psi(R, Z, T) e^{i\varphi} \ , \tag{4}$$

in which $\psi = (u', v', w', p', \theta')$, $\Psi = (U, V, W, P, \Theta)$ and $\varphi = kr + nz - \omega t$, with $k = \dfrac{\partial \varphi}{\partial r}$ and $n = \dfrac{\partial \varphi}{\partial z}$ representing the wavenumber radially and vertically, respectively, and $\omega = -\dfrac{\partial \varphi}{\partial t}$ for the wave frequency. Putting (4) into (2) yiesds

$$\begin{cases} -i\omega U+\varepsilon\dfrac{\partial U}{\partial T}-f_1V+\dfrac{1}{\rho}\left(ikP+\varepsilon\dfrac{\partial P}{\partial R}\right)=0\ , \\ f_1\left(-i\omega V+\varepsilon\dfrac{\partial V}{\partial T}\right)+F^2U+M^2W=0\ , \\ -i\omega W+\varepsilon\dfrac{\partial W}{\partial T}+\dfrac{1}{\rho}\left(inP+\varepsilon\dfrac{\partial P}{\partial Z}\right)-\dfrac{g}{\theta}\Theta=0\ , \\ \dfrac{g}{\theta}\left(-i\omega\Theta+\varepsilon\dfrac{\partial\Theta}{\partial T}\right)+M^2U+N^2W=0\ , \\ ikU+inW+\varepsilon\left(\dfrac{1}{R}\dfrac{\partial RU}{\partial R}+\dfrac{\partial W}{\partial Z}\right)=0\ . \end{cases} \tag{5}$$

Ψ is expanded, through the WKB scheme, into a power series of ε in the form

$$\Psi=\Psi_0+\varepsilon\Psi_1+\varepsilon^2\Psi_2+\ \cdots\cdots\ . \tag{6}$$

Substitution of (6) into (5) gives the zero approximations

$$\begin{cases} -i\omega U_0-f_1V_0+\dfrac{1}{\rho}ikP_0=0\ , \\ -f_1i\omega V_0+F^2U_0+M^2W_0=0\ , \\ -i\omega W_0+\dfrac{1}{\rho}inP_0-\dfrac{g}{\theta}\Theta_0=0\ , \\ -i\omega\dfrac{g}{\theta}\Theta_0+M^2U_0+N^2W_0=0\ , \\ ikU_0+inW_0=0 \end{cases} \tag{7}$$

and therefrom we have

$$U_0=-\frac{n}{k}W_0\ , \tag{8}$$

$$f_1V_0=-\frac{i}{\omega}\left(M^2-\frac{n}{k}F^2\right)W_0\ , \tag{9}$$

$$\frac{1}{\rho}P_0=\frac{1}{n}\left[\omega-\frac{1}{\omega}\left(N^2-\frac{n}{k}M^2\right)\right]W_0\ , \tag{10}$$

$$\frac{g}{\theta}\Theta_0=-\frac{i}{\omega}\left(N^2-\frac{n}{k}M^2\right)w_0\ . \tag{11}$$

By inserting (8)–(11) into the first of (7) we get the local dispersion relation

$$\omega^2=\frac{1}{K^2}\left(n^2F^2+k^2N^2-2knM^2\right)\ , \tag{12}$$

or

$$\omega=\pm\left[\frac{1}{K^2}\left(n^2F^2+k^2N^2-2knM^2\right)\right]^{1/2}=G(R,Z,T,k,n) \tag{13}$$

with $K^2=k^2+n^2$.

From (12) we find the expressions for group velocities

$$C_{gr}=\frac{1}{\omega K^4}\left(kn^2N^2+k^2nM^2-n^3M^2-kn^2F^2\right)\ , \tag{14}$$

$$C_{gz}=\frac{1}{\omega K^4}(-k^2nN^2+kn^2M^2-k^3M^2+k^2nF^2)\ , \tag{15}$$

from which we know that the radial and vertical group velocities satisfy the relation

$$kC_{gr}+nC_{gz}=0\ ,$$

a derivation that is congruent with that reported by Huang and Cao (1980) and Chen (1984).

From the relation for k,n and ω we get

$$\frac{\partial k}{\partial T}=-\frac{\partial \omega}{\partial R}\ , \tag{16}$$

$$\frac{\partial n}{\partial T}=-\frac{\partial \omega}{\partial Z}\ , \tag{17}$$

$$\frac{\partial k}{\partial Z}=\frac{\partial n}{\partial R}\ . \tag{18}$$

Also, we have, in the column coordinates, the kinematic relations

$$\frac{\partial \omega}{\partial T}+C_{gr}\frac{\partial \omega}{\partial R}+C_{gz}\frac{\partial \omega}{\partial Z}=\left(\frac{\partial G}{\partial T}\right)_{R,Z,k,n} \tag{19}$$

$$\frac{\partial k}{\partial T}+C_{gr}\frac{\partial k}{\partial R}+C_{gz}\frac{\partial k}{\partial Z}=\left(\frac{\partial G}{\partial R}\right)_{Z,T,k,n} \tag{20}$$

$$\frac{\partial n}{\partial T}+C_{gr}\frac{\partial n}{\partial R}+C_{gz}\frac{\partial n}{\partial Z}=\left(\frac{\partial G}{\partial Z}\right)_{R,T,k,n} \tag{21}$$

IV. ENERGETIC EQUATION OF THE STUDY WAVE

The first approximations of the power series expansion of (2) are in the form

$$\begin{cases} -i\omega U_1+\frac{\partial U_0}{\partial T}-f_1V_1+\frac{1}{\rho}\left(ikP_1+\frac{\partial P_0}{\partial R}\right)=0\ , \\ -if_1\omega V_1+f_1\frac{\partial V_0}{\partial T}+F^2U_1+M^2W_1=0\ , \\ -i\omega W_1+\frac{\partial W_0}{\partial T}+\frac{1}{\rho}\left(inP_1+\frac{\partial P_0}{\partial Z}\right)-\frac{g}{\theta}\Theta_1=0\ , \\ \frac{g}{\theta}\left(-i\omega\Theta_1+\frac{\partial \Theta_0}{\partial T}\right)+M^2U_1+N^2W_1=0\ , \\ ikU_1+inW_1+\frac{1}{R}\frac{\partial RU_0}{\partial R}+\frac{\partial W_0}{\partial Z}=0\ . \end{cases} \tag{22}$$

Form its 5th, 4th, 3rd and lst expressions we obtain, respectively

$$U_1=-\frac{n}{k}W_1+\frac{i}{k}\left(\frac{1}{R}\frac{\partial RU_0}{\partial R}+\frac{\partial W_0}{\partial Z}\right)\ ,$$

$$-\frac{g}{\theta}\Theta_1=-\frac{i}{\omega}\left(\frac{n}{k}M^2-N^2\right)W_1-\frac{M^2}{k\omega}\left(\frac{1}{R}\frac{\partial RU_0}{\partial R}+\frac{\partial W_0}{\partial Z}\right)+\frac{i}{\omega}\frac{g}{\theta}\frac{\partial \Theta_0}{\partial T}\ ,$$

$$\frac{i}{\bar{\rho}}nP_1 = \left[i\omega + \frac{i}{\omega}\left(\frac{n}{k}M^2 - N^2\right)\right]W_1 + \frac{M^2}{k\omega}\left(\frac{1}{R}\frac{\partial RU_0}{\partial R} + \frac{\partial W_0}{\partial Z}\right)$$
$$-\frac{\partial W_0}{\partial T} - \frac{ig}{\bar{\theta}\omega}\frac{\partial \Theta_0}{\partial T} - \frac{1}{\bar{\rho}}\frac{\partial P_0}{\partial Z},$$
$$-f_1 V_1 = i\omega\left[-\frac{n}{k}W_1 + \frac{i}{k}\left(\frac{1}{R}\frac{\partial RU_0}{\partial R} + \frac{\partial W_0}{\partial Z}\right)\right]$$
$$-\frac{\partial U_0}{\partial T} - \frac{1}{\bar{\rho}}\frac{\partial P_0}{\partial R} - \frac{ki}{\bar{\rho}}P_1,$$

whose substitution into the 2nd yeilds

$$-\frac{1}{k\omega}\left(\omega^2 + \frac{k}{n}M^2 - F^2\right)\left(\frac{1}{R}\frac{\partial RU_0}{\partial R} + \frac{\partial W_0}{\partial Z}\right) =$$
$$\frac{\partial U_0}{\partial T} + \frac{if_1}{\omega}\frac{\partial V_0}{\partial T} - \frac{k}{n}\frac{\partial W_0}{\partial T} - \frac{k}{n}\frac{g}{\bar{\theta}}\frac{i}{\omega}\frac{\partial \Theta_0}{\partial T} + \frac{1}{\bar{\rho}}\frac{\partial P_0}{\partial R} - \frac{k}{n}\frac{1}{\bar{\rho}}\frac{\partial P_0}{\partial Z}.$$

Then (8)–(11) are put into the resulting equation, followed by the use of (12) for local dispersion relation and multiplication of both sides of the equation by W_0, thus leading to

$$\frac{K^2\omega}{kn}\frac{D_g W^2}{DT} = W_0\left\{\frac{K^2}{kn}\frac{\partial\omega}{\partial T} + \frac{1}{k^2 n\omega}(k^2\omega^2 + k^2N^2 - knM^2)\frac{\partial\omega}{\partial R}\right.$$
$$-\frac{1}{kn^2\omega}(k^2\omega + k^2N^2 - knM^2)\frac{\partial\omega}{\partial Z} + \frac{1}{k^2 n\omega}(n^2\omega^2 + n^2F^2 - knM^2)\frac{\partial k}{\partial T}$$
$$+\frac{1}{k^3 n}(n^2\omega^2 - n^2F^2)\frac{\partial k}{\partial R} + \frac{M^2}{kn}\frac{\partial k}{\partial Z} - \frac{1}{kn^2\omega}(n^2\omega^2 + n^2F^2 - knM^2)\frac{\partial n}{\partial T}$$
$$-\frac{1}{k^2n^2}(n^2\omega^2 + k^2\omega^2 + 3knM^2 - n^2F^2 - k^2N^2)\frac{\partial n}{\partial R}$$
$$+\frac{1}{n^3 k}(k^2\omega^2 - k^2N^2)\frac{\partial n}{\partial Z} + \frac{2}{\omega}\frac{\partial M^2}{\partial T} + \frac{1}{k}\frac{\partial M^2}{\partial R}$$
$$-\frac{1}{n}\frac{\partial M^2}{\partial Z} - \frac{k}{n\omega}\frac{\partial N^2}{\partial T} - \frac{1}{n}\frac{\partial N^2}{\partial R} + \frac{k}{n^2}\frac{\partial N^2}{\partial Z}$$
$$\left.-\frac{n}{k\omega}\frac{\partial F^2}{\partial T}\right\} - \frac{K^2\omega}{kn}C_{gr}\frac{W_0^2}{R}, \tag{23}$$

where

$$\frac{D_g}{DT} = \frac{\partial}{\partial T} + C_{gr}\frac{\partial}{\partial R} + C_{gz}\frac{\partial}{\partial Z}.$$

With the aid of (14) through (18), (23) reduces to

$$\frac{\partial W_0^2}{\partial T} + \frac{1}{R}\frac{\partial RC_{gr}W_0^2}{\partial R} + \frac{\partial C_{gz}W_0^2}{\partial Z} =$$

$$W_0^2\Big\{\frac{1}{\omega}\Big(\frac{\partial G}{\partial T}\Big)_{R,Z,k,n}-\frac{2}{k}\frac{n^2-k^2}{k^2+n^2}\Big(\frac{\partial G}{\partial R}\Big)_{Z,T,k,n}$$
$$-\frac{2}{n}\frac{k^2-n^2}{k^2+n^2}\Big(\frac{\partial G}{\partial Z}\Big)_{R,T,k,n}+\frac{2kn}{K^2\omega^2}\frac{\partial M^2}{\partial T}+\frac{kn}{K^2\omega}\Big[\frac{1}{k}+\frac{1}{knK^2}(k^2n-n^3)\Big]\frac{\partial M^2}{\partial R}$$
$$+\frac{kn}{K^2\omega}\Big[-\frac{1}{n}+\frac{1}{knK^2}(kn^2-k^3)\Big]\frac{\partial M^2}{\partial Z}-\frac{n^2}{K^2\omega^2}\frac{\partial F^2}{\partial Z}$$
$$-\frac{kn^2}{K^4\omega}\frac{\partial F^2}{\partial R}+\frac{k^2n}{K^4\omega}\frac{\partial F^2}{\partial Z}-\frac{k^2}{K^2\omega^2}\frac{\partial N^2}{\partial T}$$
$$+\frac{kn}{K^2\omega}\Big(-\frac{1}{n}+\frac{n}{K^2}\Big)\frac{\partial N^2}{\partial R}-\frac{k^2n}{K^2\omega}\Big(\frac{1}{n^2}+\frac{1}{K^2}\Big)\frac{\partial N^2}{\partial Z}\Big\}, \tag{24}$$

which takes on the form, after manipulation based on (13),

$$\frac{\partial W_0^2}{\partial T}+\frac{1}{R}\frac{\partial RC_{gr}W_0^2}{\partial R}+\frac{\partial C_{gz}W_0^2}{\partial Z}=$$
$$W_0^2\Big[\frac{1}{K^2\omega^2}\Big(kn\frac{\partial M^2}{\partial T}-\frac{n^2}{2}\frac{\partial F^2}{\partial T}-\frac{k^2}{2}\frac{\partial N^2}{\partial T}\Big)$$
$$+\frac{n^2}{K^4\omega^2}C_r\Big(2kn\frac{\partial M^2}{\partial R}-n^2\frac{\partial F^2}{\partial R}-k^2\frac{\partial N^2}{\partial R}\Big)$$
$$+\frac{n^2}{K^4\omega^2}C_z\Big(-2kn\frac{\partial M^2}{\partial Z}+n^2\frac{\partial F^2}{\partial R}+k^2\frac{\partial N^2}{\partial Z}\Big)\Big], \tag{25}$$

in which $C_r=\frac{\omega}{k}$ ($C_z=\frac{\omega}{n}$) signifies the phase velocity horizontally (vertically), in view of the wave energy $E\propto W_0^2$, we have

$$\frac{\partial E}{\partial T}+\frac{1}{R}\frac{\partial RC_{gr}E}{\partial R}+\frac{\partial C_{gz}E}{\partial Z}=$$
$$+\frac{E}{K^2\omega^2}\Big[kn\frac{\partial M^2}{\partial T}-\frac{n^2}{2}\frac{\partial F^2}{\partial T}-\frac{k^2}{2}\frac{\partial N^2}{\partial T}$$
$$+\frac{1}{K^2}\Big(2knC_r\frac{\partial M^2}{\partial R}-n^2C_r\frac{\partial F^2}{\partial R}-k^2C_r\frac{\partial N^2}{\partial R}\Big)$$
$$+\frac{1}{K^2}\Big(-2knC_z\frac{\partial M^2}{\partial Z}+n^2C_z\frac{\partial F^2}{\partial Z}+k^2C_z\frac{\partial N^2}{\partial Z}\Big)\Big], \tag{26}$$

which is the energetic equation of the wave of interest. It follows that the energy is conserved as M^2, F^2, and N^2 are constant. We thus come to a conclusion that the spatial / temporal variations of the instability parameters serve as the source / sink of symmetric disturbance development for baroclinic vorticies.

V. DISCUSSIONS

Integration is accomplished of (26) inside a particular area with the boundary perturbation set zero and the systems like typhoons and cyclones are associatied with the

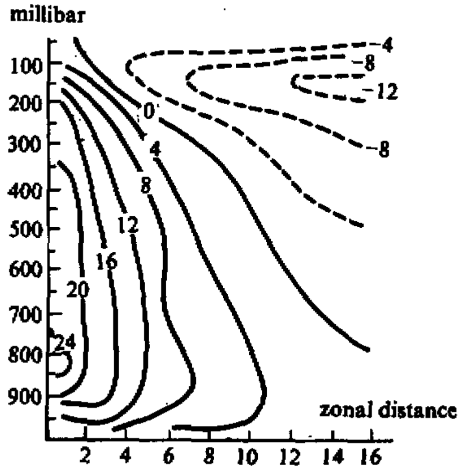

Fig. 1. Radial distribution of averaged tangential windspeed in a symmetrical flowfield of a typhoon. [Lin et al, 1988]

transportation of warm and moist flow, thereby making for the humidity high enough to approach saturation inside such that M^2 and N^2 of (26) can be replaced with the moist intertial instability parameter M_w^2 and moist stratification instability parameter N_w^2, respectively, thus resulting in a modified version of (26) (Emanuel, 1983) in the form

$$\frac{\partial}{\partial T}\int_\tau E d\tau=\int_\tau \frac{E}{K^2\omega^2}\left(kn\frac{\partial M_w^2}{\partial T}-\frac{n^2}{2}\frac{\partial F^2}{\partial T}-\frac{k^2}{2}\frac{\partial N_w^2}{\partial T}\right)d\tau$$
$$+\int_\tau \frac{n^2E}{K^4\omega^2}\left(2knC_r\frac{\partial M_w^2}{\partial R}-n^2C_r\frac{\partial F^2}{\partial R}-k^2C_r\frac{\partial N_w^2}{\partial R}\right)d\tau$$
$$+\int_\tau \frac{n^2E^2}{K^4\omega^2}\left(-2knC_z\frac{\partial M_w^2}{\partial Z}+n^2C_z\frac{\partial F^2}{\partial Z}+k^2C_z\frac{\partial N^2}{\partial Z}\right)d\tau\ , \qquad (27)$$

the rths of which can be regarded as the source / sink for disturbance development with in the integrated sector.

1. *Effect of Temporal Variation in the Instability Parameters upon the Wave Energy Perturbation over the Integrated Sector*

(1) $\frac{\partial}{\partial T}\int_\tau E d\tau \propto kn\frac{\partial M_w^2}{\partial T}$ denotes the role of the baroclinicity of the environmental field on the variation of wave energy over the integrated area and bears the relation to the positive / negative wavenumber (k,n) in the perturbation field.

We take a typhoon for example to illustrate the role of this term under the assumption that $\omega^2 > 0$. By referring to the distribution of mean tangential and radial velocities in such a storm (Fig. 1) or to its characteristic warm-core sturcture, it is seen from the expression for M^2 that the conditions of the troposphere around the typhoon's center meet $M^2 < 0$

except in the near-surface layer. $\frac{\partial M^2}{\partial T} < 0$, when M^2 gets enhanced as a function of time. Hence $kn < 0$ is indispensable for developing disturbance.

It is well-known that a symmetrically, unsteadily disturbed streamfield is composed of convective cells with the equiphase surface sloping towards the cold sector (Bennetts et al., 1979). For a typhoon, such cells are expected to be inclined outward of the center, and $k > 0$ and $n < 0$ will occur if disturbance propagates outward. For this reason, only when these cells are inclined outward is the perturation made to get vigorous. In contrast, for $\frac{\partial M^2}{\partial T} > 0$, the inward-inclined convective cells will permit disturbance to develop.

Previous studies on typhoons have focus on vertical convection, ignoring slanting cases (symmetric instability). The present work and the preceding one of the authors (1996) show that symmetrical instability can act as a mechanism for disturbance invigoration of a typhoon. Observations reveal that one or more spiral cloud / rain bands are common in a typhoon and their genesis / development are likely to agree with the rising branches of the symmetrically unstable perturbation flowfield.

(2). $\frac{\partial}{\partial T}\int_{\tau} E\tau d \propto -\frac{\partial F^2}{\partial T}$ represents the effect of environmental inertial instability on wave energetics over an integrated area.

Evidently, disturbance will intensify as the instability increases. As a metter of fact, the mid / lower troposphere inside a typhoon is normally in a state of inertial stability (absolute vorticity > 0), with the instability happening solely at its upper levels. Moreover, it is seen from the expression of F^2 that, if positive absolute vorticity decreases at mid / lower levels of the storm and negative grows at its higher levels, then the disturbance will be strengthened, a result that is in accord with synoptic analysis on an operational basis.

(3). $\frac{\partial}{\partial T}\int_{\tau} Ed\tau \propto -\frac{\partial N_w^2}{\partial T}$ indicates the influence of ambient static stability on the wave energetics in an integrated sector.

As stratification instability is enhanced, so is the disturbance, a fact that has been borne out both in theoretical and operational aspects (Cao, 1980). A typhoon is known to be at lower latitudes where the environmental atmosphere is of high temperature and humidity so that stratification instability is expected to be satisfied more often than not.

2. *Effect of Spatial Variation in the Instability Parameters upon Wave Energy (Disturbance) over an Integrated Sector*

It is clear from (27) that the nonuniform distribution of values of all the parameters in the environmental field has influence on the wave energy in an integration area, which is actually produced by phase velocities (C_r, C_Z)-caused advection, vertical and horizontal, of all these parameters.

(1) Impact on the disturbance of horizontal variation in the parameters

For M^2, the warm-core feature gets strongest in the vicinity of the typhoon's center, where the horizontal temperature gradients become so as well (Chen and Ding, 1979), thus leading to a maximum of wave energy decline (negative magnitudes). For the outward inclined and shifted perturbation ($kn < 0$), this will prevent its growth, that is, its energy reduces as the distrubance migrates outward from the center.

For F^2 and N^2, the perturbance will diminish whilst travelling from a strongly unstable to a weakly unsteady region, and vice versa.

(2) Influence upon the disturbance of vertical variation in these parameters

In view of the fact that the typhoon's warm-core feature gets most vigorous in the neighborhood of 250 hPa (Chen and Ding, 1979), so does the moist baroclinicity parameter M_w^2 at the upper levels. This shows that, during baroclinic perturbation propagating upward ($k < 0$, $n > 0$), the disturbance becomes reinforced and vice versa.

It follows from (27) that F^2 and N_w^2 have opposite sign for their vertical and horizontal variations, suggesting the reversed effect on the perturbation.

For $\omega^2 < 0$, we have results as opposed to the above statement. And under this condition, wave will be unsteady, only with local convection available.

VI. CONCLUDING REMARKS

Criteria for symmetric instability of baroclinic gyres are derived in terms of constant values of the parameters M^2, F^2 and N^2 (Fei and Lu, 1996). This paper presents the energetic equation for wave of the baroclinic vortex (typhoon) through the agency of the WKB technique, whereby the effects are investigated of the spatial / temporal variations in these parameters upon the disturbance development of the typhoon as the example.

Main results achieved are as follows:

(1) For constant M^2, F^2, and N^2, the wave energy is conserved with their space / time variations acting as the energy source / sink for the perturbation in the baroclinic vortex.

(2) With the growth of baroclinicity of the gyre (typhoon) as a function of time, the disturbance develops, migrating outward from the center. If, on the other hand, the perturbation travelling horizontally outward becomes feeble, its vertical propagation gets enhanced.

(3) The disturbance becomes intense as the ambient inertial instability is reinforced versus time. Different from the case of being advected vertically, the perturbation is intensified when advected horizontally as phase velocity from a weakly into a vigorously unsteady stratification segment.

(4) The disturbance happening due to the temporal growth of the environmental stratification instability follows the same pattern as in the case of the ambient inertial instability.

It should be noted that whether the perturbation is intensified in a baroclinic vortex depends on the integrated effects of all these factors. As the combined unstable energy increases (decreases) to certain extent, the disturbance (displayed as spiral cloud rain bands) will be enhanced (enfeebled). It is believed that these findings are more or less enlightening in dealing qualitatively with the issue of the development of perturbation in a vortex.

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