Symmetric Development of Meso Perturbation in Zonally Curved Basic Flow①

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ABSTRACT

Addressed is a problem as to meso perturbation wave ensemble development in a curved basic flow in the context of a f-plane non-hydrostatic equilibrium acoustic wave filtering model in natural coordinates with the aid of the WKJB and energetic approaches. Results show that the symmetric development depends crucially on the matching of structures of the disturbance wave and background field, and for a smooth (curved) basic flow the wave ensemble evolution hinges upon the spatial imhomogeneity of nonthermal wind of the background field (under nongradient wind balance). Finally, presented is the wave ensemble evolution in relation to the thermal curvature vorticity in the background field.

Key words: Curved basic flow, Symmetric development, Curvature vorticity

I. INTRODUCTION

Numerous studies show meso symmetric instability to be one of the mechanisms for mesosystem genesis. Review includes Hoskins (1974), Emamuel (1982), Zhang (1988) and many others that are devoted to detailed investigation of the symmetric instability and development of geostrophic smooth flow, thereby enriching the theories on perturbation stability at this scale. Observational studies, however, reveal that basic flow controlling the disturbance tends to be cyclonically or anticyclonically curved instead of smooth (Huang et al., 1982; Ding et al., 1992), the example being mostly anticyclonically-curved upper-air jet responsible for summertime rainstorm amplification over the Jiang-Huai drainage. This paper concerns meso wave ensemble development in a curved basic flow in terms of a f-plane non-hydrostatic-equilibrium, ageostrophic, adiabatic and acoustic wave filtering model in natural coordinates by means of multi-scale and WKJB techniques.

II. BASIC EQUATIONS

It has been demonstrated that the f-plane, non-hydrostatic, acoustics removing model is optimal (Zhang, 1980). To deal with the impact of nonzonal flow on meso-system behavior, we assume natural coordinates for horizonal dimensions, s for basic flow $\overline{U}$ direction and the normally-used z vertically so that we have a system of perturbation equations of $\left(\frac{\partial}{\partial s}=0\right)$ symmetric with respect to s:

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$$\frac{\partial u}{\partial t} - f_\alpha v + \overline{U}_z w = 0 \ , \tag{1}$$

$$\frac{\partial v}{\partial t} + f_\beta u + \frac{\partial p}{\partial n} = 0 \ , \tag{2}$$

$$\frac{\partial w}{\partial t} - \theta + \frac{\partial p}{\partial z} = 0 \ , \tag{3}$$

$$\frac{\partial \theta}{\partial t} - F^2 v + N^2 w = 0 \ , \tag{4}$$

$$\frac{\partial v}{\partial n} + \frac{\partial w}{\partial z} = 0 \ , \tag{5}$$

where $(u, v, w, \theta) \equiv \overline{\rho}(u', v', w', \frac{g}{\theta_0}\theta')$ with the prime denoting the perturbation quantity and u', v', and w' as the components of the field in **s**, **n**, and z directions, respectively. Consequently,

$$F^2 = -\frac{g}{\theta_0}\frac{\partial \overline{\theta}}{\partial n} \equiv f\overline{U}_\theta \ , \tag{6}$$

$$N^2 = -\frac{g}{\theta_0}\frac{\partial \overline{\theta}}{\partial z} \ , \tag{7}$$

$$f_\alpha = f - \frac{\partial \overline{u}}{\partial n} \ , \tag{8}$$

$$f_\beta = f + 2K_s \overline{U} \ , \tag{9}$$

where θ_0 represents a standard value of potential temperature, $\overline{U}(n, z)$ and $\overline{\theta}(n, z)$ the basic wind and potential temperature field, respectively, $\overline{U}_\theta$ the thermal wind of the basic temperature field; K_s the curvature of the basic wind, assumed to be a constant.

Introduction of streamfunction $\psi(n, z, t)$ into (5) leads to

$$w = -\frac{\partial \psi}{\partial n} \ , \quad v = \frac{\partial \psi}{\partial z} \tag{10}$$

which is then substituted into (1)–(4) by referring to (6)–(9). Thus we find

$$\frac{\partial^2}{\partial t^2}\nabla^2\psi + f_\alpha f_\beta \frac{\partial^2 \psi}{\partial z^2} + (f_\beta \overline{U}_z + F^2)\frac{\partial^2 \psi}{\partial n \partial z} + N^2 \frac{\partial^2 \psi}{\partial n^2}$$
$$+ \left[\frac{\partial}{\partial z}(f_\alpha f_\beta) + \frac{\partial F^2}{\partial n}\right]\frac{\partial \psi}{\partial z} + \left[\frac{\partial}{\partial z}(f_\beta \overline{U}_z) + \frac{\partial N^2}{\partial n}\right]\frac{\partial \psi}{\partial n} = 0 \ , \tag{11}$$

with $\nabla^2 = \frac{\partial^2}{\partial n^2} + \frac{\partial^2}{\partial z^2}$.

The problem of eigenvalues of (11) has been examined under the condition of smooth flow thermal wind equilibrium (Hoskins; Zhang) whilst approach to the solution of (11) in the case of the nonthermal wind balance documented by Sun et al., (1989) by dint of the WKBJ scheme. However, their conclusion applies only to southward–inclined phase–isopleths of the perturbation, leaving something to be explored. We shall now investigate in the following section the solutions of (11) in the case of a curved flow by virtue of the WKBJ method, with some additional evidence presented to the article of Sun et al. (1989).

III. DYNAMICS OF SLOWLY—VARYING WAVETRAIN

As we know, a wave group consists of I) highfrequency carrier wave, changing relatively fast as a function of space and time, thus displaying shift in phase of perturbation and II) lower –frequency wave packet that varies relatively slowly spatially and temporally, referred to as slowly–varying wavetrain, exhibits change in amplitude. As a result, two kinds of space / time scales are available in a wave group, for which a multi–scale method is employed.

$$\text{Let} \quad y = n, \quad z = z, \quad t = t \ , \tag{12}$$

$$Y = \varepsilon y, \quad Z = \varepsilon z, \quad T = \varepsilon t \ , \tag{13}$$

in which $\varepsilon = \dfrac{\text{perturbation space / time scales}}{\text{back ground} - \text{field space / time scales}} \ll 1.$

To the examined wavetrain the WKBJ approximation is applied such as we set

$$\psi = \Phi(Y, Z, T)e^{i\frac{\theta}{\varepsilon}} \ , \tag{14}$$

$$\text{with} \quad \Phi = \sum_{j=0}^{\infty} \Phi_j \varepsilon^j \ , \tag{15}$$

$$\Theta = kY + mZ - \omega T \ . \tag{16}$$

If the basic flow is assumed to be steady and a function of Y and Z only, then ω、k、m are functions of Y, Z, T, respectively. Putting (12)–(16) into (11), we get the zero– and first–order approximations of ε in the form

$$\varepsilon^0: \quad \omega^2 k^2 = m^2 f_\alpha f_\beta + km(f_\beta \overline{U}_z + F^2) + k^2 N^2 \ , \tag{17}$$

$$\begin{aligned} \varepsilon^1: \quad & 2\omega\frac{\partial}{\partial T}(K^2\Phi_0) + K^2\Phi_0\frac{\partial \omega}{\partial T} - \omega^2\Phi_0\left(\frac{\partial k}{\partial Y} + \frac{\partial m}{\partial Z}\right) \\ & + f_\alpha f_\beta \frac{\partial m}{\partial Z}\Phi_0 + \frac{\partial \Phi_0}{\partial Y}[-2\omega^2 k + m(f_\beta \overline{U}_z + F^2) + 2kN^2] \\ & + \frac{\partial \Phi_0}{\partial Z}[-2\omega^2 m + k(f_\beta \overline{U}_z + F^2) + 2mf_\alpha f_\beta] \\ & + (f_\beta \overline{U}_z + F^2)\Phi_0\frac{\partial m}{\partial Y} + N^2\Phi_0\frac{\partial k}{\partial Y} + \left(\frac{\partial f_\alpha f_\beta}{\partial Z} + \frac{\partial F^2}{\partial Y}\right)m\Phi_0 \\ & + \left(\frac{\partial f_\beta \overline{U}_z}{\partial Z} + \frac{\partial N^2}{\partial Y}\right)k\Phi_0 = 0 \ , \end{aligned} \tag{18}$$

where $K^2 = k^2 + n^2$. From the dispersion relation (17) we obtain the expressions of group velocity, viz.,

$$C_{gY} = \frac{\partial \omega}{\partial k} = \frac{k}{2\omega}\lambda^2 \ , \tag{19}$$

$$C_{gZ} = \frac{\partial \omega}{\partial m} = -\frac{k}{2\omega} \cdot \frac{k}{m}\lambda^2 \ , \tag{20}$$

$$\text{with} \quad \lambda^2 = \frac{1}{K^2}[(N^2 - \omega^2) + \frac{m^2}{k^2}(\omega^2 - f_\alpha f_\beta)] \ . \tag{21}$$

Following He (1991), for the actual atmosphere, $f_\alpha f_\beta < \omega^2 < N^2$ is normally met, leading to $\lambda^2 > 0$.

By the definition of phase velocity

$$\vec{C}=\frac{\omega}{K^2}\vec{K}\ , \tag{22}$$

we find, from (19), (20) and (22),

$$\vec{C}\cdot\vec{C}_g=0 \tag{23}$$

for group velocity perpendicular to phase velocity as one characteristic feature of gravity–inertia wave. In (23) the group velocity is given as $\vec{C}_g=C_{gY}\vec{j}+C_{gZ}\vec{k}$.

Write $C_{py}=\frac{k^2}{K^2}\frac{\omega}{k}$, and $C_{pz}=\frac{m^2}{K^2}\frac{\omega}{m}$ to stand for the projections of total phase velocity (22) on the Y and X axis, respectively. It is easy to discover that C_{py} has the same sign as C_{gY}, suggesting that both phase velocity and wave energy are propagating in the Y direction, and C_{pz} is opposite in sign to C_{gz}, implying their inverse migration in the Z direction. Fig.1(a,b) portrays the relation between phase and group velocities at different inclinations phase isolines.

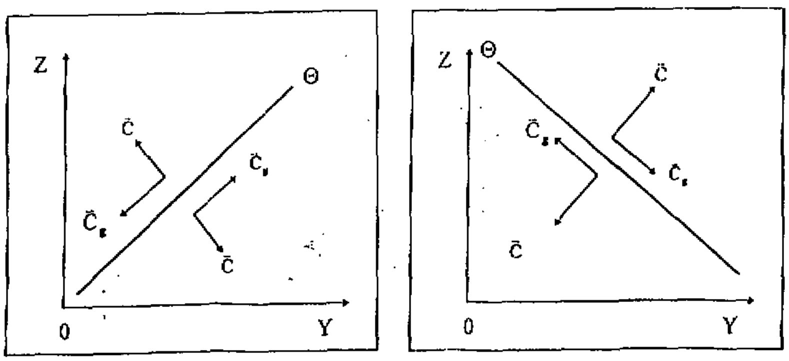
In Putting (19)–(20) into (18) in such a way as to make re–insertion based on the following

$$\frac{\partial\omega}{\partial Y}=-\frac{\partial k}{\partial T}\ ,\quad \frac{\partial\omega}{\partial Z}=-\frac{\partial m}{\partial T}\ ,\quad \frac{\partial m}{\partial Y}=-\frac{\partial k}{\partial Z}\ , \tag{24}$$

with the resulting form multiplied on both sides by Φ_0 (but by a complex conjugate for a complex Φ_0), we come to

$$\frac{\partial}{\partial T}(K^2\Phi_20\ \ )+\nabla\cdot(K^2\Phi_0^2\mathrm{C})+\frac{\varphi_0^2}{2\omega}\Big[m\Big(\frac{\partial ff_\alpha}{\partial Z}+\frac{\partial F^2}{\partial Y}\Big)$$
$$+k\Big(\frac{\partial f\overline{U}_z}{\partial Z}+\frac{\partial N^2}{\partial Y}\Big)\ \ \Big]+\frac{\Phi_0^2}{\omega}\Big[m\frac{\partial(K_s\overline{U}f_\alpha}{\partial Z}+k\frac{\partial(K_s\overline{UU}_z}{\partial Z}\Big]=0\ , \tag{25}$$

which is a formulation of perturbation wave energy evolution.



(a) $\frac{m}{k}<0$ (b) $\frac{m}{k}>0$

Fig.1. Relation between phase and group velocities for two inclinations. phase isolines.

IV. CONDITIONS FOR WAVE ENSEMBLE SYMMETRIC DEVELOPMENT

Denote

$$\Delta\overline{U}_\theta = \overline{U}_\theta - \overline{U}_z \tag{26}$$

$$\Delta\overline{U}_{\theta f} = \Delta\overline{U}_\theta - (K_s\overline{U}^2)_z / f , \tag{27}$$

where $\Delta\overline{U}_\theta$ refers to nonthermal wind, and $\Delta\overline{U}_{\theta f}$ to the same wind in basic flow of nongradient wind equilibrium. And for $K_s = 0$, $\Delta\overline{U}_{\theta f} = \Delta\overline{U}_\theta$ results.

Substitution of (6), (9), (26) and (27) into (25) yields

$$\frac{\partial}{\partial T}(K^2\Phi_0^2) + \nabla \cdot (K^2\Phi_0^2\mathbf{C}) = \frac{f\Phi_0^2}{2C_{gY}C_z}\mathbf{C}\cdot\mathbf{p} \tag{28}$$

with

$$\mathbf{p} = \nabla(\Delta\overline{U}_{\theta f}) + 2(K_s\overline{U})_z\mathbf{j} , \tag{29}$$

in which $\mathbf{p}$ is the environmental vector associated with energy variation; the first term on the rhs the vectors for the thermal wind distribution; the second the component vector for thermal curvature vorticity in the Y direction; $C_z = \frac{\omega}{m}$ the Z phase velocity. From (17), (19) and C_z we have

$$C_{gY}\cdot C_z = \frac{k}{2m}\lambda^2 . \tag{30}$$

Substituting (30) into (28) yields

$$\frac{\partial}{\partial T}(K^2\Phi_0^2) + \nabla \cdot (K^2\Phi_0^2\mathbf{C}) = -\frac{m}{k}\frac{f\Phi_0^2}{\lambda^2}\mathbf{C}\cdot\mathbf{p} , \tag{31}$$

which is integrated across the perturbation domain, and for the amplitude $\Phi_0 = 0$ at the fringe we arrive at the energy expression of the from

$$\frac{\partial E}{\partial T} = -\iint_\Sigma \frac{m}{k}\frac{f\Phi_0^2}{\lambda^2}\mathbf{C}\cdot\mathbf{p}\,dYdZ , \tag{32}$$

where $E = \iint_\Sigma K^2\Phi_0^2\,dYdZ$ signifies the $(Y-Z)$-plane energy amount of the pocket under consideration.

The perturbation is enhanced as the energy is increasing versus time, and v.v.

4.1. *Conditions of the Examined Energy Conservation*

For smooth basic flow ($K_s = 0$) and thermal equilibrium ($\Delta\overline{U}_\theta = 0$), from (29) $\mathbf{p} = 0$ follows, giving, after substituted into (28),

$$\frac{\partial E}{\partial T} = 0 , \tag{33}$$

suggesting that the energy neither intensifies nor falls. We hence arrive at the conditions of $K_s = 0$ and $\Delta\overline{U}_\theta = 0$ for the energy conservation.

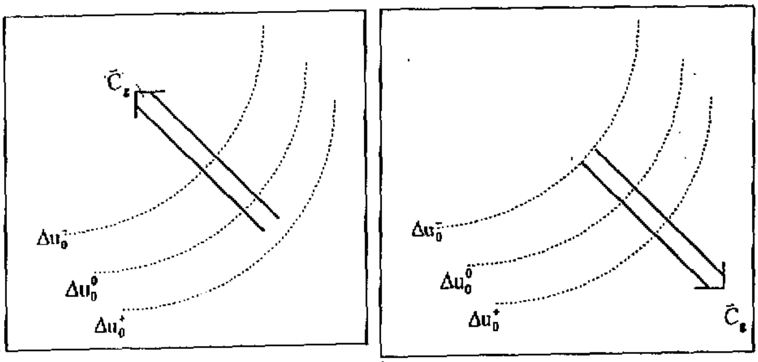


Fig.2. Models of the strengthening and attenuation of the study wave ensemble at $\frac{m}{k} > 0$.

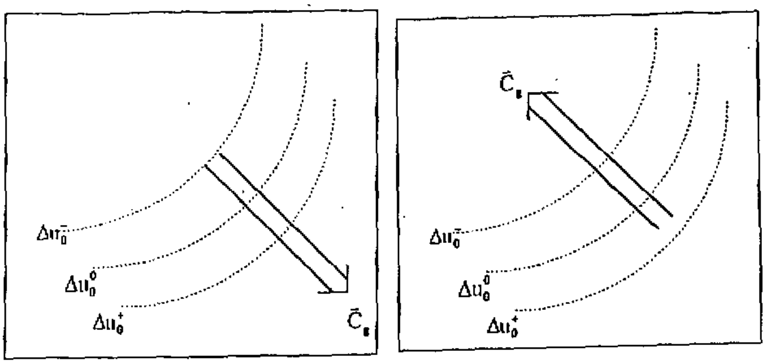


Fig.3. As in Fig.2. but at $\frac{m}{k} < 0$.

4.2. *Conditions of Wave Ensemble Development*

To facilitate analysis, we set the Y–axis directed to the due north so that $\frac{m}{k} > 0$ (<0) implies the perturbation phase isopleths inclined south–(northward).

4.2.1. *Effect of smooth flow on the wave development*

With $K_s = 0$, suggestive of smooth flow, we get, from (29), $\mathbf{p} = \nabla(\Delta \overline{U}_\theta)$, thereby leading (30) to have the form which clearly shows that, to the extent that the spatial heterogeneity of the thermal difference between the basic flow and temperature fields is responsible for the energy change, the wave packet depends for development on the matching of the phase isograms with the nonthermal ($\Delta \overline{U}_\theta$) structures

We therefore come to the conclusions as follows:

For the south–(northward)inclined phase isolines, i.e., $\frac{m}{k} > 0 (< 0)$ and the wave group velocity having the same direction as the nonthermal gradients (the upgradients), the development reaches its peak, as seen from (34),as opposed to the case when the group velocity is directed likewise as the upgradients (gradients)

It is evident from the foregoing evidence that for $K_s = 0$ the development is related to the pattern of the perturbation phase isolines as well as the group velocity direction and basic flow nonthermal wind configuration. Sun et al., (1989) however, reported on the findings only with $\frac{m}{k} > 0$. As we know, in the realistic atmosphere such isopleths tend to slope towards a cold area when symmetric instability emerges. As such, the results for $\frac{m}{k}$ are presented here as our modest contribution to the problem.

4.2.2. *Influence on perturbation development of gradient balance*

For a curved flow ($K_s \neq 0$), we have

$$\Delta \overline{U}_{\theta f} = 0 \ , \tag{35}$$

When the flow satisfies the relation of gradient wind balance, in which case (32) can be rewritten as

$$\frac{\partial E}{\partial T} = -2\iint\limits_{\Sigma} \frac{m}{k}\frac{\Phi_0^2}{\lambda^2} fC_{gY}\frac{\partial}{\partial Z}(K_s\overline{U})dYdZ \ . \tag{36}$$

Thus, $\frac{\partial}{\partial Z}(K_s\overline{U}) > 0$ is available underneath cyclonically curved jet stream and as gradient-wind equilibrium field exists if $\frac{m}{k} > 0 (< 0$, see Fig.4.).

4.2.3. *Impact of curved flow upon perturbation development*

When Ks does not equal zero nor satisfies the relation of gradient wind balance,.(32) has its two terms on the rhs, viz.,

$$\frac{\partial E}{\partial T} = -\iint\limits_{\Sigma} \frac{m}{k}\frac{\Phi_0^2}{\lambda^2} f\mathbf{C}\cdot\nabla\overline{U}_{\theta f}dYdZ - 2\iint\limits_{\Sigma} \frac{m}{k}\frac{\Phi_0^2}{\lambda^2} fC_{gY}\frac{\partial}{\partial Z}(K_s\overline{U})dYdZ \tag{37}$$

maintained where the first term denotes the role in the development of the nonthermal

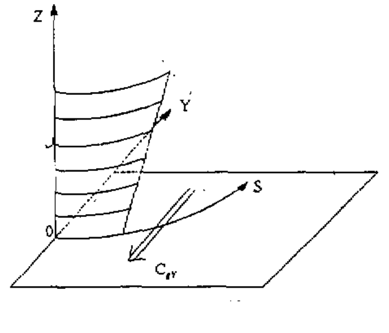


Fig.4. Wave ensemble development under the effect of cyclonically curved jet at $\frac{m}{k} > 0$.

inhomogeneity throughout the nongradient equilibrium basic flow due to the effects in vertical originating from the imbalance of pressure gradient, inertial centrifugal and Coriolis forces in combination, and the second term describes the role of thermal curvature vorticity in development of perturbation wave ensemble in question. For their detailed analyses the reader is referred to the previous subsection.

V. CONCLUDING REMARKS

Based on the wave ensemble development presented the highlights are as follows:

1. For a zonally smooth basic flow and thermal wind balance, the wave ensemble is not enhanced nor weakened, i.e., the energy is conserved.
2. For $K_r = 0$ but thermal imbalance has its development dependent on the matching of the phase isopleth pattern with nonthermal structure throughout the flow. These isolines sloping towards a cold region will give rise to the reinforcement of the wave propagating southward, an outcome that agrees more closely with observed synoptic events.
3. For a zonally curved flow, energy for the perturbation wave intensification has its origin from nonthermal heterogeneity throughout the flow under nongradient equilibrium.
4. Vertical shear of the flow curvature vorticity makes positive contribution to the development.

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