Influences of Vorticity Source and Momentum Source on Atmospheric Circulation①

Fang Juan (方 娟) and *Wu Rongsheng* (伍荣生)
Laboratory of Mesoscale Severe Weather, Nanjing University, Nanjing 210093
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ABSTRACT

A simplified one-dimensional barotropic vorticity equation is used to study the influences of the vorticity and the momentum source on the large scale wave. Both vorticity source and momentum source can cause the formation of the large scale wave, but the former can produce large scale wave only under the condition that there is apparent basic flow acting on it, while the latter can produce the large scale wave even when $\bar{u}$ tends to be zero. Furthermore, the amplitude of steady wave caused by the former is proportional to $\sqrt{\bar{u}}$, while the amplitude caused by the latter has no relation to $\bar{u}$, instead it depends only on the magnitude of the perturbation of momentum.

Key words: Vorticity source, Momentum source, Basic flow, Beta effect

I. INTRODUCTION

Yet (1949) showed that the injection of vorticity at upstream will produce the steady waves downstream. This work provided an important foundation for the dynamics of the general circulation in meteorology. Since then, this problem has been studied by many scientists. For example, Xu et al.(1984) studied the problem by changing the way of the injection of vorticity, Lu (1984) and Zhao(1988) considered the influences of the diabatic heating, i.e., heating sources on the long waves. Recently, Holt et al.(1991) indicated that a localized continuous forcing of anomalies of momentum and potential temperature in a symmetrically stable baroclinic atmosphere would lead to a steady-state flow being of a "lens" of zero P.V.. Shutts et al.(1994) also pointed out that momentum sources would make the contribution to the problem of geostrophic adjustment. Furthermore, most of the synoptic meteorologists, like Shou(1994), have the experiences that the upward and downward flux of the horizontal momentum may cause changes of the upper layer and lower layer circulation. Therefore, the injection of the momentum may be another important factor which has not been studied in details so far. The aim of the present work is to study the contribution of the momentum to the general circulation and to compare its effect to the case of the vorticity sources.

For simplicity, the present work is confined to one-dimensional problem. Although the conclusion drawn from it has some limitation due to the simple model, it is still helpful for understanding the realistic synoptic processes.

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II. THE EFFECTS OF A VORTICITY SOURCE

One dimensional barotropic non-divergence vorticity equation has the form as follows (Yet, 1949):

$$\frac{\partial^2 v}{\partial t \partial x} + \bar{u}\frac{\partial^2 v}{\partial x \partial x} + \beta v = 0, \tag{1}$$

where $\bar{u}$ is average west wind, v is north-south disturbance velocity, β is Beta parameter or Rossby parameter. Yet (1949) considered the formation of the large scale wave caused by the injection of the steady vorticity ζ_0 at $x=0$. In the present paper, we consider the vorticity source at $x=0$ is varying with the time in order to avoid the discontinuity of the initial value at $x=0$. Then, the corresponding initial and boundary conditions are expressed as:

$$\begin{aligned} &t=0, \quad v=0, \\ &x=0, \quad v=0, \quad \frac{\partial v}{\partial x} = \zeta_0(t), \end{aligned} \tag{2}$$

where $\zeta_0(0)=0$. Using Laplace-transform, we can obtain the solution of (1) and (2) as

$$v(x,t) = \bar{u}\int_{t-\frac{x}{\bar{u}}}^{t} \zeta_0(\tau) J_0(2\sqrt{\beta(t-\tau)[x-\bar{u}(t-\tau)]})\, d\tau, \tag{3}$$

where J_0 is the zero-order Bessel function. Eq. (3) is the same form as that obtained by Yet(1949) when ζ_0 is constant. Fig.1 shows the variation of the disturbance velocity v with time at $x=170$ km under the condition that ζ_0 is constant and the function of time, respectively. It shows that, in the former case, v changes rapidly at first, then keeps steady after $t=2$ hr., there is a discontinuity of $\partial v / \partial t$ at the time of v turns to be steady. While, in the latter case, ζ_0 is varying with time, v changes smoothly, no discontinuity of $\partial v / \partial t$ appears. This is more reasonable physically. However, this discrepancy due to the initial condition will gradually disappear when the integration of time is long enough.

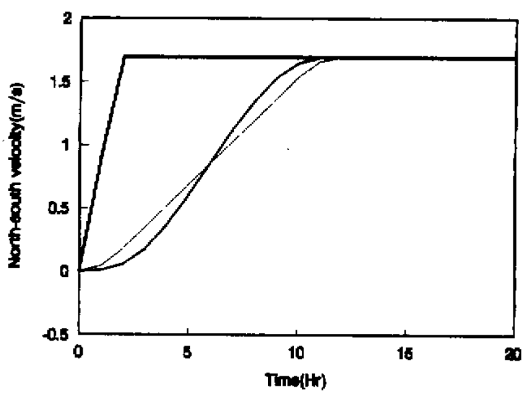

Fig.1 Variation of the north-south velocity caused by the different vorticity sources at $x=170$ km. Curves from thick to thin represent: $\zeta_0=1.e-5$. $\zeta_0=0.5e-5(1-\cos(\omega t))$ $(0 \leqslant t \leqslant T_0)$; $\zeta_0=0.5e-5(1-\cos(\omega T_0))$ $(t \geqslant T_0)$. $\zeta_0=1.e-5t$ $(0 \leqslant t \leqslant T_0)$; $\zeta_0=1.e-5T_0$ $(t \geqslant T_0)$.

From Eq. (3) one knows that the perturbation excited by the vorticity source relates to the basic flow $\bar{u}$ and the time dependence of the vorticity source. To describe this clearly, we set the vorticity source as:

$$\begin{aligned}&\zeta_0(t)=A(1-\cos(\omega t_0)),\quad 0\leqslant t\leqslant T_0,\\&\zeta_0(t)=A(1-\cos(\omega T_0))=\zeta_1=\text{const},\quad t\geqslant T_0.\end{aligned}\tag{4}$$

where A is constant. This means that the vorticity source varies with a period of $2\pi/\omega$ when $t\leqslant T_0$, and it reaches steady-state finally. This may be looked as there is vorticity perturbation(which may be caused by topography or diabatic heating) at a certain region in the atmosphere, it changes with time first and then becomes steady. Then Eq. (3), correspondingly, has the form:

$$v(x,t)=\begin{cases}\bar{u}\int_0^t\zeta_0(\tau)J_0(2\sqrt{\beta(t-\tau)[x-\bar{u}(t-\tau)]})d\tau, & t<\frac{x}{\bar{u}} \quad\text{(a)}\\ \bar{u}\int_{t-\frac{x}{\bar{u}}}^t\zeta_0(\tau)J_0(2\sqrt{\beta(t-\tau)[x-\bar{u}(t-\tau)]})d\tau, & \frac{x}{\bar{u}}\leqslant t<\frac{x}{\bar{u}}+T_0 \quad\text{(b)}\\ \bar{u}\int_{t-\frac{x}{\bar{u}}}^t\zeta_1 J_0(2\sqrt{\beta(t-\tau)[x-\bar{u}(t-\tau)]})d\tau & t\subset\frac{x}{\bar{u}}+T_0 \quad\text{(c)}\end{cases}\tag{5}$$

Eq. (5) indicates that the perturbation excited by the vorticity source depends on the basic flow. After some manipulating and simplifying on (5c), just like what was done by Yet (1949), the velocity v can be expressed as:

$$v(x,t)=\zeta_1\sqrt{\frac{\bar{u}}{\beta}}\sin(\sqrt{\frac{\beta}{\bar{u}}}x),\quad t\geqslant\frac{x}{\bar{u}}+T_0\tag{6}$$

Therefore, velocity v is steady when $t\geqslant\frac{x}{\bar{u}}+T_0$, and the amplitude and wavelength of the steady wave depend on the magnitude of $\bar{u}$, i.e., they are all directly proportional to $\sqrt{\bar{u}}$. The smaller is the $\bar{u}$, the smaller is the amplitude and the shorter is the wavelength. The

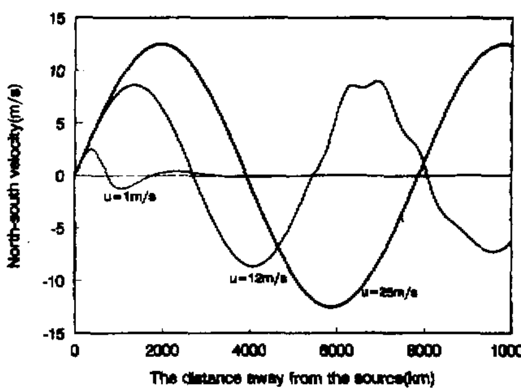

Fig.2 The north–south velocity caused by vorticity source at $t=8$ day. Curves from thick to thin represent: $\bar{u}=25$ m / s, 12 m / s, 1 m / s. The horizontal coordinate represents the distance away from the source (km). The vertical coordinate represents the north–south velocity (m / s).

unsteady perturbation of the large scale wave expressed as (5a,b) also implies that the stronger is the basic flow, the more prominent velocity perturbation is excited downstream. All these characteristics are confirmed by Fig.2, which shows that the large scale disturbance caused by the vorticity source under the different basic flow: $\bar{u}$ being 25 m / s, 12 m / s, 1 m / s respectively. It indicates that the amplitude of the large scale wave decreases quickly and the wavelength, accordingly, becomes short due to the reduction of $\bar{u}$. According to this point, the excitation of large scale waves due to the vorticity source may be explained in the following manner: Due to the existence of the basic flow, the disturbance vorticity of the source is advected downward and then causes an increasing of vorticity downstream of the source. As a result, the north-south velocity is produced, which combining with the Beta effect will finally lead to the formation of the Rossby wave. In this process, the basic flow plays an important role. When $\bar{u}$ tends to zero, no apparent large scale waves can be excited because the downstream transfer of vorticity also tends to zero and Beta effect cannot make significant contribution to the formation of the large scale waves for the weak north-south velocity.

III. THE EFFECT OF THE MOMENTUM SOURCE

Mesoscale systems are usually developed with association of the vertical flux of horizontal momentum. When such systems are weakening, momentum may be transferred to the mean flow and causes changes of the atmospheric circulation. Shutts et al.(1994) has adopted the mass source as forcing term and studied its effects on the motion of the atmosphere. In the present work, the influence of the momentum source on the large scale waves is considered. For easy comparison with the vorticity source, Eq.(1) is also taken as the basic equation. The corresponding initial and boundary conditions are put forward as:

$$\begin{aligned} &t=0, \quad v=0, \\ &x=0, \quad v=v_0(t); \quad \frac{\partial v}{\partial x}=0 \end{aligned} \tag{7}$$

where $v_0=0$. This initial condition can be interpreted as that there is an injection of momentum which is varying with time at the longitude $x=0$.

The explicit expression of $v_0=0$ is taken as:

$$\begin{aligned} &v_0(t)=B(1-\cos(\omega t)), \quad 0 \leqslant t \leqslant T_0, \\ &v_0(t)=B(1-\cos(\omega T_0))=v_1=\text{const}, \quad t \geqslant T_0. \end{aligned} \tag{8}$$

where B is constant. The solution of the problem is

$$v(x,t)=\begin{cases} v_0(t)-\int_0^t v_0(\tau)\dfrac{\sqrt{\beta[x-\bar{u}(t-\tau)]}}{\sqrt{t-\tau}}J_1(2\sqrt{\beta(t-\tau)[x-\bar{u}(t-\tau)]})d\tau & \text{(a)} \\ v_0(t)-\int_{t-\frac{x}{\bar{u}}}^t v_0(\tau)\dfrac{\sqrt{\beta[x-\bar{u}(t-\tau)]}}{\sqrt{t-\tau}}J_1(2\sqrt{\beta(t-\tau)[x-\bar{u}(t-\tau)]})d\tau & \text{(b)} \\ v_1-v_1\int_{t-\frac{x}{\bar{u}}}^t \dfrac{\sqrt{\beta[x-\bar{u}(t-\tau)]}}{\sqrt{t-\tau}}J_1(2\sqrt{\beta(t-\tau)[x-\bar{u}(t-\tau)]})d\tau\ . & \text{(c)} \end{cases} \tag{9}$$

Where J_1 is the first order Bessel function. Fig.3 shows the large scale perturbation excited by the momentum source under different basic flow ($\bar{u}$= 25 m / s, 12 m / s, 1 m / s). It is obvious that the momentum source can produce large scale perturbation downstream. The amplitudes of the perturbation are the same for $\bar{u}$=25 m / s and $\bar{u}$= 12 m / s, in both cases,

$v=10$ m / s. This is different from the case of vorticity source in which the amplitude of the perturbation is varying with $\bar{u}$. (This will be mentioned later). Furthermore, when $\bar{u}$ tends to zero, prominent large scale waves still can be excited, while it is impossible for the case of the injection of vorticity. Therefore, in contrast to the injection of the vorticity, the basic flow $\bar{u}$ is not very important to the formation of the apparent large scale waves in the case of momentum source. The following model is used to make further state on his point.

When $\bar{u}=0$, the governing equation and corresponding initial condition can be rewritten as:

$$\frac{\partial^2 v}{\partial t \partial x} + \beta v = 0,$$
$$v|_{t=0} = 0, \quad v|_{x=0} = v_0(t). \tag{10}$$

The solution to the problem is

$$v(x,t) = v_0(t) - \int_0^t v_0(\tau)\frac{\sqrt{\beta x}}{\sqrt{t-\tau}} J_1(2\sqrt{\beta x(t-\tau)})d\tau. \tag{11}$$

Eq.11 can be simplified as follows finally

$$v(x,t) = \int_0^t v'_0(\tau)(2\sqrt{\beta x(t-\tau)})d\tau. \tag{12}$$

In Eq.(12), $v'_0(\tau)$ is used to express $\frac{dv_0(\tau)}{d\tau}$. It is obvious that the large scale perturbation is dependent on $v'_0(\tau)$. Thus even though there is no significant basic flow, the large scale waves can still be produced when the momentum source is varying with the time. The dotted line in Fig.3 denotes the large scale wave caused by the momentum source when there is no basic flow. Therefore, the effects of the injection of momentum and vorticity on atmospheric circulation are different. The mechanism of this can be interpreted as follows: The influence of

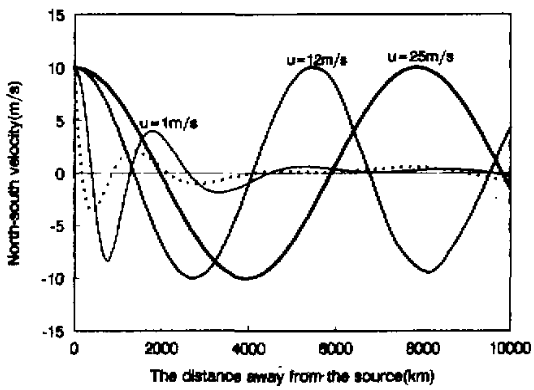

Fig.3 The north–south velocity caused by momentum source at $t=8$ day. Solid curves from thick to thin represent: $u=25$ m / s, 12 m / s, 1 m / s. Dotted line represents the case that $\bar{u}=0$ m / s. The coordinates are the same as in Fig.2.

the vorticity source on the downstream region depends on the advection of the mean flow, which transfers the disturbance vorticity downstream and causes the north–south motion of the atmosphere. The combination of the north–south motion and Beta effect will excite large scale waves. While, with the injection of momentum, the momentum can affect the atmosphere directly through the combination with Beta effect instead of the advection of the mean flow.

Although the momentum source can excite large scale waves without the basic flow, it cannot lead to the formation of steady waves. The mean flow plays a determinant role in the formation of the steady wave. According to (9c), the north–south velocity may be written as:

$$v(x,t) = v_1 \cos\left(\sqrt{\frac{\beta}{\bar{u}}}x\right) \qquad t \geqslant \frac{x}{\bar{u}} + T_0 . \tag{13}$$

This shows that the momentum source also can excite a steady wave when the basic flow exists, but the amplitude of the steady wave is independent on the magnitude of the mean flow. In Fig.3, the amplitudes of the steady wave are all equal to 10 m / s for the different basic flows: $\bar{u} = 12$ m / s, 25 m / s. However, the wavelengths of the steady wave in both cases are proportional to $\sqrt{\bar{u}}$, the smaller is the $\bar{u}$, the shorter is the wavelength. When $\bar{u}$ tends to zero, the wavelength tends to zero too. As a result, no steady wave can be excited. Therefore, the basic flow is very important to the formation of the steady wave.

IV. CONCLUSION

One–dimensional barotropic non–divergence vorticity equation is used to study the effect of vorticity source and momentum source on the atmospheric circulation. The conclusions are summarized as follows: Firstly, the vorticity source cannot excite the prominent large scale waves downstream unless the advection of the mean flow is significant. While, the formation of the large scale wave is possible for the injection of the momentum even there is no mean flow. Secondly, the amplitude of the steady wave excited by the vorticity source is in direct proportion to $\sqrt{\bar{u}}$. On the contrary, it has nothing to do with the basic flow when the wave is excited by the injection of the momentum. Finally, the mean flow is important to the formation of the steady wave.

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