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The Regional Dynamical Model of the Atmospheric Ozonosphere

Wang Weiguo (王卫国), *Xie Yingqi* (谢应齐)
Department of Earth Science, Yunnan University, Kunming 650091
Qiu Jinhuan (邱金桓) and *Liu Qing* (刘 青)
Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029

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ABSTRACT

The TOMS zonal average total ozone data in the Northern Hemisphere are decomposed with the empirical orthogonal function (EOF) method. According to the features of the spatial characteristic vectors, the characteristic vectors that have been obtained with EOF method can be used as the ordered orthogonal radixes to unfold the phase space. After the corresponding time functions are embedded in phase space, the traces of the state vectors of the regional ozonosphere dynamical system are constructed, and can be used to describe the attractor integral information of the asymptotic state of the regional ozonosphere system and the dynamical features of the regional ozonosphere system, and then the embedded saturation dimension of the regional ozonosphere system attractor is successfully obtained. Based on these works mentioned above, by using the time function series we solve a problem contrary to the numerical solution and retrieve the control parameters of the state equations in which quadratic nonlinear terms are included, and then the dynamical models that can objectively reflect the temporal variation of the regional ozonosphere system are finally established.

Key words: Ozonosphere, Field time series, Empirical orthogonal function method, Control parameter, Dynamical system

I. INTRODUCTION

The ozonosphere system is a nonlinear dissipation one, which has variable feature caused by the complex effect of force and freely variable environment, and by the interaction between inner unstable feature and feedback mechanisms. Now, the generation loss and transport mechanisms about ozone are not well understood, there is larger deviation between theoretical result and observation, so it is not easy to give the objective explanation to the evolutionary law of ozonosphere system. In order to understand and then to solve the problem of complex phenomena, we need a new mode of thinking that has been provided by nonlinear science (Chou Jifan et al., 1994; Yi Chuixiang, 1995). That the nonlinear dynamical model is built with observational time series data is an important field in which the theory and method of reconstructing nonlinear dynamical system are applied. In contrast with the traditional linear dynamical model, the nonlinear dynamical model retains the nonlinear feature of the studied system as much as it can do, and reflects to a certain extent the nonlinear feedback mechanism of the studied system. It is also different from the traditional numerical model, the mechanism of the nonlinear dynamical model need not be perfectly understood.

The spatial distribution and the temporal variation of the ozonosphere are very complex (Wang Weiguo et al., 1994). In order to describe the attractor characteristics that the dynamical system of regional ozonosphere may have, the dynamical system of regional ozonosphere needs to be unfolded in phase space. However, the traditional method of unfolding phase space, which obtains the optimum phase space radix of linear independence with a certain time lag τ, can only deal with the single variate time series. Because this method demands sample series Showd be long enough, generally the orthogonality of phase space radix cannot be totally met. Meanwhile, because there is limited information in the single variate data, the dynamical model that has been built cannot perfectly reflect the integral temporal feature of the regional ozonosphere system. To remedy the defect of the traditional method, we combine the observational data series that distribute in different spatial locations, follow the principle of the multiple variate and field time series analysis, and use EOF method to unfold the ozonospheric phase space, then obtain the integral temporal information and potential dynamical features that exist in the ozonosphere system, and finally build the dynamical model that can objectively reflect the integral temporal feature of the regional ozonosphere system.

II. THE PRINCIPLE AND METHOD OF UNFOLDING PHASE SPACE WITH EOF METHOD

In M observatories, any time series of objective element obtained from N observations at time t can be described by an $M \times N$ matrix F that consists of N array vectors (Wu Guoxiong and Liu Hui, 1995), that is

$$F \equiv (f_{mn})_{M \times N} \qquad \begin{matrix} m = 1, 2, \cdots, M \\ n = 1, 2, \cdots, N \end{matrix} \tag{1}$$

where the matrix element f_{mn} is the observation obtained at mth observatory at nth time. Obviously, using EOF method, from F matrix we can obtain M spatial function assemblage X whose characteristic is that any two of these space functions are orthogonal each other, and time function assemblage T. It is well known that the temporal variations of any two independent variates are also independent, and the time function corresponding to space function can be used to describe the temporal variation of relative spatial type. Therefore, the evolution of spatial element field with time is reflected in the temporal variation of time function assemblage, and every observation vector F_n can be linearly composed of M spatial functions X_m according to different functions weight, i.e.

$$F = XT, \quad \text{or} \quad F_n = T_{1n} X_1 + T_{2n} X_2 + \cdots + T_{mn} X_m + \cdots + T_{Mn} X_M \ . \tag{2}$$

There is orthogonality between any two spatial functions, and once the spatial functions that are just the functions of relative spatial coordinate are determined, they do not change with time, so, from the mathematical viewpoint, the M orthogonal spatial functions can be used as other orthogonal radixes, which gives a new meaning to the orthogonal space functions $\{X_1, X_2, \cdots, X_M\}$. These spatial functions that are chosen as a set of the ordered orthogonal radixes can be used to unfold the relative phase space. Every row in relative time function matrix T that is $\{T_{1n}, T_{2n}, \cdots, T_{Mn}\}$ can be regarded as a state point that has M coordinate components in M dimension orthogonal phase space. The temporal variation of the state point can be described with Eq. (2). These make it change the study of the relative time function into the study of the spatial element field, and make the temporal variation of the regional dynamical system be shown in M–dimension phase space. Moreover, the EOF method has the strong convergence and can effectively concentrate the information of the space element field. Using a few state components obtained by EOF method, the spatial and temporal condition of the regional dynamical system can be described. The EOF method, from a new viewpoint, can be used to unfold the orthogonal phase space that is applied to describing the integral features that exist in the field time series, and effectively makes the integral information be shown in lower dimension phase space.

Thus it can be seen that when the EOF method is adopted to unfold the phase space, we can obtain orthogonal space and time functions and take space functions as the unfolded phase space radix, which makes the relative phase space have orthogonal characteristics, and the nonlinear feature and theory of regional system in larger space scale can be studied. It is worthy to pay attention to the action of concentrating information of regional dynamical system that the EOF method has, and after the space element field is unfolded, the action makes the main information of regional dynamical system exist in a few state components. So, we are quite sure to determine the dimensions of phase space that can reflect the main feature of the studied system effectively, and can easily obtain useful information with little careless omission (Zhu Yufeng et al., 1995).

III. THE BUILDING OF THE REGIONAL OZONOSPHERIC DYNAMICAL SYSTEM MODEL

Use zonal average day–to–day variation series of total ozone at 7 latitudes in the Northern Hemisphere (Goddard Ozone Processing Team, 1992) and the time period is from November 1978 to December 1991, i.e., the samples of every grid N_0 are 4804, and the regional range is from 5°N to 65°N with 10° intervals, i.e., the numbers of spatial grids M are 7, the time series of total ozone of every grid is handled into relative anomaly time series, and the matrix of total ozone $F_{7\times 4804}$ is formed.

1. *The Dimension of the Studied Dynamical System*

The covariance matrix, eigenvalue, characteristic vector, time function matrices of matrix $F_{7\times 4804}$ are easily obtained with EOF method. Because EOF method is of the characteristic of faster contraction, the eigenvalue of covariance matrix is ordered from large to small, and relative characteristic vectors $\{X_1, X_2, \cdots, X_7\}$ form the ordered orthogonal radixes of phase space, which is shown in Table 1.

Table. 1. The Ordered Orthogonal Radixes Reflected by the Characteristic Vectors in Phase Space

Latitude (°N)	X_1	X_2	X_3	X_4	X_5	X_6	X_7
65	0.664678	−0.195562	−0.650194	−0.213446	−0.179144	0.138976	−0.015492
55	0.545485	−0.035659	0.185272	0.461912	0.591249	−0.311430	0.083203
45	0.440486	0.113748	0.602506	0.083021	−0.356625	0.523458	−0.148115
35	0.241488	0.412880	0.196261	−0.348286	−0.356811	−0.614189	0.326878
25	0.059989	0.620933	−0.106349	−0.274814	0.334661	0.027123	−0.641310
15	−0.037261	0.550829	−0.214080	0.117959	0.189025	0.447064	0.632343
5	−0.057601	0.296826	−0.290258	0.723495	−0.464875	−0.178489	−0.230031

Their variance contribution and the accumulated variance contribution rates are shown in Table 2.

Table. 2. Variance Contribution and the Accumulated Variance Contibution Rates of Ozone Layer

m	1	2	3	4	5	6	7
$\lambda_m / \sum_{i=1}^{7} \lambda_i$	85.8166	10.6667	2.0305	0.8208	0.4130	0.2045	0.0005
$\sum_{j=1}^{m} \lambda_j / \sum_{i=1}^{7} \lambda_i$	85.8166	96.4833	98.5138	99.3346	99.7476	99.9521	100

In time function matrix $T \equiv (T_{mn})_{M \times N}$, every row determines a state point, which can be reflected by the state vector $\vec{T}_m(n) = (T_{1n}, T_{2n}, \cdots, T_{mn})$. So, $N_0 = 4804$ points are generated with time function series in M dimension phase space. If the phase point $\vec{T}_m(i)$ is chosen as a fiducial point, the distances $|\vec{T}_m(i) - \vec{T}_m(j)|$ between the chosen phase point and other $N-1$ points $\vec{T}_m(j)$ can be calculated, and the number of phase points that locate in the sphere whose center is at $\vec{T}_m(i)$ and whose radius is r can be counted. For all phase points, the process mentioned above is repeated, the correlation function

$$C_m(r) = \frac{1}{N^2} \sum \theta(r - |\vec{T}_m(i) - \vec{T}_m(j)|) \tag{3}$$

can be obtained, where θ is step function. The estimated value of attractor correlation dimension is

$$D_2(m) = \frac{|\ln C_m(r)|}{|\ln r|} \quad . \tag{4}$$

The embedded dimension m is continuously increased until D_2 has reached the saturation. The saturation value is determined as the attractor dimension. For a certain m, a curve that reflects the correlation between $\ln C_m(r)$ and $\ln r$ can be drawn in Fig. 1 whose coordinates are logarithmic coordinate. In Fig. 2, it is obviously found that the slope of the curve changes until $m = 5$. So, the correlation dimension of the chaos attractor of the regional ozonosphere system, that is $D_2 = 2.9$, can be easily obtained. These results indicate that if the dynamical model of the regional ozonosphere system is established, at least three independent state variables are needed.

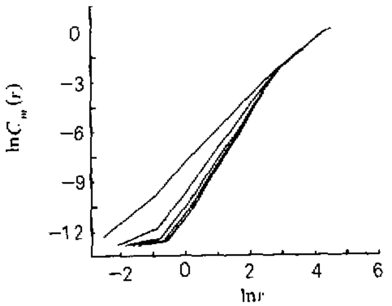

Fig. 1. The relation of $\ln(C_m(r)$ and $\ln r$ for the regional ozonosphere series.

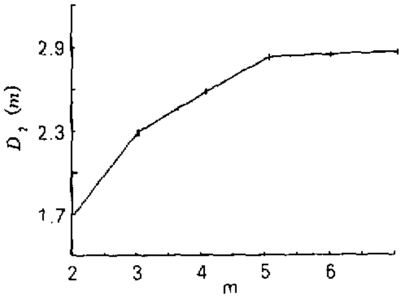

Fig. 2. The relation of attractor's dimension $D_2(m)$ and the phase space's dimension.

2. *The Control Parameters of the Dynamical System*

The advantage of the EOF method is that the main information of the time series of total element field can be concentrated in the first several spatial and temporal functions, and their spatial fluctuation and temporal variation can reflect the main temporal variation law of the ozonospheric spatial field. According to the information of the attractor dimension obtained, 3~6 independent variables are needed to establish the nonlinear dynamical model of the regional ozonosphere system, and to study the evolution characteristics of the zonal average regional ozonospheric dynamical system in phase space and the inner interaction relationship of the regional ozonospheric dynamical system. In the ozonosphere system, there are various time scales and spatial scales, and the waves complex interactions exist among different scales, and the different spatial scales indicate different time-scale evolutions. The interactions among different time scales and the inner relationships among temporal and spatial scales form the multiple hierarchy structures of the ozonosphere system, and they reflect the phenomena, nature and evolution law of the ozonosphere system from different aspects. Viewing from larger scale, the accumulating effect of the photochemical reaction influences the temporal variation of the ozonosphere system. So, the evolution of the global ozone concentration is determined by the photochemical reaction. However, viewing from synoptic scale, the short-term variation of regional ozone content is controlled by dynamical cause. The first three characteristic vectors that have high contribution rates, and can reflect the main information of the day to day variation of the regional ozonosphere system is applied to construct the phase space. According to Fig. 3, the meridional fluctuations of the zonal average ozonospheric data that are described by the characteristic vectors have obvious significance. The first characteristic vector reflects the meridional gradient variation of total ozone from low latitude to high latitude. The relation is that there is anticorrelation between the total ozone in low latitude area and that in middle-high latitude area. This relation indicates that the ozone content zonal distribution is influenced by the dynamical transport process. The reason is that the mean motion and the eddy motion make the stratospheric air with the new generated ozone move from the low latitude where the ozone content is generated by photochemical process to the north pole. The transporting result of air makes inevitably the ozone content redistribute with latitude. The second characteristic vector presents the anticorrelation between the total ozone in polar area and that in low-middle latitude area. Because of the effect of polar vortex, the strong stratospheric potential vorticity gradient resists the ozone meridional exchange, and makes the ozone that exists in polar area be difficult to be supplied, and then decreases, however, the ozone that exists in low latitude area, especially in subtropical area increases. The third characteristic vector presents the correlation between the ozone content in polar area and that in low latitude area, and the anticorrelation between the ozone content in polar area or low latitude area and that in middle latitude area, which is modulated by the polar vortex and subtropical high. It is obviously found that the spatial fluctuations reflected by the three characteristic vectors mentioned above present the meridional effect feature of the total ozone in different latitudes, and show that the ozone layer dynamics is related to the complex process of the ozone generating, losing, and inner feedback mechanism, and they are the embodiment of the complex nonlinear ozonosphere system.

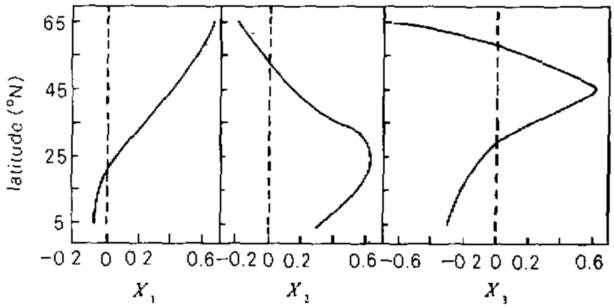

Fig. 3. The spatial model of regional ozonosphere reflected by the first three characteristics vectors.

If T_1, T_2 and T_3 are separately applied to represent the state variates in phase space, the nonlinear dynamical equation can be written as

$$\frac{dT_i}{dt} = h_i(T_1, T_2, T_3; p_1, \cdots, p_9) \qquad i = 1,2,3 \tag{5}$$

where it is assumed that h_i is the functions of the linear and quadratic nonlinear terms of T_1, T_2 and T_3, and p_k is control parameters of relative terms. Equation (5) that dominates the evolution law of system can be particularly written as

$$\begin{cases} \dfrac{dT_1}{dt} = a_1 T_1 + a_2 T_2 + a_3 T_3 + a_4 T_1^2 + a_5 T_2^2 + a_6 T_3^2 + a_7 T_1 T_2 + a_8 T_1 T_3 + a_9 T_2 T_3 \\ \dfrac{dT_2}{dt} = b_1 T_1 + b_2 T_2 + b_3 T_3 + b_4 T_1^2 + b_5 T_2^2 + b_6 T_3^2 + b_7 T_1 T_2 + b_8 T_1 T_3 + b_9 T_2 T_3 \\ \dfrac{dT_3}{dt} = c_1 T_1 + c_2 T_2 + c_3 T_3 + c_4 T_1^2 + c_5 T_2^2 + + c_6 T_3^2 + c_7 T_1 T_2 + c_8 T_1 T_3 + c_9 T_2 T_3 \end{cases} \tag{6}$$

Consider all phase points that have been determined by time series T_1, T_2 and T_3 as the particular solution assemblage of Eq. (6), and use the inverse theory and method (Chou Jifan, 1986), the value of the control parameter can be objectively retrieved, and then it can also be researched that the temporal variation of the regional ozonospheric dynamical system is relatively contributed by each retrieved control parameter. The results are shown in Table 3.

Table. 3. The Value of Every Control Parameter and the Relative Variance Contribution in the Retrieved Regional Ozonospheric Dynamical Equations

K	dT_1/dt		dT_2/dt		dT_3/dt	
	a_k	R_k	b_k	R_k	c_k	R_k
1	-1.194821×10^{-3}	7.636199×10^{-3}	2.214989×10^{-3}	1.865261×10^{-2}	2.748268×10^{-3}	2.262396×10^{-3}
2	-6.405816×10^{-5}	2.445929×10^{-4}	3.277467×10^{-4}	4.433195×10^{-3}	-4.350311×10^{-2}	8.916157×10^{-1}
3	-5.559208×10^{-4}	8.531055×10^{-2}	5.326175×10^{-3}	8.596333×10^{-1}	-9.396235×10^{-5}	5.350961×10^{-4}
4	7.869681×10^{-5}	1.121413×10^{-2}	3.345724×10^{-5}	1.530223×10^{-3}	4.586958×10^{-4}	2.279380×10^{-2}
5	4.227953×10^{-5}	9.221625×10^{-2}	-3.128056×10^{-5}	4.510232×10^{-2}	-4.955197×10^{-5}	8.521346×10^{-4}
6	-8.604859×10^{-6}	1.490419×10^{-1}	-2.910454×10^{-7}	1.534516×10^{-5}	-1.923604×10^{-6}	2.315436×10^{-3}
7	-2.689605×10^{-4}	1.728326×10^{-1}	-2.676899×10^{-5}	2.113321×10^{-3}	-7.528185×10^{-4}	2.474291×10^{-2}
8	-6.782298×10^{-5}	8.668026×10^{-2}	-8.067792×10^{-5}	2.294757×10^{-2}	1.505846×10^{-4}	5.405866×10^{-2}
9	6.660860×10^{-5}	3.948228×10^{-1}	-4.961111×10^{-5}	4.557198×10^{-2}	-1.839952×10^{-5}	8.227500×10^{-4}

According to the inverse results that are the value of every control parameter and the relative variance contribution Rk, the control parameters can be rationally chosen, and the dynamical governing equations that describe the temporal variation law of regional ozone layer system can be finally obtained.

3. *Dynamical System Model*

The temporal variation of the regional ozonospheric dynamical system is a complex process. It reflects the inner unstability and the complex characteristic of the studied system. The regional ozonosphere system is controlled by the processes of the energy conversion and the matter conversion, which makes the temporal variation of system state be determined by internal cause and various positive or negative feedback mechanisms and the external forcing, and makes the system become unstable or fluctuating. The external forcing can affect the system state via internal cause, and force the system inner characteristics adjustable. This variation connects with the inner nonlinear process of the system, and causes the nonlinear interaction of the state components of the regional ozonosphere system. In order to understand the integral characteristic of the regional ozonospheric dynamical system as a whole, the temporal variation of every state component and the strong interactions among state components need to be considered. According to Table 3, it can be found that the values of the control parameters are different. However, the significance of a certain control parameter may not be totally determined by the value of the control parameter. To determine the terms that will be finally chosen, the relative variance contribution needs to be considered. So, the dynamical system model that describes the temporal variation of the zonal average ozone layer system, i.e.,

$$\begin{cases} \dfrac{dT_1}{dt} = a_3 T_3 + a_6 T_3^2 + a_7 T_1 T_2 + a_9 T_2 T_3 \\ \dfrac{dT_2}{dt} = b_1 T_1 + b_3 T_3 + b_5 T_2^2 + b_8 T_1 T_3 + b_9 T_2 T_3 \\ \dfrac{dT_3}{dt} = c_2 T_2 + c_4 T_1^2 + c_7 T_1 T_2 + c_8 T_1 T_3 \end{cases} \tag{7}$$

can be finally obtained, where, $a_3 = -5.559208 \times 10^{-4}$, $a_6 = -8.604859 \times 10^{-6}$, $a_7 = -2.689605 \times 10^{-4}$, $a_9 = 6.660860 \times 10^{-5}$, $b_1 = 2.214989 \times 10^{-3}$,$b_3 = 5.326175 \times 10^{-3}$, $b_5 = -3.128056 \times 10^{-5}$, $b_8 = -8.067792 \times 10^{-5}$, $b_9 = -4.961111 \times 10^{-5}$, $c_2 = -4.350311 \times 10^{-2}$, $c_4 = 4.586958 \times 10^{-4}$, $c_7 = -7.528185 \times 10^{-4}$, and $c_8 = 1.505846 \times 10^{-4}$

For the study on the evolution of ozonosphere system, the dynamical system model should theoretically reflect the system features and the temporal variation characteristics, and the simulation results should have the same variation tendency as the observations have. In order to understand the ability that the retrieved dynamical model describes the evolution of the zonal ozonosphere system in every day, we made the single step simulation experiment for each state component $T_i(t)$ of three-dimensional phase space with Eq. (7). Because every time function series is long, we choose 400 phase points that are from $T_i(4400)$

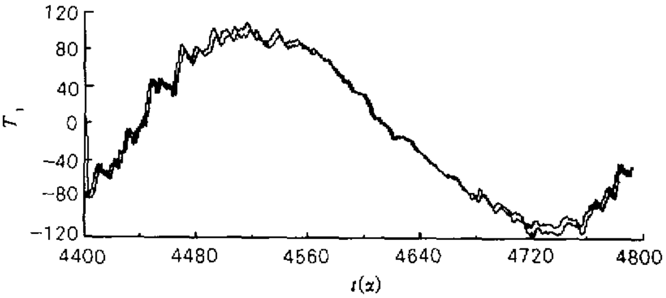

Fig. 4. The contrast of the first state component T_1 and the simulation result.

to $T_i(4800)$ to carry out the experiment. The correlation coefficients between the state components of the time function, i.e., T_1, T_2 and T_3 and relative simulation results are obtained, and shown in Table 4.

Table. 4. The Correlation Coefficients between the State Components of the Time Function and Relative Simulation Results in three Dimension Phase Space

State component	T_1	T_2	T_3
Correlation coefficient	0.9948	0.9959	0.6156

These correlation coefficients indicate that the simulation results have mainly reflected the information that exists in the time functions of the regional ozonosphere system, and the retrieved dynamical model can objectively describe the system states.

To contrast the temporal variation of the time function series T_1 with the temporal variation of its simulation result (Fig. 4), it can be obviously found that the variation tendency of the regional ozone layer system can be effectively described by the retrieved dynamical system model, the variation of the real state component $T_1(t)$ does synchronize with the variation of its simulation data in time, and the fine structures of amplitude variation are very close. These results also indicate that the retrieved dynamical system models obtained by EOF method can rationally, objectively, and meticulously describe the short-term variation of the regional ozonospheric dynamical system.

IV. CONCLUSION

1. Use EOF method, let the day to day time series of the Northern Hemispheric zonal average ozone layer of 7 latitudes be unfolded in phase space, and the orthogonal time and spatial functions are obtained. These characteristic functions can be used as other orthogonal functions, and then the radixes in state space can be replaced by the spatial functions. When the time functions are embedded in m dimension phase space, it can be found that the dimension of system attractor saturates in $m = 5$, and the saturation value $D_2 = 2.9$. The value indicates that at least three independent state variates are needed to build the dynamical model of the regional ozonosphere system.

2. The excellent features of the characteristic functions are that the characteristic functions can effectively concentrate the main information of the regional ozone layer system in space and time, and can be considered as independent variates of the temporal variation of the regional ozonosphere system. Each characteristic vector corresponds to a relative time series that describes the temporal variation of a spatial type. So, the time function series includes abundant information about the temporal variation of the regional ozonosphere system, and records the results that are caused by the interaction of various effect elements. It has important significance to study the qualitative information that exists in the time function series and the dynamical features, and to use time function series to rebuild the nonlinear dynamical model of the regional ozone layer system.

3. The temporal variation of the regional ozone layer system is a very complex process. As a preliminary study, in the embedded three dimension phase space, the state phase points are used to construct the nonlinear state equations in which linear terms and quadratic terms are included, then we can successfully retrieve the control parameters, and finally obtain the dynamical system model that can objectively describe the regional ozonosphere system with a few state variates. By comparing the simulation results with the real data of the state components, i.e., T_1, T_2 and T_3, it can be found that the nonlinear interaction terms can rationally, objectively and meticulously describe the integral characteristics of the short-term temporal variation of the regional ozonosphere system. This indicates that the inner interaction mechanism is the nature and the law of the regional ozonosphere system, and it can be reflected through the feedback relations. These feedback relations determine the function of the inner structure of the regional ozonosphere system and the multiplicity of the evolution process of the regional ozonosphere system.

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