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Topographically Forced Three–Wave Quasi–Resonant and Non–Resonant Interactions among Barotropic Rossby Waves on an Infinite Beta–Plane

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ABSTRACT

In this paper, we first apply the assumption $h = \varepsilon h'$ of topographic variation (h is the nondimensional topographic height and is a small parameter) to obtain nonlinear equations describing three–wave quasi–resonant and non–resonant interactions among Rossby waves for zonal wavenumbers 1—3 over a wavenumber–two bottom topography (WTBT). Some numerical calculations are made with the fourth–order Rung–Kutta Scheme. It is found that for the case without topographic forcing, the period of three–wave quasi–resonance (TWQR) is found to be independent of the zonal basic westerly wind, but dependent on the meridional wavenumber and the initial amplitudes. For the fixed initial data, when the frequency mismatch is smaller and the meridional wavelength is moderate, its period will belong to the 30–60–day period band. However, when the wavenumber–two topography is included, the periods of the forced quasi–resonant Rossby waves are also found to be strongly dependent on the setting of the zonal basic westerly wind. Under the same conditions, only when the zonal basic westerly wind reaches a moderate extent, intraseasonal oscillations in the 30–60–day period band can be found for zonal wavenumbers 1–3. On the other hand, if three Rossby waves considered have the same meridional wavenumber, three–wave non–resonant interaction over a WTBT can occur in this case.When the WTBT vanishes, the amplitudes of these Rossby waves are conserved. But in the presence of a WTBT, the three Rossby waves oscillate with the identical period.The period, over a moderate range of the zonal basic westerly wind, is in the intraseasonal, 30–60–day range.

Key words: Three–wave quasi–resonant, Non–resonant interactions

I. INTRODUCTION

Since the poineering study of three–wave resonance among barotropic Rossby waves was done by Longuet–Higgins and Gill (1967), considerable works on this problem were made in Geophysical fluid dynamics (Loesch, 1974; Jones, 1977; Pedlosky, 1987). Loesch (1974) and Egger (1978) showed that nonlinear interactions in a resonant Rossby wave triad could be considered as a prototypical mechanism for the onset of atmospheric blocking situation. However, because the large–scale topography plays an important role in the variability of the general atmospheric circulation, the topographically forced nonlinear interaction theory has long been an important topic. Recently, Cree and Swaters (1991) made a theoretical study on the large–scale orographic modulation of interacting Rossby wave triads by allowing the bottom topography to be the constant slope one, and showed that in the presence of topographic forcing, the nonlinear energy exchange between the members of triad will lead to resonance conditions being no longer satisfied. The numerical and rotating annulus experimental studies of Marcus et al. (1994) and Li (1993) indicated that topographic forcing plays

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a key role in producing the NH extratropical intraseasonal oscillation. For this reason, more recently, Luo (1994) investigated the effect of wavenumber-two topography on three-wave quasi-resonant interaction under the assumption of $h=\varepsilon^2 h'$ and tried to explain the occurence of intraseasonal oscillation in the NH extratropics. But this result cannot explain the travelling components of wavenumber-two intraseasonal oscillations. However, Tung and Lindzen (1979) and Szoeke (1983) pointed out that the topographically induced disturbance should be the same order as the free linear Rossby disturbance. In this case, the assumption $h=\varepsilon h'$ seems to be more appropriate, while the assumptions $h=\varepsilon^2 h'$ and $h=\varepsilon^3 h'$ (Jin and Ghil, 1991; Nathan and Barcilon 1994; Luo 1994) seem to underestimate the role of the large-scale topography in the intraseasonal variability of atmospheric circulation. Consequently, it is helpful to further investigate three-wave quasi-resonant and non-resonant interactions over a WTBT under the assumption $h=\varepsilon h'$. More recently, Luo (1997) investigated the interaction between single-mode topography and single wave with the same wavenumber in a near-resonant uniform westerly wind, and found that the low-frequency oscillation can be excited through this interaction.

In this paper, the effect of a WTBT on three-wave quasi-resonance is reexamined in the case of $h=\varepsilon h'$, and three-wave non-resonant interaction among barotropic Rossby waves for the same meridional wavenumber and zonal wavenumbers 1—3 is also studied. The basic model we used is the non-dimensional quasi-geostrophic potential vorticity equation associated with the rigid-lid shallow water equations. The outline of this paper is as follows. In Section 2, the nonlinear equations describing three-wave quasi-resonant and non-resonant interactions among barotropic Rossby waves for zonal wavenumbers 1—3 over a WTBT are obtained under the assumption of $h=\varepsilon h'$, which are remarkably different from those derived by Luo (1994) under the assumption of $h=\varepsilon^2 h'$. The numerical results of three-wave quasi-resonant and non-resonant interactions with and without the forcing of WTBT are presented in Sections 3 and 4, respectively. The main conclusions are given in Section 5.

II. DERIVATION OF THE WAVE-WAVE INTERACTION EQUATIONS

1. *Derivation of Three-wave Quasi-resonance Equation over a WTBT*

The inviscid and non-dimensional quasi-geostrophic potential vorticity equation associated with the rigid-lid shallow water equations can be written in the form

$$\frac{\partial}{\partial t}\nabla^2\psi + J(\psi, \nabla^2\psi + h) + \beta\frac{\partial\psi}{\partial x} = 0 \ , \tag{1}$$

where, $\beta=\beta_0\frac{L^2}{U}$ and $\beta_0=\frac{2\omega_0}{a_0}\cos(\varphi_0)$, in which ω_0 is the angular frequency of the earth's rotation, a_0 the earth's radius and φ_0 the latitude, h is the bottom topographic distribution, $U=10$ m / s and $L=10^6$ m are the horizontal velocity and length scales respectively, and the characteristic height of WTBT has been taken to be 1000 m, and the other notation can be seen in Pedlosky's (1987) book.

As pointed out in the introduction, the bottom topography has been supposed to be the same order as the linear disturbance term. Based on Szoeke's (1983) treatment, we introduce a new topographic amplitude paramter ε in the nondimensional form, so that

$$h=\varepsilon h' \ , \tag{2}$$

where $0 < r_0 \ll \varepsilon \ll 1$ has been assumed with the Rossby number $r_0 = \frac{U}{2\omega_0 L\sin(\varphi_0)} \approx 0.1$.

The assumption (2) is different from that used in Cree and Swaters (1991) and Luo (1994). However, the derivation of the three–wave interaction equation using the multiple–scale method when no forcing is present is well known (Pedlosky, 1987). In order to discuss the effect of a WTBT on three–wave quasi–resonance among Rossby waves under the assumption of $h = \varepsilon h'$, we will use a multiple–scale procedure to derive the topographically–forced three–wave quasi–resonance equation. Accordingly, we introduce the slow time variable in the form

$$\tau = \varepsilon t.$$

The solution to (1) can be obtained in a straightforward asymptotic expansion of the form

$$\psi = -\bar{u}y + \varepsilon\psi_1(x,y,t,\tau) + \varepsilon^2\psi_2(x,y,t,\tau) + \cdots , \tag{4}$$

where $\bar{u}$ is the zonal basic westerly wind and a constant.

If (2)—(4) are substituted into (1), we can obtain a set of perturbation equations in the following form:

$$L(\psi_1) = \left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\nabla^2(\psi_1) + \beta\frac{\partial(\psi_1)}{\partial x} = -\bar{u}\frac{\partial h'}{\partial x} , \tag{5}$$

$$L(\psi_2) = -\frac{\partial}{\partial \tau}\nabla^2\psi_1 - J(\psi_1, \nabla^2\psi_1 + h') . \tag{6}$$

In the Northern Hemispheric midlatitudes, the large–scale mountain ridges are approximate to a zonal wavenumber–2 distribution (Charney and Devore, 1979; Bernardet et al., 1990), in this case, the bottom topographic distribution is assumed to be

$$h = h'_0 \exp[i(k_2 x + m_2 y)] + cc , \tag{7}$$

where h'_0 denotes the topographic amplitude, $i = \sqrt{-1}$, $k_2 = 2k$, $m_2 = m$ and $m = \frac{\pi}{Ly}$ (Ly is called "meridional wavelength"), $k = \frac{1}{6.371\cos(\varphi_0)}$ is the zonal wavenumber of one wave and cc represents the conjugate of it's previous terms.

Since (5) is a linear equation including the topographic forcing, it permits a solution consisting of the superposition of three Rossby waves in which zonal wave 2 includes a travelling part and a stationary part forced by a WTBT in the form

$$\psi_1 = \sum_{n=1}^{3} A_n(\tau)e^{i\theta_n} + h_A h'_0 \exp[i(k_2 x + m_2 y)] + cc , \tag{8}$$

where $\theta_n = k_n x + m_n y - \omega_n t$ ($n = 1, 2, 3$), k_n and m_n are the zonal and meridional wavenumbers of the nth Rossby wave respectively, ω_n is the frequency of the nth Rossby wave, and $A_n(\tau)$ denotes the complex amplitude of the n Rossby wave. We can also note that the amplitude of travelling part of zonal wave 2 is $A_2(\tau)$, while h_A represents the stationary part of zonal wave 2 which is induced by a WTBT. If one removes the second term in the right–hand side of (8), it reduces to the usual expression of three–wave resonance solution, for example, in Longuett–Higgins and Gill (1967), Pedlosky (1987), Cree and Swaters (1991) and Luo (1994). On the other hand, we note that when the assumptions $h = \varepsilon^2 h'$ and $h = \varepsilon^3 h'$ are used, the second term in the right–hand side of (8) vanishes. In this case, the period of the three–wave resonance does not depend on the setting of the background westerly wind.

Substitution of (7) and (8) into (5) yields the linear dispersion relation of three travelling Rossby waves and an expression of h_A in the form

$$\omega_n = \bar{u}k_n - \frac{\beta k_n}{|\vec{K}_n|} , \tag{9}$$

$$h_A = -\frac{1}{\frac{\beta}{\bar{u}} - |\vec{K}_2|} , \tag{10}$$

where $\vec{K}_n = \vec{i}k_n + \vec{j}m_n$, $\vec{i}$ and $\vec{j}$ are the unit vectors in the zonal and meridional directions.

When $\omega_2 = 0$, then $\bar{u} = \frac{\beta}{|\vec{K}_2|}$ and $h_A \to \infty$. In this case, the travelling part of zonal wave 2 becomes stationary, while its stationary part becomes infinite. This shows that the stationary part of zonal wave 2 is a resonant wave forced by a WTBT, which resembles the resonant theory of Tung and Lindzen (1979). This implies that a travelling part of zonal wave 2 seems to exist besides a stationary part forced by a WTBT if the relation $\bar{u} = \frac{\beta}{|\vec{K}_2|}$ is not satisfied. In the work of Luo (1994), the second term in the right–hand side of (8) does not exist because the assumption $h = \varepsilon^2 h'$ was used. In this paper, we only consider the case of $\bar{u} \neq \frac{\beta}{|\vec{K}_2|}$. In order to investigate the effect of a WTBT on three–wave quasi–resonance, as a simple example we can take $(k_1, m_1) = (k, 2m)$, $(k_2, m_2) = (2k, -m)$ and $(k_3, m_3) = (-3k, -m)$ as the zonal and meridional wavenumbers of three Rossby waves for zonal wavenumbers 1–3. Consequently, when the three Rossby waves in (8) satisfy the following conditions

$$\vec{K}_1 + \vec{K}_2 + \vec{K}_3 = 0, \quad \omega_1 + \omega_2 + \omega_3 = \Delta\omega , \tag{11}$$

for a small frequency mismatch $\Delta\omega$, three–wave quasi–resonance among these three Rossby waves can occur. The condition (11) is the well–known quasi–resonance conditions derived by Craik (1985). For example, if we choose $Ly = 3 \sim 3.5$ (3000~ 3500 km in the dimensional form), then we have $\Delta\omega = 0.0634 \sim 0.0365$ at 45°N. In this case, three–wave quasi–resonant interaction is permitted.

Substitution of (7) and (8) into the right–hand side of (6) yields the appearance of secular terms in the solution for ψ_2. Consequently, for the smaller parameters $\Delta\omega$ and $\Delta\Omega = \omega_1 + \omega_3$, if we assume $\Delta\omega = \varepsilon\Delta\omega_0$ and $\Delta\Omega = \varepsilon\Delta\Omega_0$, the nonsecularity condition requires that

$$\frac{dA_1}{d\tau} = S_1 A_2^* A_3^* e^{i\Delta\omega_0\tau} + \left[-\frac{b_1}{|\vec{K}_1|} + S_1 h_A\right] h'_0 A_3^* e^{i\Delta\Omega_0\tau} , \tag{12}$$

$$\frac{dA_2}{d\tau} = S_2 A_1^* A_3^* e^{i\Delta\omega_0\tau} , \tag{13}$$

$$\frac{dA_3}{d\tau} = S_3 A_1^* A_2^* e^{i\Delta\omega_0\tau} + \Big[-\frac{b_3}{|\vec{K}_3|} + S_3 h_A\Big] h'_0 A_1^* e^{i\Delta\Omega_0\tau} \ , \tag{14}$$

where $S_1 = \frac{b_1(|\vec{K}_2| - |\vec{K}_3|)}{|\vec{K}_1|}$, $S_2 = \frac{b_2(|\vec{K}_3| - |\vec{K}_1|)}{|\vec{K}_2|}$, $S_3 = \frac{b_3(|\vec{K}_1| - |\vec{K}_2|)}{|\vec{K}_3|}$

$b_1 = k_3 m_2 - k_2 m_3 = b_2 = k_1 m_3 - k_3 m_1 = b_3 = k_2 m_1 - k_1 m_2, \Delta\Omega = \omega_1 + \omega_3$ and A_n^* denotes the complex conjugate of A_n.

Note that in Eqs.(12)–(14), $\Delta\Omega$ must be required to be smaller. For example, if $\bar{u} = 1.35 \sim 1.5$ is allowed, we have $\Delta\Omega = 0.0194 \sim -0.0472$ at 45°N for $Ly = 3$, while when $\bar{u} = 1.75 \sim 1.86$ is chosen, $\Delta\Omega = -0.024 \sim -0.073$ exists for $Ly = 3.5$. Thus, over a moderate range of $\bar{u}$, equations (12)–(14) are valid for describing three–wave quasi-resonance over a WTBT.

Equations (12)–(14) are the three–wave quasi–resonance equations over a WTBT, which are different from the topographically forced interaction equations derived by the previous investigators (for example, Cree and Swaters, 1991; Luo, 1994). It can be noted that topographic forcing acts directly on the A_1 and A_3 equations under the assumption $h = \varepsilon h'$. However, for the assumption $h = \varepsilon^2 h'$ the topographic forcing only acts on the A_2 equation (Luo, 1994). When the WTBT is absent, (12)–(14) reduce to the quasi–resonant interaction equations derived by Craik (1985), its analytical solution can be expressed in terms of the Jacobian elliptic functions. However, if the forcing of WTBT is included in (12)–(14), their solutions cannot be obtained directly. In this case the fourth–order Rung–Kutta scheme may be used to compute Eqs.(12)–(14), and their numerical results will be presented in Section 3.

2. *Derivation of the Three–wave Non–resonant Interaction Equations of Zonal Waves 1—3 with the Same Meridional Wave Number over a WTBT*

In the above section, if the meridional wavenumbers of the three Rossby waves are the same, the three Rossby waves don't permit quasi–resonance. For this situation the three–wave non–resonant interactions among Rossby waves can only occur in the higher order nonlinear terms, which equations will be obtained by using a formal multiple–scale asymptotic expansion in this section. In order to derive the topographically–forced wave–wave non–resonant interaction equations, we will introduce a new slow–time variable T to replace the slow–time variable τ in (3) in the form

$$T = \varepsilon^2 t \ , \tag{15}$$

and the solution of (1) may be expanded as

$$\psi = \bar{u}y + \varepsilon\psi_1(x,y,t,T) + \varepsilon^2\psi_2(x,y,t,T) + \varepsilon^3\psi_3(x,y,t,T) + \cdots \tag{16}$$

If (15)—(16) are introduced into (3), we have

$$L(\psi_1) = \Big(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\Big)\nabla^2(\psi_1) + \beta\frac{\partial(\psi_1)}{\partial x} = -\bar{u}\frac{\partial h'}{\partial x} \ , \tag{17}$$

$$L(\psi_2) = -J(\psi_1, \nabla^2\psi_1 + h') \ , \tag{18}$$

$$L(\psi_3) = -\frac{\partial}{\partial T}\nabla^2\psi_1 - J(\psi_2, \nabla^2\psi_1 + h') - J(\psi_1, \nabla^2\psi_2) \ . \tag{19}$$

In this section, we assume that the topographic distribution and the solution to (1) have the same forms as given in (7) and (8), but the main difference is that the zonal and meridional wavenumbers of zonal waves 1—3 considered here will be taken to be $(k_1, m_1) = (k, m)$, $(k_2, m_2) = (2k, m)$ and $(k_3, m_3) = (3k, m)$ so that three Rossby waves in (8) don't satisfy the quasi-resonance condition (11).

If (7) and (8) are substituted into (17), we can obtain the same dispersion relation and h_A as given in (9) and (10), but replaced by $m_n = m$.

Substituting (7) and (8) with $(k_1, m_1) = (k, m)$, $(k_2, m_2) = (2k, m)$ and $(k_3, m_3) = (3k, m)$ into the right-hand side of (18), we have

$$\begin{aligned} L(\psi_2) = & -S_{1,2}A_1A_2e^{i(\theta_1-\theta_2)} - S_{1,3}A_1A_3e^{i(\theta_1+\theta_3)} - S_{2,3}A_2A_3e^{i(\theta_2+\theta_3)} \\ & + S_{1,2}A_1^*A_2e^{i(-\theta_1+\theta_2)} + S_{1,3}A_1^*A_3e^{i(-\theta_1+\theta_3)} + S_{2,3}A_2^*A_3e^{i(-\theta_2+\theta_3)} \\ & - \sum_{p\neq 2} A_pr_ph'_0e^{i(\theta_p+k_2x+my)} + \sum_{p\neq 2} A_p^*r_ph'_0e^{i(-\theta_p+k_2x+my)} + cc \ , \end{aligned} \tag{20}$$

where $\omega_n = \bar{u}k_n - \frac{\beta k_n}{|\vec{K}_n|}$, $h_A = -\frac{1}{\frac{\beta}{\bar{u}} - |\vec{K}_n|}$, $\vec{K}_n = \vec{i}k_n + \vec{j}m$, $S_{p,q} = m(k_p - k_q)$ $(|\vec{K}_q| - |\vec{K}_p|)$ and $r_p = m(k_p - k_2)[(|\vec{K}_2| - |\vec{K}_p|)h_A - 1]$.

It can be noted that all terms in the right-hand side of (20) are the non-resonant terms. Thus, the solution of (20) is able to exist for this case.

Clearly, the solution of (20) can be given by

$$\begin{aligned} \psi_2 = & i\Omega_{1,2}A_1A_2e^{i(\theta_1+\theta_2)} + iQ_{1,3}A_1A_3e^{i(\theta_1+\theta_3)} + iQ_{2,3}A_2A_3e^{i(\theta_2+\theta_3)} \\ & + iZ_{1,2}A_1^*A_2e^{i(-\theta_1+\theta_2)} + iZ_{1,3}A_1^*A_3e^{i(-\theta_1+\theta_3)} + iZ_{2,3}A_2^*A_3e^{i(-\theta_2+\theta_3)} \\ & + i\sum_{p\neq 2} E_pA_ph'_0e^{i(\theta_p-k_2x+my)} + i\sum_{p\neq 2} F_pA_p^*h'_0e^{i(-\theta_p+k_2x+my)} + cc \ , \end{aligned} \tag{21}$$

where $Q_{p,q}$, $Z_{p,q}$, E_p and F_p are defined by

$$Q_{p,q} = \frac{S_{p,q}}{\beta(k_p + k_q) - [\bar{u}(k_p + k_q) - (\omega_p + \omega_q)][(k_p + k_q)^2 + 4m^2]} \ , \tag{22}$$

$$Z_{p,q} = \frac{S_{p,q}}{\beta(-k_p + k_q) - [\bar{u}(-k_p + k_q) - (-\omega_p + \omega_q)][(-k_p + k_q)^2]} \ , \tag{23}$$

$$E_p = \frac{r_p}{\beta(k_p + k_2) - [\overline{u}(k_p + k_2) - \omega_p][(k_p + k_2)^2 + 4m^2]} \;, \tag{24}$$

$$F_p = -\frac{r_p}{\beta(-k_p + k_2) - [\overline{u}(k_p + k_2) + \omega_p][(-k_p + k_2)^2]} \;. \tag{25}$$

Substitution of (7), (8) and (21) into (19) leads to the appearance of secular terms in the solution for ψ_3. When $Ly=3$, we have $\omega_2 = -0.178 \sim -0.089$ for $\overline{u} = 0.85 \sim 1.05$ and $\omega_2 = 0.112 \sim 0.2$ for $\overline{u} = 1.5 \sim 1.7$. While when $ly = 3.5$, we have $\omega_2 = -0.21 \sim -0.095$ for $\overline{u} = 1.15 \sim 1.4$ and $\omega_2 = 0.105 \sim 0.24$ for $\overline{u} = 1.85 \sim 2.15$. Accordingly, for these cases if $\varepsilon = 0.34$ is allowed, then when we assume $\omega_2 = \varepsilon^2 \omega_{20}$ and preclude the secular terms, the topographically forced three-wave non-resonant interaction equations can be obtained as

$$\frac{dA_1}{dT} = i(a|A_2|^2 A_1 + a_2|A_3|^2 A_1 + a_3 A_1 A_2^* h'_0 e^{i\omega_{20}T} + a_4 A_1 A_2 h'_0 e^{-i\omega_{20}T} + a_5 h'_0 A_1) \;, \tag{26}$$

$$\frac{dA_2}{dT} = i(b_1|A_1|^2 A_2 + b_2|A_3|^2 A_2 + b_3|A_1|^2 h'_0 e^{i\omega_{20}T}) + b_4|A_3|^2 h'_0 e^{i\omega_{20}T}) \;, \tag{27}$$

$$\frac{dA_3}{dT} = i(c_1|A_1|^2 A_3 + c_2|A_2|^2 A_3 + c_3 A_2^* A_3 h'_0 e^{i\omega_{20}T} + c_4 A_2 A_3 h'_0 e^{-i\omega_{20}T} + c_5 h'^2_0 A_3) \;, \tag{28}$$

where $|A_n|^2 = A_n A_n^*$ $(n = 1,2,3)$ and the coefficients of (26)–(28) are given in the Appendix A. (26)–(28) are the three–wave non–resonant interaction equations over a WTBT, which have not yet been derived by the previous investigators. Note that ω_2 must be required to be smaller so that Eqs.(26)–(28) are valid. When the forcing of a WTBT is absent, we have $\frac{d|A_n|}{dT} = 0$ $(n = 1, 2, 3)$. It means that the energy of each Rossby wave is conserved. This reason is that when the three Rossby waves have the same meridional wavenumber, the three–wave quasi–resonant interaction (nonlinear interaction) does not occur. This is why the amplitudes of the three Rossby waves are conserved when the WTBT is absent. However, when the forcing of WTBT is considered in (26)–(28), we have $\frac{d|A_n|}{dT} \neq 0$. This shows that the forcing of WTBT could drive the three non–resonant Rossby waves to produce oscillations. In order to obtain the oscillatory characters of the three Rossby waves, the numerical solutions of (26)—(28) will be obtained by using the fourth–order Rung–Kutta scheme in Section 4.

III. NUMERICAL RESULTS OF THREE-WAVE QUASI-RESONANT INTERACTION WITHOUT AND WITH THE FORCING OF WTBT

1. *Without Forcing*

If we define $B_n(t)=\varepsilon A_n(\varepsilon t)$, and use the transformation $t=\frac{\tau}{\varepsilon}$, Eqs.(12)–(14) can be written in the form

$$\frac{dB_1}{dt}=S_1B_1^*B_2^*e^{i\Delta\omega t}+[-\frac{b_1}{|\vec{K}_1|}+S_1h_A]h_0B_3^*e^{i\Delta\Omega t}\ , \tag{29}$$

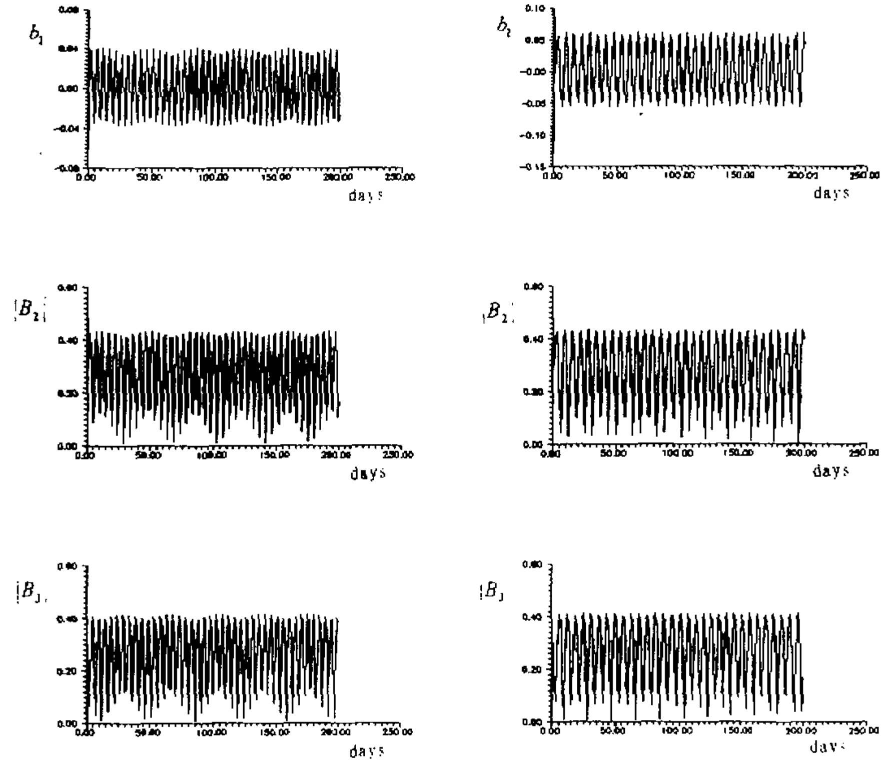
$$\frac{dB_2}{dt}=S_2B_1^*B_3^*e^{i\Delta\omega t}\ , \tag{30}$$

$$\frac{dB_3}{dt}=S_3B_1^*B_2^*e^{i\Delta\omega t}+[-\frac{b_3}{|\vec{K}_3|}+S_3h_A]h_0B_1^*e^{i\Delta\Omega t}\ , \tag{31}$$

where $h_0=\varepsilon h_0'$ and $2h_0$ denotes the height of WTBT because of $h=\varepsilon h'=\varepsilon h'_0\exp[i(k_2x+m_2y)]+cc=2h_0\cos(k_2x+m_2y)$.

Here we use $|B_1|$,$|B_2|$ and $|B_3|$ to represent the real amplitudes of three Rossby waves for zonal wavenumbers 1—3. In order to see qualitatively how WTBT has an influence upon three-wave quasi-resonance, we at first consider the case without forcing. In the numerical experiment we take $b_1=(|B_1|-0.3)\times 10$ as a measure of the amplitude of zonal wave 1. We may assume, without loss of generality, that initially $B_1(0)=B_2(0)=B_3(0)=0.3$. Consequently, the time-varying amplitudes of three Rossby waves for zonal wavenumbers 1–3 without forcing for $Ly=3$ and 3.5 at 45°N are, respectively, shown in Fig. 1.

Fig. 1a shows that for the case without forcing the three quasi-resonant Rossby waves for zonal wavenumbers 1–3 possess a long period oscillation besides a short period, and the periods are found to be dependent on the initial amplitudes and the meridional wavelength but not dependent on the zonal basic westerly wind. For $Ly=3$ (3000 km in the dimensional form), the long period is about 38 days, while the short period is close to 5 days. However, for $Ly=3.5$, the long period becomes 19 days, while the short period becomes about 7 days. These results are limited to small frequency mismatch $\Delta\omega$, which can be approximatly used to explain 21-day and 45-day oscillations in the Northern Hemisphere (NH) extratropics observed by Ghil and Mo (1990). In addition, we note that the long period of zonal wave 1 is not too clear. More recently, Branstator (1987) found that the 21-day oscillation has a large-scale wavenumber-two and slightly smaller wavenumber-three components. This can be basically explained by the three-wave quasi-resonance without forcing for $Ly=3.5$. It should be pointed out, however, that the results obtained here differ from those obtained by Luo (1994), who showed that for the case without forcing only 10-day oscillation was found when the frequency mismatch is taken to be relatively large in comparison with that given here. More recently, the numerical experiments by Marcus et al. (1994) using the UCLA general atmospheric circulation demonstrate that 42-day oscillation was found to arise in the NH extratropics of the standard topography. By contrast, no intraseasonal (36-60-day) oscillations can be found in the three no-mountain experiments. This indicates that the



(*a*) (*b*)

Fig. 1. Time-varying amplitudes of three quasi-resonant Rossby waves for zonal wavenumbers 1-3 without forcing at 45°N for $B_1(0) = B_2(0) = B_3(0) = 0.3$, $Ly = 3$ and 3.5, in which b_1 ($|B_1| - 0.3) \times 10$); (a) $Ly = 3$; (b) $Ly = 3.5$.

forcing of topography is very important. Certainly, the forcing of WTBT has an impotant effect on the oscillation periods of three-wave quasi-resonance. This problem will be discussed in the following subsection.

2. *With the Forcing ofWTBT*

In this subsection, as an example we choose $2h_0 = 1$ in our subsequent calculation, i.e., the height of wavenumber-two mountain ridges is chosen to be 1 km. If we take $Ly = 3$, under the same initial conditions as in Fig. 1 the time-varying amplitudes of three quasi-resonant Rossby waves for zonal wavenumbers 1-3 for $\bar{u} = 1.35$ and 1.5 (13.5 m / s and 15 m / s in the dimensional form), at 45°N, are shown in Fig. 2, respectively.

It is found from the numerical calculations here that when the WTBT is included, the periods of the envelope amplitudes of three quasi-resonant Rossby waves are found to be dependent on the zonal basic westerly wind. For $\bar{u} = 1.35$, the three Rossby waves propagate eastward and exhibit 31-day oscillation, in which zonal wave 2 (its phase speed is 0.1 m / s) is

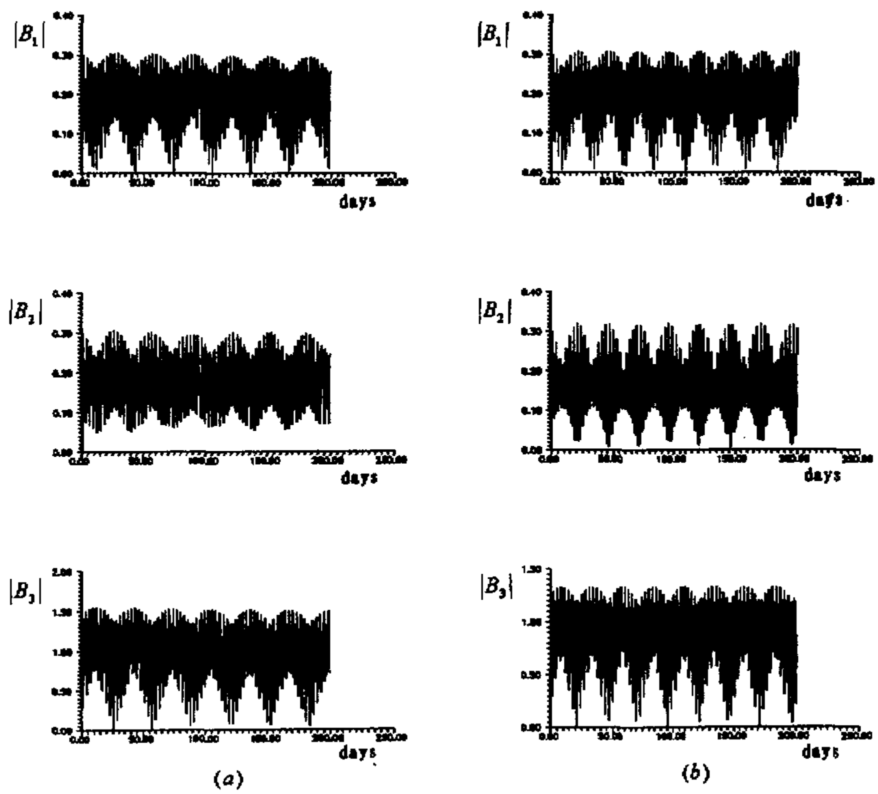


Fig. 2. Time-varying amplitudes of three quasi-resonant Rossby waves for zonal wavenumbers 1–3 with the forcing of WTBT for $h_0 = 0.5$, $Ly = 3$ and various value of $\bar{u}$, in which the initial data are the same as in Fig. 1: (a) $\bar{u} = 1.35$; (b) $\bar{u} = 1.5$.

slower than the other two waves. While for $\bar{u} = 1.5$, the envelope amplitudes of the three quasi-resonant Rossby waves possess 25-day period. In addition, by comparing Fig. 2 with Fig. 1 we note that when the forcing of WTBT is considered and when the zonal basic westerly wind is moderate, intraseasonal oscillations are found to be more clear than those found for the case without forcing. This shows that the forcing of WTBT and the moderate setting of zonal basic westerly wind favor the appearance of intraseasonal oscillations in the NH midlatitudes. However, for $Ly = 3.5$, a similar calculation is made. We can see in Fig. 3 that the envelope amplitudes of zonal waves 1 and 3 possess 50-day period, while the envelope amplitude of zonal wave 2 undergoes 34-day variation.These conclusions coincides roughly with the observational results of Ghil and Mo (1991), who showed that the 45-day waves at mid-latitudes are dominated by wavenumber one-through-three. Moreover, Tribbia and Ghil(1990) and Jin and Ghil (1991) have shown that the intraseasonal oscillation appears to arise from the nonlinear interaction with the topography of the large-scale flow field in the NH extratropics. However, our study here indicates that three-wave quasi-resonant interaction over a WTBT seems to be advantageous to intraseasonal oscillations in the NH

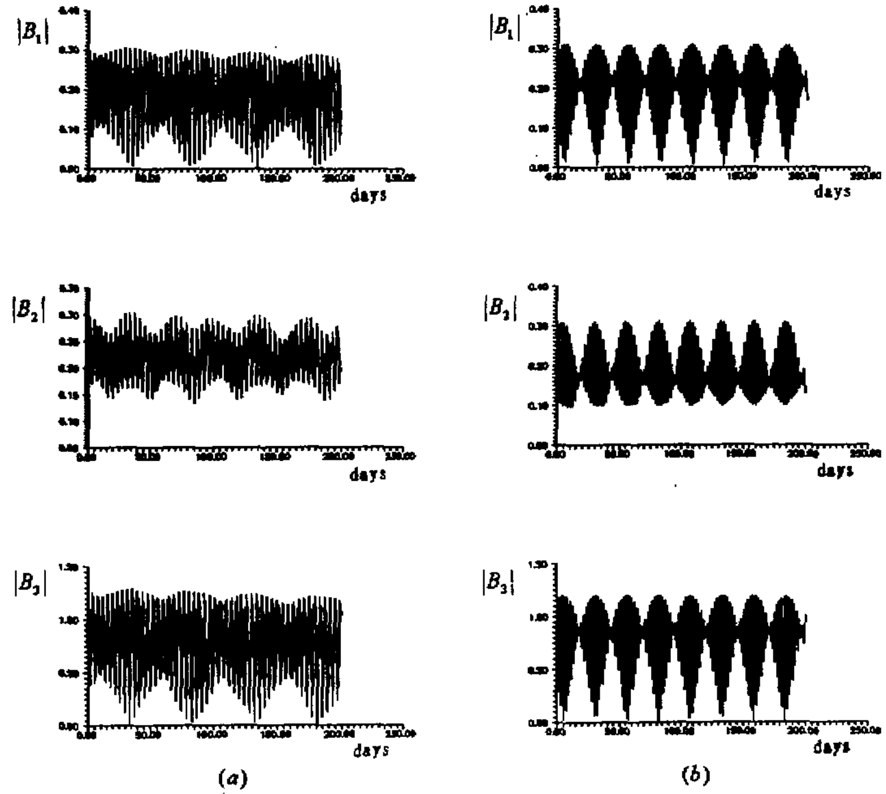


Fig. 3. Time-varying amplitudes of three quasi-resonant Rossby waves for zonal wavenumbers 1–3 with the forcing of WTBT for Ly = 3.5 and various value of $\bar{u}$, in which the other parameters are the same as in Fig. 2: $(a)\bar{u}$ = 1.75; $(b)\bar{u}$ = 1.86.

extratropics, which has been suggested by Luo (1994). But his reslut is only appropriate for the lower topography.

IV. NUMERICAL RESULTS OF THREE–WAVE NON–RESONANT INTERACTION WITH THE FORCING OF WTBT

In Section 2.2, we have shown that for the same meridional Rossby waves, if the topography vanishes, the three non–resonant Rossby waves don't oscillate. However, if the forcing of WTBT is considered, can the forcing of WTBT drive the three non–resonant Rossby waves to produce periodic oscillation? what is its property? These problems deserve to be studied. To examine the problems, if we let $B_n(t) = \varepsilon A_n(\varepsilon^2 t)$ and use $t = \frac{T}{\varepsilon^2}$, we rewrite the evolution equations (26)—(28) as follows:

$$\frac{dB_1}{dt} = i(a_1|B_2|^2B_1 + a_2|B_3|^2B_1 + a_3B_1B_2^*h_0e^{i\omega_2t} + a_4B_1B_2h_0e^{-i\omega_2t} + a_5h_0^2B_1) , \quad (32)$$

$$\frac{dB_2}{dt} = i(b_1|B_1|^2B_2 + b_2|B_3|^2B_2 + b_3|B_1|^2h_0e^{i\omega_2t} + b_4|B_3|^2h_0e^{i\omega_2t}) , \quad (33)$$

$$\frac{dB_3}{dt} = i(c_1|B_1|^2B_3 + c_2|B_2|^2B_3 + c_3B_2^*B_3h_0e^{i\omega_2t} + c_4B_2B_3h_0e^{-i\omega_2t} + c_5h_0^2B_3) . \quad (34)$$

Similarly, here we still use $|B_1|$, $|B_2|$ and $|B_3|$ to represent the real amplitudes of three non-resonant Rossby waves. If we choose the parameters $B_l(0) = B_2(0) = B_3(0) = 0.3$, $Ly = 3$, and $h_0 = 0.5$, the time-varying amplitudes of three-nonresonant Rossby waves over a WTBT for $\bar{u} = 0.85$ and 1.05 are shown in Fig. 4, respectively.

Fig.4 shows the dependence of the same period of three-nonresonant Rossby waves at 45°N, under the forcing of WTBT, on the setting of zonal basic westerly wind for $Ly = 3$. It is noted that when $\bar{u}$ is between 0.85 and 1.05, the three topographically forced nonresonant waves travel westward and their period is between 37 and 63 days. While when $\bar{u}$ is between

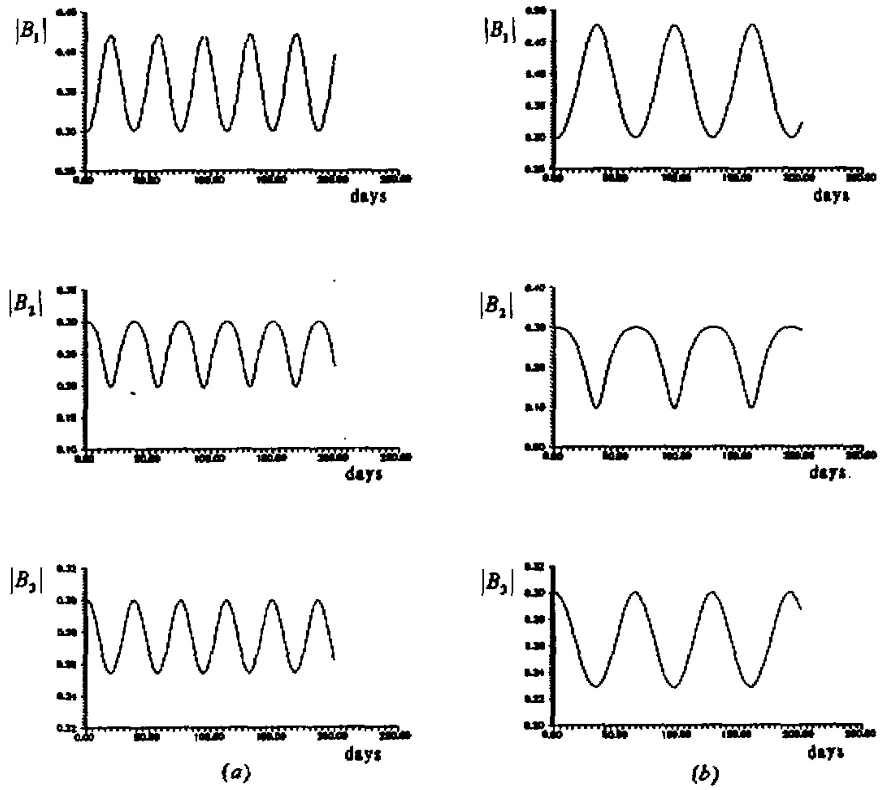


Fig. 4. Time-varying amplitudes of three non-resonant Rossby waves for zonal wavenumbers 1–3 with the forcing of WTBT for $Ly = 3$ and various value of $\bar{u}$ in which the other parameters are the same as in Fig. 2: (a) $\bar{u} = 0.85$; (b) $\bar{u} = 1.05$.

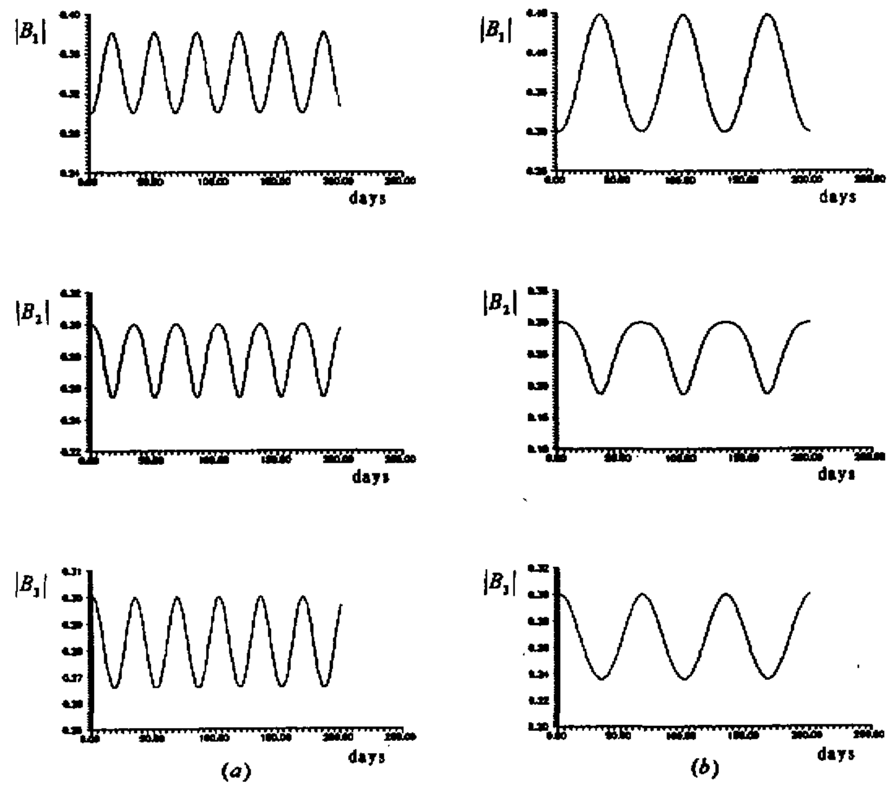


Fig. 5. Same as Fig. 4, but for $Ly = 3.5$ and different $\bar{u}$: (a) $\bar{u} = 1.15$; (b) $\bar{u} = 1.4$.

1.5 and 1.7, the three Rossby waves travel eastward and possess the same period in the range from 60 to 36 days. Fig. 5 corresponds to the case of $Ly = 3.5$. Clearly, for this case, there is the same property as that for $Ly = 3$. It is found that when $\bar{u}$ is between 1.15 and 1.4, the westward travelling 33–66–day oscillation can be found for zonal waves 1–3. While when $\bar{u}$ is between 1.85 and 2.15, the eastward travelling low frequency oscillation with the period of 66 to 30 days can occur. However, when the topography vanishes, no intraseasonal oscillation can be found. This indicates that topographic forcing is an origin of the NH 30–60–day low frequency oscillations with the same meridional scale. This result was supported by the numerical experiment of Marcus et al. (1994), who showed that in the standard topography run, 42–day oscillation was found to occur in the NH extratropics, but in the three no–mountain experiments, no intraseasonal oscillation can be found. Ghil and Mo (1991) found that 48–day oscillation has wavenumbers 1—3 structure, in which wavenumber two is dominant and has both travelling and standing components. However, it is worth noting that becuase the assumptions $h = \varepsilon^2 h'$ and $h = \varepsilon^3 h'$ are used, the previous theories can only explain the standing component of wavenumber–two intraseasonal oscillation, but cannot explain the travelling component (Jin and Ghil 1990; Luo 1994). But in this paper, becuase the assumption $h=\varepsilon h'$ is used, our result can explain the travelling component of wavenumber–two intraseasonal oscillation. Actually, in this paper when $Ly=3.0\sim3.5$, it denotes the meridional wavelength of Rossby wave being 6000~ 7500 km (the nondimensional meridional wavelength of Rossby wave is $2Ly=6.0\sim 7.0$), which is consistent with the observational facts of low–frequency oscillations obtained by Ghil and Mo (1991). It should be pointed out that when the assumption $h=\varepsilon h'$ is used, the amplitude of the topographically induced stationary wave depends on the background westerly wind. When it interacts with the three quasi–resonant waves, the periods of the three quasi–resonant waves depend on the setting of the background westerly wind. This is why our result is different from the previous results.

V. CONCLUSIONS

In this paper, we have investigated three–wave quasi–resonant and nonresonant interactions among barotropic Rossby waves over a WTBT under the assumption $h=\varepsilon h'$. It is found that for the case without forcing, three–wave quasi–resonance can induce low frequency oscillations, whose periods depend on the initial amplitudes and the meridional wavelength. However, under the forcing of WTBT, the low frequency periods of the topographically forced quasi–resonant Rossby waves are also found to be dependent on the zonal basic westerly wind. When the zonal basic westerly wind is in a moderate range, the topographically forced low frequency oscillation is found to be more apparent than that without forcing. Thus, the forcing of WTBT and the moderate setting of zonal basic westerly wind favor the appearance of intraseasonal oscillation in the NH extratropics. On the other hand, if the meridional wavenumbers of the three Rossby waves are the same, three–wave non–resonant interaction can take place. When topography vanishes, no oscillation can be found. While when a WTBT is considered, the three–wave non–resonant interaction can lead to low frequency oscillation in the 30—60–day period band, whose period length depends strongly on the setting of zonal basic westerly wind.Therefore, it appears that topographic forcing is an origin of the NH extratropical 30–60–day oscillation. This result has been confirmed by the numerical experiment of Marcus et al. (1994).

In this paper, some physical processes, for example, dissipation, baroclinic effect and thermal forcing have been ignored. These problems deserve further investigation.

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APPENDIX A

The coefficients of (26)—(28) are defined by

$$a_1=m(-k_1+k_2)[|\vec{K}_2|^2(\Omega_{1,2}+Z_{1,2})-|\vec{K}_1+\vec{K}_2|^2\Omega_{1,2}-|-\vec{K}_1+\vec{K}_2|^2Z_{1,2}]/|\vec{K}_1|^2\ , \tag{A.1}$$

$$a_2=m(-k_1+k_3)[|\vec{K}_3|^2(\Omega_{1,3}+Z_{1,3})-|\vec{K}_1+\vec{K}_3|^2Q_{1,3}-|-\vec{K}_1+\vec{K}_3|^2Z_{1,3}]/|\vec{K}_1|^2\ , \tag{A.2}$$

$$a_3=m(-k_1+k_2)[|\vec{K}_2|^2E_1-(-|\vec{K}_2|^2h_A+1)Z_{1,2}-|\vec{K}_1+\vec{K}_2|^2E_1-|-\vec{K}_1+\vec{K}_2|^2Z_{1,2}h_A]/|\vec{K}_1|^2\ , \tag{A.3}$$

$$a_4=m(-k_1+k_2)[|\vec{K}_2|^2F_1-(-|\vec{K}_2|^2h_A+1)Q_{1,2}-|\vec{K}_1+\vec{K}_2|^2Q_{1,2}h_A-|-\vec{K}_1+\vec{K}_2|^2F_1]/|\vec{K}_1|^2\ , \tag{A.4}$$

$$a_5=m(-k_1+k_2)[-(-|\vec{K}_2|^2h_A+1)(E_1+F_1)-|\vec{K}_1+\vec{K}_2|^2E_1h_A-|-\vec{K}_1+\vec{K}_2|^2F_1h_A]/|\vec{K}_1|^2\ , \tag{A.5}$$

$$b_1=m(-k_1+k_2)[|\vec{K}_1|^2(Z_{1,2}-Q_{1,2})+|\vec{K}_1+\vec{K}^2|^2Q_{1,2}-|-\vec{K}_1+\vec{K}_2|^2Z_{1,2}]/|\vec{K}_2|^2\ , \tag{A.6}$$

$$b_2 = m(-k_3 + k_2)[|\vec{K}_3|^2(Z_{2,3} - Q_{2,3}) + |\vec{K}_3 + \vec{K}^2|_2 Q_{2,3} - |-\vec{K}_3 + \vec{K}_2|^2 Z_{2,3}]/|\vec{K}_2|^2 \ , \tag{A.7}$$

$$b_3 = m(-k_1 + k_2)[|\vec{K}_1|^2(F_1 - E_1) + |\vec{K}_1 + \vec{K}_2|^2 E_1 - |-\vec{K}_1 + \vec{K}_2|^2 F_1]/|\vec{K}_2|^2 \ , \tag{A.8}$$

$$b_4 = m(-k_3 + k_2)[|\vec{K}_3|^2(F_3 - E_3) + |\vec{K}_3 + \vec{K}_2|^2 E_3 - |-\vec{K}_3 + \vec{K}_2|^2 F_3]/|\vec{K}_2|^2 \ , \tag{A.9}$$

$$c_1 = m(-k_1 + k_3)[|\vec{K}_1|^2(Q_{1,3} + Z_{1,3}) - |\vec{K}_1 + \vec{K}_3|^2 Q_{1,3} - |-\vec{K}_1 + \vec{K}_3|^2 Z_{1,3}]/|\vec{K}_3|^2 \ , \tag{A.10}$$

$$c_2 = m(-k_3 + k_2)[|\vec{K}_2|^2(Q_{2,3} + Z_{2,3}) - |\vec{K}_3 + \vec{K}_2|^2 Q_{2,3} - |-\vec{K}_3 + \vec{K}_2|^2 Z_{2,3}]/|\vec{K}_3|^2 \ , \tag{A.11}$$

$$c_3 = m(-k_3 + k_2)[|\vec{K}_2|^2 E_3 + (-|\vec{K}_2|^2 h_A + 1)Z_{2,3} - |\vec{K}_3 + \vec{K}_2|^2 E_3 + |-\vec{K}_3 + \vec{K}_2|^2 Z_{2,3} h_A]/\vec{K}_3 \quad ^2 \tag{A.12}$$

$$c_4 = m(-k_3 + k_2)[|\vec{K}_2|^2 F_3 + (-|\vec{K}_2|^2 h_A + 1)Q_{2,3} - |\vec{K}_3 + \vec{K}_2|^2 Q_{2,3} - |-\vec{K}_3 + \vec{K}_2|^2 F_3]/\vec{K}_3 \quad ^2 \ , \tag{A.13}$$

$$c_5 = m(-k_3 + k_2)[-(E_3 + F_3)(-|\vec{K}_2|^2 h_A + 1) - |\vec{K}_2 + \vec{K}_3|^2 h_A E_3 - |-\vec{K}_3 + \vec{K}_2|^2 h_A F_3]/|\vec{K}_3|^2 \ , \tag{A.14}$$

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