Vol.15 No.1 Advances in Atmospheric Sciences February 1998

Atmospheric Refractive Turbulence Effect on Diffraction–Limited Infrared Coherent Lidar①

Chen Wuhe (陈武喝),② *Situ Da* (司徒达) and *Zhong Xubin* (钟旭滨)
Department of Physics, South China Normal University, Guangzhou 510631

Received February 16, 1996; revised July 28, 1997

ABSTRACT

This paper, based on the Kavaya–Suni format, discusses the signal–to–noise ratio equation of the diffraction–limited coherent CO_2 lidar in detail, which is applied to atmospheric turbulence. The cumulative SNR and relative SNR, which are all affected by the nonlinear effects of the diffraction–limited Gaussian beam, atmospheric molecule and atmospheric turbulence, are simulated by microcomputer. Six instructions for the optimal design of IR CO_2 Coherent Lidar System, are provided.

Key words: Coherent lidar, Atmospheric refractive turbulence, Turbulent structural parameter, Signal–to–noise ratio (SNR), Diffraction–limited

I. INTRODUCTION

In this paper, we refer to the new formats of the normalized cumulative SNR and relative SNR deduced by Kavaya and Suni in 1991 (Kavaya and Sani, 1991) and apply it to atmospheric refractive turbulence. The cumulative SNR, relative SNR and percentile–range–resolution, which are related to the aperture's diameter, focal–length, aerosol back–scattering, absorption and extinction of atmospheric moleules and atmospheric turbulence, are evaluated numerically. Both cases of the coherent lidar's equations for diffraction–limited Gaussian beams in atmospheric turbulent medium, transmitting along vertical direction and horizontal direction, are also discussed.

II. THE HETERODYNE LIDAR'S EQUATION

The SNR's equation of heterodyne lidar could be given by the modified Fraunhofer–Fresnel diffraction formula (Kavaya and Sani, 1991; Kavaya et al., 1989). If the diameter of the telescope's aperture is sufficiently larger than the backward local oscillator beam, the equation of heterodyne lider may be inferred as

$$SNR(R)=\frac{\eta_Q P_t}{h\nu B}\int_0^R \frac{\pi(D/2)^2[K(R)]^2\beta(R)dR}{R^2[1+(1-R/F)^2(KD^2/8R)^2+(D/2\rho_0)^2]}\ , \qquad (1)$$

where η_Q is the quantum efficiency of detector, and $h = 6.626 \times 10^{-34}$ [Js] is Planck's constant. ν [Hz] is the optical frequency. B [Hz] is the detector band–width. P_t [W] is the laser's

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①The project is supported by the National Natural Science Foundation of China and the Science Committee of Guangdong Province.

②Now Chen Wuhe works at Jiaying University, Meizhou, Guangdong Province.

transmitting power. $K(R)$ is the function of atmospheric absorption and extinction. $\beta(R)$ is the aerosol back-scattering coefficient. R [m] is the target-range. D [m] is the aperture's diameter of telescope. F [m] is the focal length. K is wave number ($=2\pi/\lambda$). ρ_0 [m] is the transversal (or lateral) coherent length in refractive turbulence.

III. CALCULATION OF SNR, RANGE-RESOLUTION AND PERCENTILE-VOLUME IN THE ATMOSPHERIC TURBULENCE

1. *Cumulative SNR*

We assume the beam propagating along horizontal direction, then the atmospheric turbulent structural parameter could be regarded as a constant. For the spherical wave transmitting in the weak atmospheric turbulence, the refractive turbulent structural parameter can be expressed as $C_n^2 = 3 \times 10^{-14}$ [$\mathrm{m}^{-2/3}$], and the transversal coherent length is given by $\rho_0 = (0.546 \times C_n^2 \ K^2 R)^{-3/5}$ (Chumside, 1991). The normalized cumulative SNR could be defined by

$$CUMSNR(R) = \frac{SNR(R)}{SNR(R \to \infty)} \ . \tag{2}$$

In Fig.1, the curves of *CUMSNR* easily approach saturation in the near-field for the shorter focal length. That is to say, most optical signal's field concentrates in the near-field. It gradually extends to far-field as the focal length is increasing. However, we can find an abnormal region in that the cumulative *SNR* for the longer focal length may overreach for shorter ones; that is $CUMSNR(F=3000\ \mathrm{m}) > CUMSNR(F=1000\ \mathrm{m})$. This phenomenon will be explained in next section.

2. *Relative SNR*

Eq.(1) is an integral equation, and it is integrated along the range. If the cumulative process is not considered in the integration, the relative SNR can be defined as $SNR(R)$ referring to $SNR(R=0)$, and is written as

$$SNR(R)_{\mathrm{Relat}} = 20\log\left[\frac{R_R^2}{R^2 + (1 - R/F)^2 (R_R)^2 + (\lambda/\pi)(R_R/\rho_0^2)R^2}\right]\ (\mathrm{dB})\ . \tag{3}$$

R_R [m]$=\pi D^2/4\lambda$ is called Rayleigh range, and it shows that the beam may be diffracted by an aperture of diameter D.

In Fig.2, it shows the plots of relative SNR versus the range R in different focal length. There is a peak at $R = 50$ m when the focal length is 50 m. The peak's position is the optimal measuring target range. However, the peak's position of relative SNR does not locate at the focus when focal length becomes large. The peak's position of different focal length first moves to the right along the increasing target range direction as focal length is increased. Then it diverts to the left, and the peak becomes more flat as the focal length is increased.

The peak inverses its migratory direction, when the focal length is larger than 740.94 m. There is a cusp point of migratory direction due to the nonlinear effect of the diffraction-limited Gaussian beam and the atmospheric turbulence. This is why the curves in Fig.2 exist in the region of abnormal behavior as the aperture's diameter is 0.1 m.

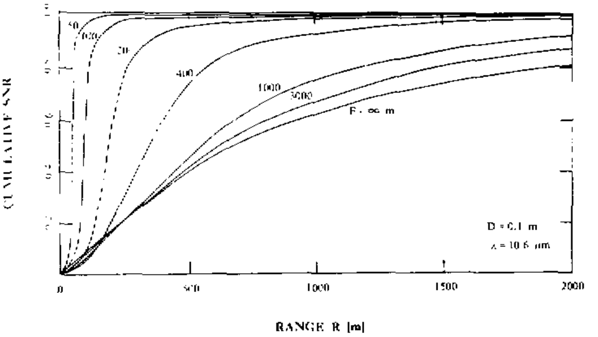

Fig. 1. Curves of cumulative SNR versus range R in the refractive turbulence, $C_n^2 = 3 \times 10^{-14}$ $m^{-2/3}$.

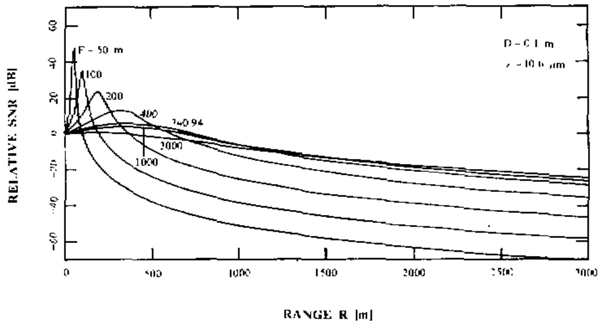

Fig. 2. Relative SNR versus range as a function of focal length in the refractive turbulence. The SNR is normalized to 1 at $R = 0$. $C_n^2 = 3 \times 10^{-14}$ $m^{-2/3}$.

On the other hand, in far-range region, the larger focal range is, the smaller relative SNR is. That is to say, the relative SNR for larger focal length could be larger than that for shorter focal length in measuring far-range object.

Now we explore the natural characters of the peak's position in Fig.2 in detail. Firstly, we calculate the location of maximal relative SNR for different aperture when the focal length does not vary. Secondly, just like above ways, but the focal length varies. A series of curves, that are shown in Fig.3, could be obtained. There is a maximal relative SNR at focus for shorter focal length as the aperture is small. That is to say, the tight focusing has been realized.

The maximal location of relative SNR locates at its focus for far-range measurement, which require to match both focal length and aperture. In other words, for the target measurement at far-range, the tight focusing does not access unless the focal length and the aperture become large. In short words, there is the optimal matching condition for both the focal length and aperture's diameter to realize the optimal target measurement.

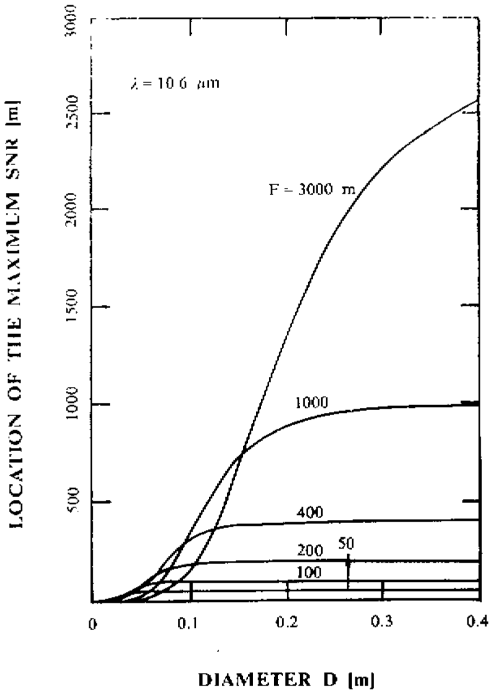

Fig. 3. Location of the maximum SNR is determined by varied aperture's diameter D in the same focal length. $C_n^2 = 3 \times 10^{-14}\ \mathrm{m}^{-2/3}$.

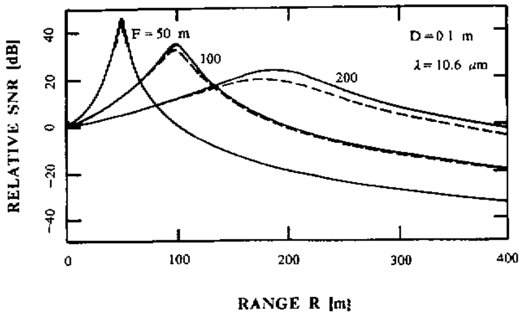

Fig. 4. Relative SNR versus range R as a function of focal length F in same aperture's diameter in different atmospheric turbulence. (——) $C_n^2 = 3 \times 10^{-14}\ \mathrm{m}^{-2/3}$, (- - -) $C_n^2 = 3 \times 10^{-12}\ \mathrm{m}^{-2/3}$.

3. *Relative SNR in the Different Atmospheric Turbulence*

We analyze the impact in different atmospheric refractive turbulence. As an example, we choose that C_n^2 identifies $3 \times 10^{-12}\ \mathrm{m}^{-2/3}$ and $3 \times 10^{-14}\ \mathrm{m}^{-2/3}$, respectively. In Fig. 4, the

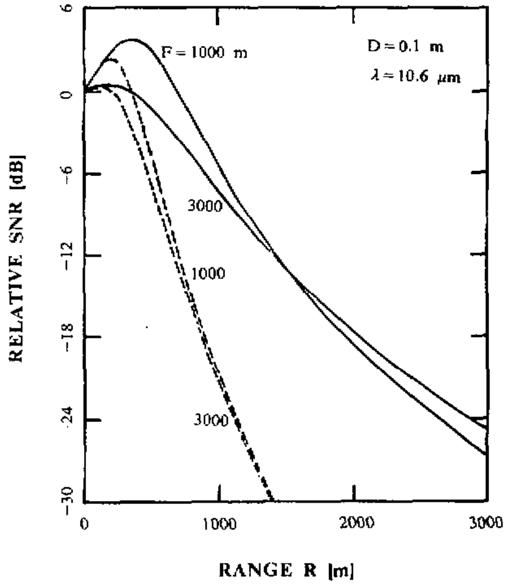

Fig.5. Relative SNR versus range R for different aperture's diameter in different atmospheric turbulence. (——) $C_n^2 = 3 \times 10^{-14}$ $m^{-2/3}$, (- - -) $C_n^2 = 3 \times 10^{-12}$ $m^{-2/3}$.

relative SNR is less affected by the weak turbulenec as the focal length is shorter. In Fig.5, that are apparently affected by the strong refractive turbulence as increasing focal length and causing the SNR decrease seriously.

In Fig.6, the ratio of SNR in different turbulence is plotted against the range as fixed aperture's diameter. If we increase the aperture's diameter, the ratio of relative SNR for smaller aperture's diameter is a little smaller than the larger aperture's diameter in near–range. That is to say, if increasing the aperture's diameter, it would reduce stronger turbulent influence in near–range. It has contrary results in far–range because the relative SNR is affected seriously by stronger turbulence, especially for larger aperture's diameter in far–range.

4. *Range–resolution*

The percentile–range $R(X\%)$ can be defined by a measuring range R corresponding to percentage $X\%$ of cumulative SNR. Apparently, at fixed percentage $X\%$ of cumulative SNR, the farther focusing is, the larger percentile–range is. That is to say, the optical signal's field splits extensively at longer and wider range for large focal length.

The percentile–range–resolution is defined by

$$W((a-b)\%) = R(a\%) - R(b\%) \ , \tag{4}$$

where $a\%$ and $b\%$ are the percentage of cumulative SNR. The equations of range–resolution could be written according to Eq.(4) as

$$W(90\%) = R(95\%) - R(5\%) \ , \quad W(80\%) = R(90\%) - R(10\%) \ , \quad W(70\%) = R(80\%) - R(10\%) \ ,$$
$$W(60\%) = R(80\%) - R(20\%) \ , \quad W(50\%) = R(75\%) - R(25\%) \ ,$$

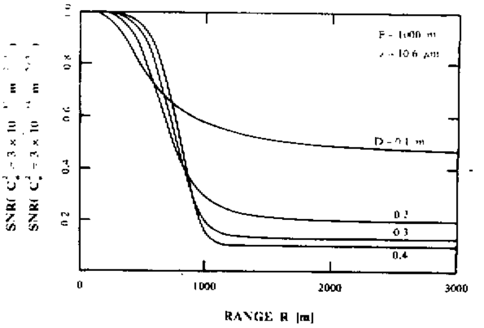

Fig. 6. Ratio of relative SNR for $C_n^2 = 3 \times 10^{-12}$ $m^{-2/3}$ and $C_n^2 = 3 \times 10^{-14}$ $m^{-2/3}$.

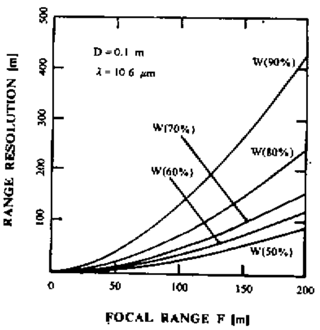

Fig. 7. Range–resolution versus focal length in turbulence. $C_n^2 = 3 \times 10^{-14}$ $m^{-2/3}$.

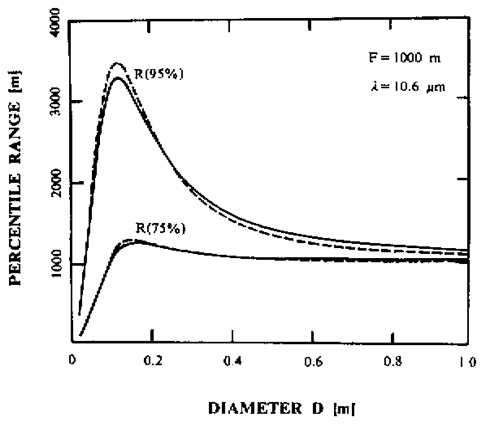

Fig. 8. Range–resolution versus aperture's diameter in different atmospheric turbulence. (– – –) $C_n^2 = 0$. (——) $C_n^2 = 3 \times 10^{-14}$ $m^{-2/3}$.

In Fig.7, we can see that the range–resolution becomes worse when increasing focal length and fixed aperture's diameter. The similar results were reported by Kavaya and Sani (1991), but they did not consider the turbulent effect.

Now we consider the variation of range–resolution, for fixed focal length and the aperture's diameter varied. In Fig.8, it shows the width of range–resolution versus the aperture's diameter. The plot of $W(90\%)$ has a peak value, which is the maximal range–resolution. This means that it is the worst to discern the range and the distribution of optical signal's field is so wide.

5. *Percentile Volume*

For the diffraction–limited beam, the percentile–volume is given by

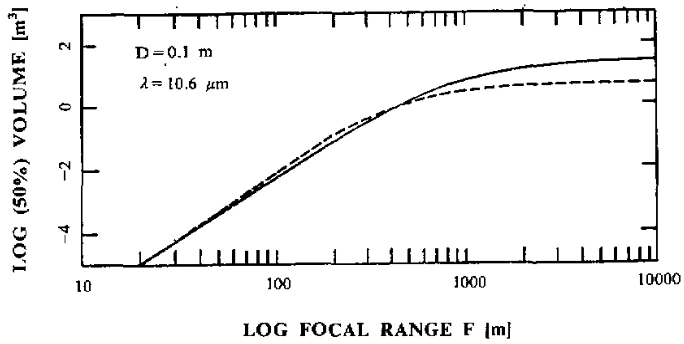

Fig.9. Percentile-volume $V(50\%)$ versus focal length in varied atmospheric turbulence. (- - -) $C_n^2=3\times10^{-12}$ $\mathrm{m}^{-2/3}$, (——) $C_n^2=3\times10^{-14}$ $\mathrm{m}^{-2/3}$.

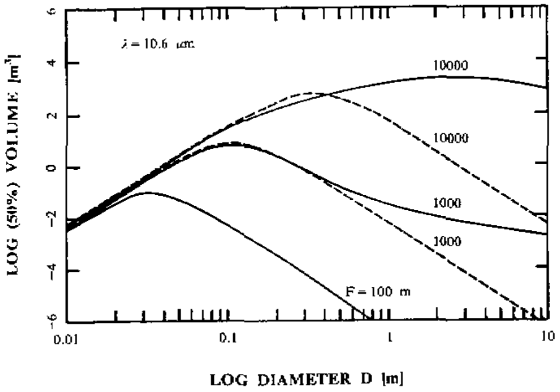

Fig. 10. Pencentile-volume V (50%) versus aperture's diameter as a function of focal length in varied atmospheric turbulence. (- - -) $C_n^2=0$,(——) $C_n^2=3\times10^{-14}$ $\mathrm{m}^{-2/3}$.

$$V=\frac{\pi D^2}{4}\int_{R_1}^{R_u}[(1-R/F)^2+(R/R_R)^2+(\lambda/\pi R_R)(R/\rho_0)^2]dR\ , \tag{5}$$

where the integral limits are R_u and R_l. As an example, the percentile-volume $V(50\%)$ is calculated from Eq.(4), and the range-resolution is $W(50\%)=R(75\%)-R(25\%)$. In Fig.9, it shows the plots for the percentile-volume $V(50\%)$ versus the focal length as diameter of 0.1 m in two kinds of turbulence conditions. It shows that the cumulative SNR is affected a little by turbulence below the focal length of 250 m, and the percentile-volume $V(50\%)$ in weak turbulence is slightly smaller than that in strong turbulence. The percentile-volume is affected more and more by the turbulence in contrary way as the focal range increases beyond 250 m.

In Fig.10, it shows the percentile-volume V(50%) versus the aperture's diameter. Apparently, the percentile volume is affected a little by turbulence for shorter focal length, but it is affected much seriously by turbulence as the focal length is larger. The percentile-volume in turbulence is larger than that in non-turbulence for large focal length and large aperture's diameter.

6. *Relative SNR and Cumulative SNR for Laser Beam Travelling along Horizontal Direction Including the Cumulative Process*

In above sections, we discuss some parametric variations in special assumptions; those are β = constant and $[K(R)]^2 = 1$, and the beam travels along horizontal direction. Now we evaluate the relative SNR and cumulative SNR, which include the cumulative contributions of absorption and extinction of the atmospheric CO_2 and H_2O at a horizontal plane of 1 m height. The function of the absorption and extinction of atmospheric CO_2 and H_2O molecules could be written as

$$[K(R)]^2 = \exp(-2 \times 8.478846 \times 10^{-5} R) \ . \tag{6}$$

The value $8.478846 \times 10^{-5}\mathrm{m}^{-1}$ is the coefficient of the absorption and extinction of atmospheric CO_2 and H_2O molecules, and it will be calculated in next section.

For uniform aerosol target, we can get the relative SNR by substituting Eq.(6) into Eq.(1), and the correspondent result is

$$SNR(R)_{\mathrm{Relat}} = 20\log[\frac{R_R^2 \exp(-2 \times 8.478846 \times 10^{-5} R)}{R^2 + (1 - R / F)^2 (R_R)^2 + (\lambda / \pi)(R_R / \rho_0^2) R^2}] \ (\mathrm{dB}) \ . \tag{7}$$

The relative SNR and cumulative SNR, which include the effects of the absorption and extinction of atmospheric CO_2 and H_2O molecules, would be achieved by Eq.(7). While the beam travels along horizontal direction, these plots are drawn in Fig.11.

IV. RELATIVE SNR AND CUMULATIVE SNR FOR BEAMS TRAVELLING ALONG THE VERTICAL DIRECTION INCLUDING CUMULATIVE PROCESS

In this section, we would discuss the relative SNR and cumulative SNR for beam travelling along the vertical direction. We would consider the aerosol back-scattering, the absorption and extinction of the atmospheric CO_2 and H_2O and the effects of atmospheric refractive turbulent structural parameter on optical signal's field. The coefficient of aerosol back-scattering could be expressed as (Kavaya et al., 1989; Steinvall et al., 1983)

$$\log\beta(h) = 3.521\exp(-h / 4967) - 9.455 \quad (\mathrm{m}^{-1}\mathrm{Sr}^{-1}) \ , \tag{8}$$

where h is the vertical height. the absorption coefficients of CO_2 and H_2O, for CO_2 lidar's line P(20), are given by (Sutton et al., 1979)

$$\alpha_{CO_2} = \frac{0.209\exp(-h / 3.421)}{1.177 + 1.592\exp(-h / 3.421) + 0.218\exp(-2h / 3.421)} \quad (\mathrm{Km}^{-1}) \ . \tag{9}$$

$$\alpha_{H_2O} = 1.484 \times 10^{-2} \exp(-h) \quad (\mathrm{Km}^{-1}) \ . \tag{10}$$

The absorption of path is

$$[K(h)]^2 = \exp[\frac{-2}{1000}\int_0^h (\alpha_{CO_2} + \alpha_{H_2O})dh]$$

$$= \exp\{\frac{-2}{1000}[14.84(1-\exp(-h/1000))-2785.831$$

$$\times (1.404+\ln\frac{0.436\exp(-h/3421)+0.364}{0.436\exp(-h/3421)+2.82})]\} \ . \qquad (11)$$

The refractive turbulent structural parameter C_n^2 is (Kavaya et al., 1989; Post, 1979)

$$C_n^2 = \begin{cases} 1.585\times 10^{-12} h^{-4/3} \quad (\mathrm{m}^{-2/3}) \quad h \leqslant 3\ \mathrm{Km} \\ 2.693928[2.2\times 10^{-23}(h/1000)^{10}\exp(-h/1000) \\ +10^{-16}\exp(-h/1500)] \quad (\mathrm{m}^{-2/3}) \quad h > 3\ \mathrm{Km} \end{cases} \qquad (12)$$

Assume that the coherent lider is located at 1 m height above the ground, and the lateral coherent length, for spherical wave case, can be expressed as

$$\rho_0 = [H_0 K^2 \int_0^H C_n^2(h)(1-h/H)^{5/3} dh]^{-3/5} \ , \qquad (13)$$

where $H_0 \approx 2.91$. The lateral coherent length can be achieved from Eq.(12) and Eq.(13). The corresponding equation of relative SNR could be written as

$$SNR(H)_{\mathrm{Relat}} = 20\log\{10^{[3.521\exp(-H/4967)-9.455]}\exp[-\frac{2}{1000}(14.84$$

$$\times (1-\exp(-H/1000))-2785.831(1.404$$

$$+\ln\frac{0.436\exp(-H/3421)+0.364}{0.436\exp(-H/3421)+2.82}))]$$

$$\times \frac{R_R^2}{H^2+(1-H/F)^2(R_R)^2+(\lambda/\pi)(R_R/\rho_0^2)H^2}[10^{-9.455}\exp(-\frac{2}{1000}$$

$$\times (14.84-2785.831(1.404+\ln\frac{0.364}{2.82})))]^{-1}\} \ (\mathrm{dB}) \ . \qquad (14)$$

The relative SNR for the beam travelling along vertical direction, decreases more quickly than that along horizontal propagation ones. The reason is that the atmospheric molecules are more rarefied in the higher atmospheric layers.

In Fig.11, it shows the curves of the relative SNR versus the range in the case of the beam vertical propagation. The total reduction of relative SNR, corresponding to aerosol backward scattering absorption and extinction of the atmospheric CO_2 and H_2O, is about 50~ 80 dB lower than horizontal measuring results at 10000 m measuring range.

On the other hand, we find that the curves corresponding to the diameter 0.2 m and 0.3 m, respectively, are so close together each other in the region beyond 2 km, and the difference between them is about a few dB. Whatever, they are valid for both propagating in weak turbulence along horizontal direction or vertical direction. In this case, if we select the aperture's diameter 0.2 m, it is more adequate for economical requirement.

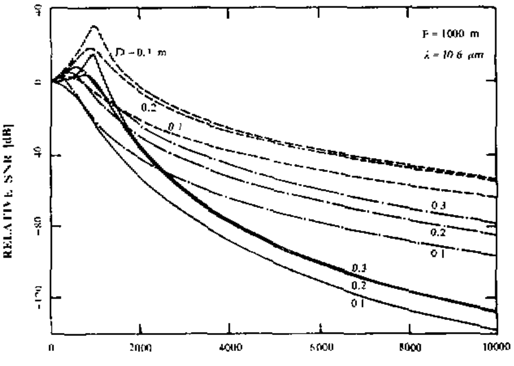

Fig. 11. Relative SNR versus range for different aperture. (A) (——) Beam travelling along vertical direction. (B) Beam travelling along horizontal direction (- - -) $C_n^2 = 3 \times 10^{-14}$ m$^{-2/3}$ (— • —) $C_n^2 = 3 \times 10^{-12}$ m$^{-2/3}$.

V. CONCLUSIONS

From the above discussions, some results can be summarized as follows:

(1) The cumulative process of coherent lidar's equation exists in the saturation phenomenon due to the nonlinear effects of diffraction–limited beam, atmospheric turbulence and cumulative process. It causes the cumulative SNR, relative SNR and percentile–range–resolution, etc., have different characteristics in the near–range and far–range.

(2) For a fixed aperture's diameter, when increasing focal length, it would make the distributive optical signal's field expand, percentile–volume increase, range–resolution worsen, and the relative SNR increase in farther range. While there are turbulent effects, the relative SNR decreases more rapidly with increasing of the focal length and the range–resolution worsens in far–range.

(3) For a fixed focal length, when increasing the aperture's diameter, it could make the diffraction–limited decrease and avails for the beams condense into far–range. It increases the percentile–volume, and could make the range–resolution worsen. There exists a diameter corresponding to the poorest range–resolution, and if the aperture is larger than this diameter; the range–resolution would be improved.

The relative SNR and range–resolution would be improved by increasing the aperture's diameter in near–range. Nevertheless, the relative SNR decreases more seriously and range–resolution worsens as increasing the measured range to far–range, even increasing the aperture's diameter.

(4) The aperture's diameter, focal length and atmospheric turbulent intensity match each other to achieve the optimal relative SNR of measurement and the needed range–resolution. The focal length and aperture's diameter could be determined by the peak's position of the relative SNR. A maximal migratory range, which confines a cusp point of the migratory range, is determined by matching the focal length and aperture's diameter.

(5) The poorest range–resolution and maximal percentile–volume have the similar mean-

ing, and they are defined with the same aperture in non-turbulence. In turbulence and fixed focal length, the range-resolution would be improved a little by small aperture, but it would worsen for the large aperture, relatively to non-turbulence.

(6) For the beam travelling along the ground vertical direction, the relative SNR reduces more seriously than that for the beam travelling along horizontal direction. The reason is that the atmospheric molecules are more rarefied in higher atmospheric layers, and it makes the return lidar's signals weaken. Therefore, it must require that lidar's receiver has higher receiving sensitivity, and the lidar's transmitter has stronger laser's transmitting power for the beam vertical measurement.

With the aid of the above analyses and instructions, an IR CO_2 Coherent Lidar System is designed and setup in our optical laboratory.

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