# The Water-Bearing Numerical Model and Its Operational Forecasting Experiments Part I: The Water-Bearing Numerical Model

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#### ABSTRACT

In first paper of articles, the physical and calculating schemes of the water-bearing numerical model are described. The model is developed by bearing all species of hydrometeors in a conventional numerical model in which the dynamic framework of hydrostatic equilibrium is taken. The main contributions are: the mixing ratios of all species of hydrometeors are added as the prognostic variables of model, the prognostic equations of these hydrometeors are introduced, the cloud physical framework is specially designed, some technical measures are used to resolve a series of physical, mathematical and computational problems arising from water-bearing; and so on.

The various problems (in such aspects as the designs of physical and calculating schemes and the composition of computational programme) which are exposed in feasibility test, in sensibility test, and especially in operational forecasting experiments are successfully resolved using a lot of technical measures having been developed from researches and tests. Finally, the operational forecasting running of the water-bearing numerical model and its forecasting system is realized stably and reliably, and the fine forecasts are obtained. All of these mentioned above will be described in second paper.

Key words: Water-Bearing, Numerical Forecasting Model, Cloud Physical Framework, Calculating Scheme

## I. INTRODUCTION

Both the cloud and precipitation forecasts are important and difficult problems in numerical weather prediction. In recent years, the simplest cloud physical schems have been introduced into some numerical models (Anthes, et al., 1987; Grell, et al., 1993) which are used in mesoscale weather research. The purpose is to calculate the complex physical processes of cloud and precipitation through a technical way approaching to the real atmosphere.

Based on the grid-nested N-level primitive equation model (Zheng, 1989) suitable for mesoscale weather research, we have developed a water-bearing numerical model (Xia, et al., 1995). The mixing ratios of all species of hydrometeors, such as cloud water, cloud ice, rain, snow and rain ice etc., are added as the prognostic variables of model. Correspondingly, the prognostic equations of these hydrometeors are introduced; the cloud physical framework is specially designed, in which almost all of the main cloud physical processes in real atmosphere are involved and the complex relations among cloud physical processes are demonstrated; and their calculating schemes are constructed and the computational programme is composed. Our final aims are to push the water-bearing numerical model into operational forecasting running, and to make the water-bearing numerical model having both the abilities of conventional and cloud field forecasts.

## II. BASIC EQUATIONS

σ is taken as the vertical coordinate of the water-bearing numerical model

$$\sigma = \frac{p - p_T}{p_s - p_T} = \frac{p - p_T}{\pi} \quad , \tag{2.1}$$

where p is the pressure;  $p_T = 100 \text{ hPa}$ , the pressure at the top of atmosphere;  $p_S$  the surface pressure. Taking the dynamic framework of hydrostatic equilibrium and considering the map factor m, the governing equations in plane-rectangular coordinate system are

$$\frac{\partial}{\partial t} \left( \frac{\pi v}{m} \right) = -m \left[ \frac{\partial}{\partial x} \left( \frac{\pi u}{m} v \right) + \frac{\partial}{\partial y} \left( \frac{\pi v}{m} v \right) \right] - \frac{\partial}{\partial \sigma} \left( \frac{\pi \dot{\sigma}}{m} v \right) - f \frac{\pi u}{m} 
- \pi \left( \frac{\partial \varphi}{\partial v} + \frac{\sigma RT}{\sigma \pi + \rho_{\pi}} \frac{\partial \pi}{\partial v} \right) + \frac{\pi}{m} F_{\nu} + \frac{\pi}{m} D_{\tau} ,$$
(2.3)

$$\frac{\partial}{\partial t} \left( \frac{\pi T}{m} \right) = -m \left[ \frac{\partial}{\partial x} \left( \frac{\pi u}{m} T \right) + \frac{\partial}{\partial y} \left( \frac{\pi v}{m} T \right) \right] - \frac{\partial}{\partial \sigma} \left( \frac{\pi \dot{\sigma}}{m} T \right)$$

$$+ \frac{\pi R T}{C_{p} (\sigma \pi + p_{T})} \left[ \frac{n \dot{\sigma}}{m} + \sigma \frac{\partial}{\partial t} \left( \frac{\pi}{m} \right) + \sigma \left( u \frac{\partial \pi}{\partial x} + v \frac{\partial \pi}{\partial y} \right) \right]$$

$$+\frac{\pi}{m}F_T + \frac{\pi}{m}D_T + \frac{\pi}{mC_p}(Q_L - Q_A)$$
, (2.4)

$$\frac{\partial}{\partial t} \left( \frac{\pi}{m} \right) = -m \int_0^1 \left[ \frac{\partial}{\partial x} \left( \frac{\pi u}{m} \right) + \frac{\partial}{\partial y} \left( \frac{\pi v}{m} \right) \right] d\sigma , \qquad (2.5)$$

$$\frac{\pi \dot{\sigma}}{m} = -m \int_{0}^{\sigma} \left[ \frac{\partial}{\partial x} \left( \frac{\pi u}{m} \right) + \frac{\partial}{\partial y} \left( \frac{\pi v}{m} \right) \right] d\sigma - \sigma \frac{\partial}{\partial t} \left( \frac{\pi}{m} \right) , \qquad (2.6)$$

$$\frac{\partial}{\partial t} \left( \frac{\partial \varphi}{\partial \sigma} \right) = -\frac{(1+1.608q_{_T})R_d}{(1+q_{_V}+q_{_W}+q_{_I}+q_{_R}+q_{_S}+q_{_H})(\sigma\pi+p_{_T})}$$

$$\times \left( \pi \frac{\partial T}{\partial t} + \frac{p_T T}{\sigma \pi + p_T} \frac{\partial \pi}{\partial t} \right), \tag{2.7}$$

where  $\dot{\sigma} = \frac{d\sigma}{dt}$  is the velocity in  $\sigma$ -direction;  $F_u$ ,  $F_v$  and  $F_T$  the vertical turbulent diffusion rates of momentum and heat, respectively;  $D_v$ ,  $D_v$  and  $D_T$  the horizontal turbulent diffusion rates of momentum and heat, respectively;  $Q_L$  the latent heating rate; and  $Q_A$  the sensible heat exchange rate. The equation of hydrostatic tendency (2.7) is derived by considering the

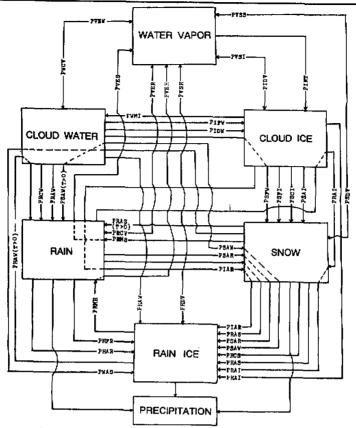


Fig. 1. The cloud physical framework of the water-bearing numerical model.

loading of water substances.  $q_V$ ,  $q_W$ ,  $q_I$ ,  $q_R$ ,  $q_S$  and  $q_H$  are the mixing-ratios of water vapor, cloud water, cloud ice, rain, snow and rain ice, respectively. Other symbols are conventional in meteorology.

The mixing-ratios of all species of hydrometeors are taken as the prognostic variables of the water-bearing numerical model. Cloud water and cloud ice are referred to as cloud particles. They suspend in the air and drift with currents. Rain, snow and rain ice are referred to as precipitation particles. They are falling with their terminal velocities. The prognostic equations of all species of water substance can be written in a unified form

$$\frac{\partial}{\partial t} \left( \frac{\pi q}{m} \right) = -m \left[ \frac{\pi u}{m} \frac{\partial q}{\partial x} + \frac{\pi v}{m} \frac{\partial q}{\partial y} \right] - \frac{\pi \dot{\sigma}}{m} \frac{\partial q}{\partial \sigma} + q \frac{\partial}{\partial t} \left( \frac{\pi}{m} \right) 
- \delta \frac{g}{m} \frac{\partial}{\partial \sigma} (\rho q W) + \frac{\pi}{m} P ,$$
(2.8)

where q is the mixing-ratio of a species of water substance;  $\delta=0$  for water vapor, cloud water and cloud ice;  $\delta=1$  for rain, snow and rain ice;  $\rho$  the air dencity; W the terminal falling velocity; and P the generation rate of a species of water substance. The precipitation substance falling out of the lowest layer of model atmosphere is accumulated as the precipitation. The latent heat release and the sensible exchange in the processes of cloud and precipitation are

Table 1. Key to Figure 1

Symbol	Meaning
PHAI	Accretion of cloud ice by rain ice (Wisner, et al., 1972; Orville, et al., 1977).
PHAR	Accretion of rain by rain ice (Wisner, et al., 1972; Orville, et al., 1977).
PHAS	Accretion of snow by rain ice (Wisner, et al., 1972; Orville, et al., 1977).
PHAW	Accretion of cloud water by rain ice (Wisner, et al., 1972; Orville, et al., 1977).
PHCS	Autoconversion of snow to form rain ice (Kessler, 1969).
PHDV	Depositional growth of rain ice (Bigg, 1953; Byers, 1965).
PHFR	Probabilistic freezing of rain to form rain ice (Bigg, 1953).
PIAR	Accretion of rain by cloud ice: produces snow or rain ice depending
	on the amounts of rain (Lin, et al., 1983; Rutledge, et al., 1984).
PIDV	Depositional growth of cloud ice (Byers, 1965; Murry, 1967).
PIDW	Depositional growth of cloud ice at expense of cloud water (Koinig, 1971; Lin, et al., 1983).
PIFW	Homogeneous freezing of cloud water to form cloud ice (Rutledge, et al., 1984).
PINT	Initiation of cloud ice (Rutledge, et al., 1983).
PRAI	Accretion of cloud ice by rain: produces snow or rain ice depending on the amounts
	of rain (Wisner, et al., 1972; Orville, et al., 1977; Rutledge, et al., 1984).
PRAS	Accretion of snow by rain; produces rain ice if rain and snow exceed threshold and $T \le 273.15$ K.
	or rain if $T > 273.15$ K (Wisner, et al., 1972; Orville, et al., 1977; Rutledge, et al., 1984).
PRAW	Accretion of cloud water by rain (Wisner, et al., 1972; Orville, et al., 1977).
PRCV	Condensational growth of rain (Byers, 1965; Orville, et al., 1977).
PRCW	Autoconversion of cloud water to form rain (Kessler, 1969).
PRMH	Melting of rain ice to form rain (Wisner, et al., 1972).
PRMS	Melting of snow to form rain (Wisner, et al., 1972).
PSAI	Accretion of cloud ice by snow (Wisner, et al., 1972; Orville, et al., 1977).
PSAR	Accretion of rain by snow: produces snow or rain ice depending on the amounts
	of snow and rain (Wisner, et al., 1972; Orville, et al., 1977; Rutledge, et al., 1984).
PSAW	Accretion of cloud water by snow; produces snow or rain ice depending on the amounts
	of snow and cloud water if $T \le 273.15$ K, or rain if $T > 273.15$ K (Wisner, et al.,
	1972; Orviffe, et al., 1977; Rutledge, et al., 1984).
PSCI	Autoconversion of cloud ice to form snow (Kessler, 1969).
PSDV	Depositional growth of snow (Byers, 1965; Lin, et al., 1983).
PSF1	Transfer rate of cloud ice to snow through growth of Bergeron process embryos
	(Hsie, et al.;1980; Lin, et al., 1983).
PSFW	Bergeron process (deposition and riming)—transfer of cloud water to form snow
DVE11	(Fletcher, 1962; Koinig, 1971; Lin, et al., 1983).
PVEH	Evaporation of melted rain ice (Rutledge, et al., 1983; Rutledge, et al., 1984).  Evaporation of rain (Byers, 1965; Orville, et al., 1977).
PVER	Evaporation of rain (byers, 1903, Orvine, et al., 1977).  Evaporation of melted snow (Rutledge, et al., 1983; Rutledge, et al., 1984).
PVES	
PVEW	Evaporation of cloud water (Asai, 1965).
PVSH	Sublimation of rain ice (Bigg, 1953; Byers, 1965).
PVSI	Sublimation of cloud ice (Byers, 1965; Murry, 1967).
PVSS	Sublimation of snow (Byers, 1965; Lin, et al., 1983).
PWAS	Accretion of snow by cloud water; produces rain ice if cloud water and
	snow exceed threshold (Rutledge, et al., 1984).
PWCV	Condensation of water vapor to form cloud water (Asai, 1965).
PWMI	Melting of cloud ice to form cloud water (Rutledge, et al., 1984).

## III. PHYSICAL PROCESSES

Some basic physical processes being considered in conventional numerical model have been included in the water-bearing numerical model, such as the horizontal turbulent diffusions and vertical turbulent transportations of momentum and heat, the turbulent exchanges of momentum and heat between air and underlying surface, grid-scale precipitation calculated by diagnostic explicit method, subgrid-scale precipitation calculated by parameterization, and so on. In the case of water-bearing, the grid-scale precipitation is calculated by prognostic explicit method. The various physical contents involved in the water-bearing numerical model can be selectively composed and used according to the requirements of prediction and research.

The comprehensive cloud physical framework in the water-bearing numerical model is specially designed according to the cloud physical image evoluting from cloud emergence to precipitation. At most, 36 cloud physical processes, e.g. condensation/evaporation, freezing/melting, deposition/sublimation, auto-conversion and accretion, etc., may be involved in the framework. Furthermore, three cloud physical schemes, i.e. warm cloud scheme, cold cloud scheme and complete cloud scheme, are constructed and may be selectively used according to various research purposes and forecasting objects. They are:

## 1. Complete Cloud Scheme

It involves 6 species of water substances, i.e. water vapor, cloud water, cloud ice, rain, snow and rain ice, and considers 36 cloud physical processes.

## 2. Cold Cloud Scheme

It involves 5 species of water substances, i.e. water vapor, cloud water, cloud ice, rain and snow, and considers 25 cloud physical processes.

## 3. Warm Cloud Scheme

It involves 3 species of water substances, i.e. water vapor, cloud water and rain, and considers 6 cloud physical processes.

The water substances are conservative in designing all of these cloud physical schemes.

The complete cloud scheme, for example, is shown in Fig 1. And the symbol presentations of transfer rates of water substances for 36 cloud physical processes are listed in Table 1. Then, the generation rates of water vapor, cloud water, cloud ice, rain, snow and rain ice  $P_V$ ,  $P_W$ ,  $P_I$ ,  $P_R$ ,  $P_S$  and  $P_H$  are given as follows:

$$P_{V} = -\alpha_{1} PHDV - \alpha_{1} PIDV - \alpha_{1} PINT - PRCV - \alpha_{1} PSDV + (1 - \alpha_{1}) PVEH + PVER + (1 - \alpha_{1}) PVES + \alpha_{2} PVEW + \alpha_{1} PVSH + \alpha_{1} PVSI + \alpha_{1} PVSS - \alpha_{2} PWCV ,$$
(3.1)

$$P_{W} = -\alpha_{2}PHAW - \alpha_{1}\alpha_{2}PIDW - \alpha_{1}(1-\alpha_{2})PIFW - \alpha_{2}PRAW -\alpha_{2}PRCW - \alpha_{2}PSAW - \alpha_{1}\alpha_{2}PSFI - \alpha_{1}\alpha_{2}PSFW -\alpha_{2}PVEW + \alpha_{2}PWCV + (1-\alpha_{1})PWMI,$$
(3.2)

$$P_{1} = -\alpha_{1}PHAI + \alpha_{1}PIDV + \alpha_{1}\alpha_{2}PIDW + \alpha_{1}(1 - \alpha_{2})PIFW + \alpha_{1}PINT - \alpha_{1}PRAI - \alpha_{1}PSAI - \alpha_{1}PSCI - \alpha_{1}PVSI - (1 - \alpha_{1})PWMI,$$
(3.3)

$$P_{R} = -\alpha_{1}PHAR + (1 - \alpha_{1})\alpha_{2}PHAW - \alpha_{1}PHFR - \alpha_{1}PIAR + (1 - \alpha_{1})PRAS + \alpha_{2}PRAW + PRCV + \alpha_{2}PRCW + (1 - \alpha_{1})PRMH + (1 - \alpha_{1})PRMS - \alpha_{1}PSAR + (1 - \alpha_{1})\alpha_{2}PSAW - (1 - \alpha_{1})PVEH - PVER - (1 - \alpha_{1})PVES ,$$
(3.4)

$$P_{S} = -PHAS - \alpha_{1}PHCS + \alpha_{1}(1 - \beta_{1})PIAR + \alpha_{1}(1 - \beta_{1})PRAI$$

$$-[1 - \alpha_{1}(1 + \beta_{2})]PRAS - (1 - \alpha_{1})PRMS + \alpha_{1}PSAI$$

$$+ \alpha_{1}(1 - \beta_{2})PSAR + \alpha_{1}\alpha_{2}(1 - \beta_{3})PSAW + \alpha_{1}PSCI + \alpha_{1}PSDV$$

$$+ \alpha_{1}\alpha_{1}PSFI + \alpha_{1}\alpha_{2}PSFW - \alpha_{1}PVSS - \alpha_{1}\alpha_{2}\beta_{3}PWAS , \qquad (3.5)$$

$$P_{H} = \alpha_{1} PHAI + \alpha_{1} PHAR + PHAS + \alpha_{1} \alpha_{2} PHAW + \alpha_{1} PHCS + \alpha_{1} PHDV + \alpha_{1} PHFR + \alpha_{1} \beta_{1} PIAR + \alpha_{1} \beta_{1} PRAI + \alpha_{1} \beta_{2} PRAS - (1 - \alpha_{1}) PRMH + \alpha_{1} \beta_{2} PSAR + \alpha_{1} \alpha_{2} \beta_{3} PSAW - \alpha_{1} PVSH + \alpha_{1} \alpha_{2} \beta_{3} PWAS ,$$

$$(3.6)$$

where

$$\alpha_1 = \begin{cases} 0 & \text{if } T > 273.15 \text{ K} \\ 1 & \text{if } T \le 273.15 \text{ K} \end{cases}$$
 (3.7)

$$\alpha_2 = \begin{cases} 1 & \text{if } T \ge 233.15 \text{ K} \\ 0 & \text{if } T \le 233.15 \text{ K} \end{cases}$$
 (3.8)

$$\beta_1 = \begin{cases} 1 & \text{if } q_R > 0.1 \text{ g} \cdot \text{kg}^{-1} \\ 0 & \text{otherwise} \end{cases}$$
 (3.9)

$$\beta_2 = \begin{cases} 1 & \text{if } q_R > 0.1 \text{ g} \cdot \text{kg}^{-1} & \text{and } q_S > 0.1 \text{ g} \cdot \text{kg}^{-1} \\ \text{otherwise} \end{cases}$$
 (3.10)

$$\beta_3 = \begin{cases} 1 & \text{if } q_S > 0.1 \text{ g} \cdot \text{kg}^{-1} & \text{and } q_W > 0.5 \text{ g} \cdot \text{kg}^{-1} \\ 0 & \text{otherwise} \end{cases}$$
 (3.11)

(3.1)+(3.2)+(3.3)+(3.4)+(3.5)+(3.6), then

$$P_{\nu} + P_{\mu\nu} + P_{I} + P_{R} + P_{S} + P_{H} = 0 . {3.12}$$

The water substances are conservative.

According to those basic facts revealed by cloud physical observations, theoratical and experimental researches, the emergent conditions for every cloud physical process are determined one by one. And the complex relations of both mutual promotion and mutual restriction among various cloud physical processes are carefully demonstrated in the arrangements of computational procedure.

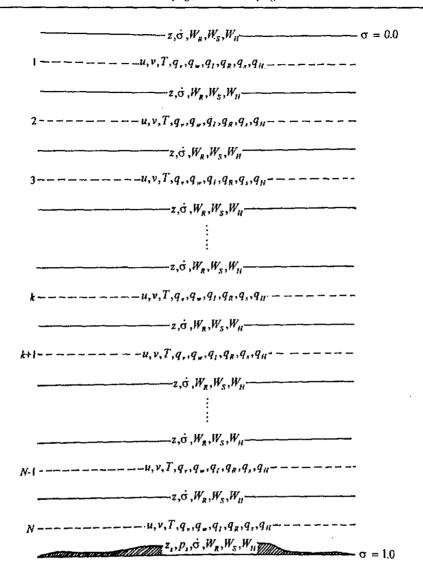


Fig. 2. The configuration of variables in σ-direction for the water-bearing numerical model.

# IV. SPACE-DISPERSED FORMS OF DIFFERENTIAL EQUATIONS

# 1. Structure of Grid Network and Configuration of Variables

The horizontal and vertical resolutions of the water-bearing numerical model can be regulated. The forecasting area can be easily changed according to the requirements. The

model atmosphere is evenly divided in  $\sigma$ -direction, and the vertical distribution of variables is staggered as shown in Figure 2. The Arakawa A scheme is temporarily taken for horizontal distribution of variables.

The duplicate grid-nested structure is arranged in horizontal. When the non grid-nested network is taken, the water-bearing numerical model or conventional numerical model may be run. And when the grid-nested structure is taken, the conventional numerical model is run in the coarse grids, and the water-bearing numerical model or the conventional numerical model is run in the fine grids.

## 2. Space-Dispers Forms of Differential Equations

Through strict mathematical derivations, the space-difference schemes for the governing equations are obtained, which are of conservative properties for mass, momentum and energy. The difference forms of dynamic-potential energy transformation terms in the equations of momentum and thermodynamics are concerted and consistent each other. The space-difference schemes of flux terms in two-, four- and six-order computational accuracy are constructed for rising the accuracy in those difference calculations of non-linear terms. These difference schemes may be used selectively.

For a variable A and in x-direction, as an example and for writing easily, the following operators are appointed:

$$\overline{A_i}^{\ \ } = \frac{1}{2} \left( A_{i+\frac{1}{2}} + A_{i-\frac{1}{2}} \right) \qquad \qquad \delta_{\ \ } A = \frac{1}{d} \left( A_{i+\frac{1}{2}} - A_{i-\frac{1}{2}} \right)$$

where d is the grid space. Then, the time-differential and space-difference forms of equations (2.2)—(2.7) are given as follows:

$$\frac{\partial}{\partial t} \left( \frac{\pi u}{m} \right) \approx -m \left[ \delta_x \left( \frac{\overline{\pi u}}{m} \overline{u}^x \right) + \delta_y \left( \frac{\overline{\pi v}}{m} \overline{u}^y \right) \right] - \delta_\sigma \left( \frac{\pi \dot{\sigma}}{m} \overline{u}^\sigma \right) + f \frac{\pi v}{m} \\
- \pi \left( \delta_x \overline{\phi}^{\sigma^{-\lambda}} + \frac{\sigma R}{\sigma \pi + p_T} \overline{T}^x \delta_x \overline{\pi}^x \right) + \frac{\pi}{m} F_u + \frac{\pi}{m} D_u \quad , \tag{4.1}$$

$$\frac{\partial}{\partial t} \left( \frac{\pi v}{m} \right) \approx -m \left[ \delta_{x} \left( \frac{\overline{\pi u}^{x}}{m} \overline{v}^{x} \right) + \delta_{y} \left( \frac{\overline{\pi v}^{x}}{m} \overline{v}^{y} \right) \right] - \delta_{\sigma} \left( \frac{\pi \dot{\sigma}}{m} \overline{v}^{\sigma} \right) - f \frac{\pi u}{m} \\
- \pi \left( \delta_{y} \overline{\overline{\phi}^{\sigma}}^{y} + \frac{\sigma R}{\sigma \pi + p_{x}} \overline{T}^{y} \delta_{x} \overline{\pi}^{y} \right) + \frac{\pi}{m} F_{y} + \frac{\pi}{m} D_{y} \quad .$$
(4.2)

$$\frac{\partial}{\partial t} \left( \frac{\pi T}{m} \right) \approx -m \left[ \delta_{x} \left( \frac{\overline{\pi u}^{x}}{m} \, \overline{T}^{x} \right) + \delta_{y} \left( \frac{\overline{\pi v}^{y}}{m} \, \overline{T}^{y} \right) \right] - \delta_{\sigma} \left( \frac{\pi \sigma}{m} \, \overline{T}^{\sigma} \right) 
+ \frac{\pi R T}{C_{\rho} (\sigma \pi + \rho_{T})} \left[ \frac{n \dot{\sigma}}{m} + \sigma \frac{\partial}{\partial t} \left( \frac{\pi}{m} \right) \right] 
+ \frac{\sigma \pi R}{C_{\mu} (\sigma \pi + \rho_{T})} \left( u \, \overline{T}^{x} \, \delta_{x} \, \overline{\pi}^{x} + v \, \overline{T}^{y} \, \overline{\delta_{y}} \, \overline{\pi}^{y} \right) 
+ \frac{\pi}{m} F_{T} + \frac{\pi}{m} D_{T} + \frac{\pi}{m C_{\rho}} (Q_{L} - Q_{A}) .$$
(4.3)

$$\frac{\partial}{\partial t} \left( \frac{\pi}{m} \right) \approx -\sum_{k=1}^{N} \left( \delta_{\chi} \frac{\overline{\pi u}^{\chi}}{m} + \delta_{\chi} \frac{\overline{\pi v}^{r}}{m} \right)_{k} \Delta \sigma . \tag{4.4}$$

$$\left(\frac{\pi\dot{\sigma}}{m}\right)_{k+\frac{1}{2}} \approx \left(\frac{\pi\dot{\sigma}}{m}\right)_{k-\frac{1}{2}} - \left[m\left(\delta_x \frac{\overline{\pi u}^x}{m} + \delta_x \frac{\overline{\pi v}^y}{m}\right)_k + \frac{\partial}{\partial t} \left(\frac{\pi}{m}\right)\right] \Delta\sigma , \qquad (4.5)$$

$$k = 1, 2, \dots, N.$$

$$\frac{\hat{c}}{\partial t} \left( \frac{\partial \varphi}{\partial \sigma} \right) \approx \left[ \frac{(1 + 1.608q_{\perp})R_d}{(1 + q_{\perp} + q_{\parallel} + q_{\parallel} + q_{\parallel} + q_{\parallel} + q_{\parallel})(\sigma \pi + p_{\perp})} \right]^{\prime} \times \left( \pi \delta_t T + \frac{p_T T}{\sigma \pi + p_{\perp}} \delta_t \pi \right).$$
(4.6)

The upstream scheme is used for the difference of advection terms in the prognostic equations of water substances. Although this space-difference scheme is of only one-order computational accuracy, the water substances can be ensured non-negative in time integrations. Then, the time-differential and space-difference form of Eq. (2.8) is

$$\frac{\partial}{\partial t} \left( \frac{\pi q}{m} \right) \approx -m \left( \frac{\overline{\pi u}}{m} \delta_{v} q + \frac{\overline{\pi v}}{m} \delta_{v} q \right) - \frac{\pi \dot{\sigma}}{m} \delta_{\sigma} q + q \frac{\partial}{\partial t} \left( \frac{\pi}{m} \right) - \frac{\delta_{m}^{g}}{m} \delta_{\sigma} (\rho q W) + \frac{\pi}{m} P . \tag{4.7}$$

For example, the upstream scheme for the advection term of water substance in x-direction is

$$\frac{\overline{\pi u}}{m} \delta_{v} q = \begin{cases}
\frac{1}{2} \left[ \left( \frac{\pi u}{m} \right)_{i-1} + \left( \frac{\pi u}{m} \right)_{i} \right] \frac{q_{i} - q_{i-1}}{d} & u > 0 \\
\frac{1}{2} \left[ \left( \frac{\pi u}{m} \right)_{i} + \left( \frac{\pi u}{m} \right)_{i+1} \right] \frac{q_{i+1} - q_{i}}{d} & u < 0
\end{cases} \tag{4.8}$$

## V. BOUNDARY CONDITIONS AND INITIAL VALUES

# 1. Boundary Conditions at Top and Bottom of Atmosphere

In  $\sigma$ -coordinate, the boundary surfaces at top and bottom of model atmosphere are dealt as substance surfaces, and the boundary conditions are

$$\dot{\sigma} = 0$$
 if  $\sigma = 1$  or 0 (5.1)

# 2. Lateral Boundary Conditions of Limited Area

Whatever the lateral boundary conditions taken for the numerical models on limited area are artificial and hardly ideal. The simplest lateral boundary condition, i.e. the fixed lateral boundary condition, is taken for the water-bearing numerical model. For a variable A, the lateral boundary condition is defined as

$$\left. \frac{\partial A}{\partial t} \right|_{\Gamma} = 0 \quad . \tag{5.2}$$

In order to diminish those abuses arising from the fixed lateral boundary condition, the following measures may be taken: making Davies boundary treatment, increasing the horizontal diffusion coefficients near the lateral boundary, making boundary smoothing, and so on.

When the grid network is nested, the initial values of conventional elements in fine-grid area are interpolated by those in coarse-grid area and the values on inner boundary are periodically replaced by the forecasting values in coarse-grid area through interpolations as finished every fixed interval of time integrations. All these interpolations mentioned above are conducted using the double-cubic spline function. So, the values on the inner boundary of forecasting area are varied with time.

## 3. Initial Values and Initializations

The primitive equation model is sensible to initial values. The problem of initial values, obviously, is more important for a limited—area model being used for short-term forecasts.

## i. Initial values of conventional elements

The high quality inital values of conventional meteorological elements, e.g. winds, gravitational potential heights, temperatures and relative humidities etc., are provided using the specially designed schemes of data-pocessing and objective analysis.

The surface pressure is calculated using the empirical relation of the potential height  $Z_0$  and the surface pressure  $p_r$  if no topography is taken into consideration, i.e.

$$p_{S} = 0.125Z_0 + 1000 {.} {(5.3)}$$

Otherwise, the quadratic relation between terrestrial hight  $Z_S$  and  $p_S$  is assumed

$$Z_S = \alpha - \beta \ln p_S + \gamma (\ln p_S)^2 ,$$

then

$$p_S = \exp\left[\frac{2(\alpha - Z_S)}{\beta + \sqrt{\beta^2 - 4\gamma(\alpha - Z_S)}}\right], \qquad (5.4)$$

where three coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  can be calculated from the potential heighs at three mandatory pressure surfaces nearest to the underlying surface.

## ii. Initial values of hydrometeors

Through researches and tests, two initial value schemes of hydrometeors are developed, based on the coventional meteorological data and on the synthetical analysis of conventional data and GMS digital data (will be described in another paper) respectively.

## iii. Initializations

There is still certain inharmony among the initial values of various elements obtained from data processing and objective analysis, and so does between those initial values and the numerical model. Generally, these inharmonies produce "noises" in the early stage of time integrations, which have no meteorological meanings and are unfavourable to the model's stable calculations and are harmful to the forecasts. So, it is necessary to make certain modifications and adjustments for the initial values of meteorological elements before time integrations. According to the adaptation theory, the high quality initial values of winds are of much importance for the research and forecast of mesoscale weather systems.

## 1) Adjustments and Modifications for Initial Values

In the water-bearing numerical model, some adjustments and modifications may be selectively conducted for temperatures and winds if necessary, e.g. the dry convective adjustments for temperature stratification; and several modifications of winds to assure the currents flowing around the terrains, to include the effects of precipitation fallen previous to the initial winds, to reach non-divergence in whole the volume, and so on.

## 2) Static Initialization

The unknown initial values of meteorological elements may be derived from the known ones using certain diagnostic relations among these meteorological elements. For instance, the initial values of wind, gravitational potential height and temperature may be derived from the relations of geostrophic wind or balance wind, and that of hydrostatic equilibrium.

## 3) Dynamic Initialization

The purposes are to reach adaptation between those initial values of various elements and the numerical model and to pull out "noises" arising from inadaptations in early stage of time integrations.

The basic equations of dry model which has no source and sink are repeatedly integrated foreward and backward around the initial time, using Euler backward difference scheme which has damping effects for short waves in time integrations. After complishing every cycle of time differences, either the initial potential height field is recovered if taking the large—scale weather systems as the objects of research and forecast, or the initial wind field is recovered if taking the mesoscale weather systems as the objects of research and forecast. Using this method mentioned above, the initial values of all other elements are adjusted to initial potential height field or wind field.

## VI. TIME INTEGRATIONS

There are two schemes of time integrations may be selectively used for the governing equations. They are the scheme of Euler backward difference and the mixing scheme of Euler backward difference and central difference. The former is capable of damping those fast moving waves but expensing too much computational time, and the latter is taken into consideration of two aspects of maintaining computational stability and saving computational time. To avoid the seperation of resolutions, the time smoothing for various forecasting variables is conducted in central difference.

The foreward difference is conducted in the time integrations of the prognostic equations

of water substances. Combining with the upstream scheme of space difference, the computations are stable.

## VII. SUMMARY

For examing the feasibility of water-bearing in numerical model, the sensibility of reflecting the variations in atmospheric environment and the computational stability of the model, a series of tests have been conducted after preliminary accomplishing the designs of physical and calculating schemes and the composition of computational programme (Xu, et al., will be published). General speaking, the results of these tests are satisfactory. The essential prerequisite is created for hereafter experiments, from which the operational forecasting running of the water-bearing numerical model is realized.

## REFERENCES

Anthes, R.A., E.-Y.Hsie, and Y.H.Kuo (1987), Description of the Penn State / NCAR mesoscale model version (MM4), NCAR / TN-282+ATR, National Center for Atmospheric Research, Boulder, Colorado, 66 pp.

Asai, T. (1965), A numerical study of the air-mass transformation over the Japan Sea in winter, J. Met. Soc. Japan, 43: 1-15.

Bigg, E.E. (1953). The supercooling of water, Proc. Phys. Soc., London, B66, 668-694.

Byers, H.R. (1965), Elements of cloud physics, The University of Chicago Press, 191 pp.

Fletcher, N.H. (1962). The physics of rain clouds, Cambridge University Press, 390 pp.

Grell, G. A., J. Dudhia, and D. R. Stauffer (1993), A description of the fifth-generation Ponn State / NCAR mesoscale model (MM5), NCAR / TN-398+1A, National Center for Atmospheric Research, Boulder, Colorado, 107 pp.

Hsie, E. Y., R. D. Farley, and H. D. Orville (1980). Numerical simulation of ice-phase convective cloud seeding, J. Appl. Met., 19: 950-977.

Kessler, E. III. (1969). On the distribution and continuity of water substance in atmospheric circulation. Meteor. Monogr., 10(32): 84 pp.

Koenig, L. R. (1971), Numerical modeling of ice deposition, J. Atmos. Sci., 28: 226-237.

Lin, Y. L., R. D. Farley, and H. D. Orville (1983), Bulk parameterization of the snow field in a cloud model, J. Appl. Met., 22: 1065-1092.

Murry, F. W. (1967), On the computation of saturation vapour pressure, J. Appl. Met., 6: 203-204.

Orville, H. D., and F. J. Kopp (1977), Numerical simulation of the life history of a hailstorm, J. Atmos. Sci., 34: 1596-1618.

Rutledge, S. A., and P. V. Hobbs (1983). The mesoscale and microscale structure and organization of cloud and precipitation in midlatitude cyclones, VIII: A model for the "seeder-feeder" process in warm-frontal rainbands, J. Atmos. Sci., 40: 1185-1206.

Rutledge, S. A., and P. V. Hobbs (1984). The mesoscale and microscale structure and organization of cloud and precipitation in midlatitude cyclones, XII: A diagnostic modeling study of precipitation development in narrow cloud-frontal rainbands, J. Atmos. Sci., 41: 2949-2972.

Wisner, C. E., H. D. Orville, and C. G. Myers (1972), A numerical model of a hail bearing cloud, J. Atmos. Sci., 29: 1160-1181.

Xia, D. Q., and Y. P. Xu (1995). The grid-nested, water-bearing, N-level primitive equation model, International Workshop on Limited-Area and Variable Resolution Models. WMO / TD-No. 699, 203-208.

Xu, Y. P., and D. Q. Xia, The water-bearing numerical model and its operational forecasting experiments, Part II: Operational forecasting experiments, Advances in Atmospheric Sciences (will be published).

Zheng, L. J. (1989). Diagnostic analysis and numerical simulations for mesoscale weather systems. China Meteorological Press, Beijing, 198 pp (in Chinese).