

An Analytical Study on the Urban Boundary Layer^①

Lin Naishi (林乃实), Zhou Zugang (周祖刚), Zhou Liufei (周六飞)

Air Force Institute of Meteorology, Nanjing 211101

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ABSTRACT

A two-layer model based on the linearized time-independent atmospheric dynamical equations is proposed in this paper. The analytical solutions of the vertical, the horizontal motions and the potential temperature field induced by the anthropogenic source of urban surface heating are obtained, therefore the heat island circulation existing in unstable boundary layer is verified theoretically. From the analytical solutions, some conclusions can be drawn.

(1) The vertical motion induced by urban heat island consists of two parts, namely, the cross-hill wave and the lee wave;

(2) The cross-hill wave only exists in the unstable boundary layer, and varies with height according to exponential function law;

(3) The vertical motion induced by heat island reaches the maximum at the top of the unstable boundary layer;

(4) The wave generated by heat island not only propagates to the downwind district but also travels to the upwind area;

(5) $\gamma \neq 0$ is not the necessary condition of the lee wave generation.

Key words: Urban boundary layer, Heat island, Cross-hill wave

1. INTRODUCTION

It is known that heat island is one of the important characteristics of the urban boundary layer. Most cities are anthropogenic sources of heat and pollution. In addition, the downtown areas are covered by a large percentage of asphalt and concrete, which are usually dry water-proof surfaces with albedoes and heat capacities that convert and store incoming radiation as sensible heat better than the surrounding countryside. Thus, surface layer air in cities is generally warmer than that of their surroundings. When isotherms are plotted on a surface weather map, the pattern looks like the topographic contours of an island, whence the term heat island. Heat island circulation exists in not only stable but also daily unstable urban boundary layers, which has been verified by a large quantity of atmospheric observation data above city (Angell, et al., 1973) and the observation analyses about convective activities (Auer, et al., 1974; Uthe, et al., 1974). Angell, et al. (1973) revealed that the atmospheric motion above city is in a state of wave, the peaks of which appear in the downwind of the city, which are induced by heat island.

It is found that the top of mixed layer is forced to go higher, the highest peak of which appears in the downwind areas of the city, in which the convective activities and precipitation phenomena are triggered based on the observations at home and abroad (Ochs, 1975; Dirks, 1974).

Sang (1988) and Li (1992) studied the heat island circulation in unstable boundary layer under some simplifications. In this paper the effects of urban heating and boundary layer

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temperature field on the formation of heat island circulation are investigated with the similar method of Sang (1988). In the process of deduction, some hypotheses distorting the solutions of the partial differential equations are avoided, thus the heat island circulation structure is analyzed theoretically. Therefore, not only an insight into the physical mechanism of the heat island circulation but also some clues useful for the field experiment and numerical modelling can be gained.

II. TWO-LAYER MODEL AND ITS ANALYTICAL SOLUTIONS

In order to deal with the heat island circulation, we consider a two-layer fluid model. Assume that the lower layer is the unstable atmospheric boundary layer and that the upper layer above the boundary layer is stable, and the interface between the above two layers is fixed at the top of the boundary layer (Fig. 1).

As to the small scale motion such as the heat island, it is reasonable to ignore the Coriolis force and the motion in y -direction, furthermore it is also assumed to be time-independent of the motion. So the linearized, two-dimensional atmospheric thermal and dynamical equations in the lower layer are as follows:

$$\begin{cases} \bar{u} \frac{\partial u}{\partial x} = -\bar{\theta} \frac{\partial \pi}{\partial x}, & (1) \end{cases}$$

$$\begin{cases} \bar{u} \frac{\partial w}{\partial x} = -\bar{\theta} \frac{\partial \pi}{\partial z} + \frac{\theta}{\bar{\theta}} g, & (2) \end{cases}$$

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, & (3) \end{cases}$$

$$\begin{cases} \bar{u} \frac{\partial \theta}{\partial x} + w \frac{\partial \bar{\theta}}{\partial z} = \frac{\bar{\theta}}{T} \frac{V}{c_p}, & (4) \end{cases}$$

where $\bar{u}, \bar{\theta}, \bar{T}$ are the mean values, and satisfying $\frac{\partial \bar{u}}{\partial x} = \frac{\partial \bar{u}}{\partial z} = \frac{\partial \bar{\theta}}{\partial x} = 0$, $\theta = \theta(z)$. u, w, π, θ are the perturbations induced by heat island. $\bar{\pi} + \pi = c_p (p/p_0)^{R/c_p}$ is Enter function, and satisfies

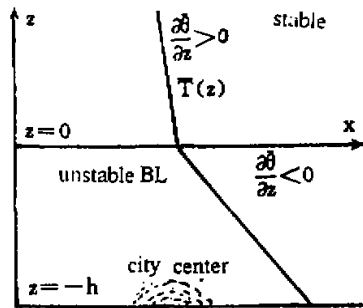


Fig. 1. Sketch of model structure.

$$-\bar{\theta} \frac{\partial \bar{\pi}}{\partial z} - g = 0.$$

With u, π, θ cancelled in above equations, the vertical speed perturbation w in unstable boundary layer can be obtained as following:

$$\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial z^2} - \frac{1}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z} \frac{\partial w_1}{\partial z} + \frac{g}{u^2} \frac{1}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z} w_1 = \frac{gV}{c_p u^2 T}. \quad (5)$$

In order to avoid distorting the above partial differential equation solution, no any simplifications here are assumed, namely maintaining the third term, which is different from the manipulation taken by Sang (1988).

The lower boundary layer is unstable, so $\frac{\partial \bar{\theta}}{\partial z} < 0$, thus assume $n^2 = -\frac{g}{u^2} \frac{1}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z}$,

$\tau_n = -\frac{1}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z} > 0$, and $\frac{gV}{c_p u^2 T} = Q \frac{b^2}{b^2 + x^2} e^{-a(z+h)}$, where Q is the heating magnitude in the city center, then Eq. (5) may be rewritten as

$$\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial z^2} + \tau_n \frac{\partial w_1}{\partial z} - n^2 w_1 = Q \frac{b^2}{b^2 + x^2} e^{-a(z+h)}. \quad (6)$$

As to upper stable layer, similar equation as Eq. (6) can be given. Considering the heating intensity is very small above the top of boundary layer, then $V = 0$, $\frac{\partial \bar{\theta}}{\partial z} > 0$, furthermore,

$l^2 = \frac{g}{u^2} \frac{1}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z}$ and $\tau_l = \frac{1}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z} > 0$ may be assumed, then we have

$$\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial z^2} - \tau_l \frac{\partial w_2}{\partial z} + l^2 w_2 = 0 \quad 0 \leq z < \infty. \quad (7)$$

Assuming $w_1 = \tilde{w}_1 e^{ikx}$, $w_2 = \tilde{w}_2 e^{ikx}$, and inserting them into Eqs. (6), (7), then

$$\frac{d^2 \tilde{w}_1}{dz^2} + \tau_n \frac{d\tilde{w}_1}{dz} - (k^2 + n^2) \tilde{w}_1 = Q b e^{-hk} e^{-a(z+h)} \quad -h \leq z \leq 0, \quad (8)$$

$$\frac{d^2 \tilde{w}_1}{dz^2} - \tau_l \frac{d\tilde{w}_2}{dz} - (k^2 - l^2) \tilde{w}_2 = 0 \quad 0 \leq z < \infty, \quad (9)$$

where $b e^{-hk}$ is the Fourier element of the function $\frac{b^2}{b^2 + x^2}$.

The general solutions of Eqs. (8), (9) are

$$\tilde{w}_1 = A e^{\lambda_1 z} + B e^{\lambda_2 z} + Q \frac{b e^{-hk}}{(a + \lambda_1)(a + \lambda_2)} e^{-a(z+h)}, \quad (10)$$

$$\tilde{w}_2 = C e^{\mu_2 z} + D e^{\mu_1 z} \quad k > l^*,$$

$$\tilde{w}_2 = e^{\frac{\tau_l}{2} z} (C_0 \cos \nu z + D_0 \sin \nu z) \quad k < l^*, \quad (11)$$

where $\lambda_{1,2} = -\frac{\tau_n}{2} \pm \sqrt{(\frac{\tau_n}{2})^2 + k^2 + n^2}$, $\mu_{1,2} = \frac{\tau_l}{2} \pm \sqrt{(\frac{\tau_l}{2})^2 + k^2 - l^2}$ (when $k > l^*$), $l^* = \sqrt{-(\frac{\tau_l}{2})^2 + l^2}$, noting $l^2 \gg (\frac{\tau_l}{2})^2$, so $\mu_1 > 0$, $\mu_2 = -\mu < 0$, $v = \sqrt{-(\frac{\tau_l}{2})^2 - k^2 + l^2} > 0$.

according to the upper boundary conditions: $z \rightarrow \infty, w_2 = 0$, (11) can be rewritten as

$$\begin{aligned} \tilde{w}_2 &= C e^{-\mu z} & k > l^*, \\ \tilde{w}_2 &= 0 & k < l^*, \end{aligned} \quad (12)$$

the constants A, B and C can be carried out depending on the following boundary conditions and interface condition, namely

$$w_1 = 0, \quad \tilde{w}_1 = 0, \quad z = -h \quad (13)$$

$$w_1 = w_2, \quad \tilde{w}_1 = \tilde{w}_2, \quad z = 0 \quad (14)$$

$$\frac{\partial \tilde{w}_1}{\partial z} = \gamma \tilde{w}_1 + \frac{\partial \tilde{w}_2}{\partial z} \quad z = 0. \quad (15)$$

Eq. (15) is the discontinuous condition deduced from the discontinuous condition of potential temperature, where the Bernoulli Theorem is used (Sang, 1988). $\gamma = \frac{g}{u^2}(\rho_1 - \rho_2)/\rho_1$,

where ρ_2 and ρ_1 are the densities of upper and lower model layers respectively.

As to the high wavenumber ($k > l^*$), inserting the solutions (10) and (12) into the boundary conditions and interface conditions (13), (14), (15), so the following equations can be obtained

$$A e^{-\lambda_1 h} + B e^{-\lambda_2 h} = -Q \frac{b e^{-hk}}{(\lambda_1 + a)(\lambda_2 + a)}, \quad (16)$$

$$A + B + Q \frac{b e^{-hk}}{(\lambda_1 + a)(\lambda_2 + a)} e^{-ah} = C, \quad (17)$$

$$\lambda_1 A + \lambda_2 B - \frac{a Q b e^{-hk}}{(\lambda_1 + a)(\lambda_2 + a)} e^{-ah} = (\gamma - \mu)C, \quad (18)$$

from Eqs. (16), (17) and (18) A, B, and C can be resolved, further being substituted into (10) and (12), then \tilde{w}_{1h} and \tilde{w}_{2h} are as follows:

$$\tilde{w}_{1h} = g_1(k) \{ f_1(k) [e^{\lambda_1(z+h)} - e^{\lambda_2(z+h)}] - e^{\lambda_1(z+h)} + e^{-a(z+h)} \} e^{-hk}, \quad (19)$$

$$\tilde{w}_{2h} = g_1(k) [f_1(k) (e^{\lambda_1 h} - e^{\lambda_2 h}) - (e^{\lambda_1 h} - e^{-ah})] e^{-\mu z}, \quad (20)$$

where the subscript h means the higher wave number relatively ($k > l^*$), and

$$g_1(k) = \frac{Qb}{(\lambda_1 + a)(\lambda_2 + a)}, \quad f_1(k) = \frac{(\gamma - \mu - \lambda_1)e^{\lambda_1 h} - (\gamma - \mu + a)e^{-ah}}{(\gamma - \mu - \lambda_1)e^{\lambda_1 h} - (\gamma - \mu - \lambda_2)e^{\lambda_2 h}}$$

As to the low wavenumber ($k < l^*$), the constants A and B can be carried out only in demand of (16) and (17) because of $\tilde{w}_2 = 0$, so the \tilde{w}_{1l} , \tilde{w}_{2l} are

$$\tilde{w}_{1l} = g_1(k) [f_2(k) e^{\lambda_1(z+h)} + f_3(k) e^{-a(z+h)}] e^{-\delta k}, \quad (21)$$

$$\tilde{w}_{2l} = 0, \quad (22)$$

where the subscript l means the wave number is relatively lower (i.e. $k < l^*$), and

$$f_2(k) = \frac{1 - e^{-(\alpha + \lambda_2)h}}{e^{(\lambda_1 - \lambda_2)h} - 1}, \quad f_3(k) = \frac{1 - e^{-(\alpha + \lambda_1)h}}{e^{(\lambda_2 - \lambda_1)h} - 1}.$$

Therefore, the vertical speeds in boundary layer and the upper stable layer can be deduced in use of Fourier integral

$$w_1 = w_{1l} + w_{1h} = Re \left(\int_0^{l^*} \tilde{w}_{1l} e^{ikx} dk + \int_{l^*}^{\infty} \tilde{w}_{1h} e^{ikx} dk \right), \quad (23)$$

$$w_2 = w_{2l} + w_{2h} = Re \left(\int_0^{l^*} \tilde{w}_{2l} e^{ikx} dk + \int_{l^*}^{\infty} \tilde{w}_{2h} e^{ikx} dk \right) = Re \left(\int_{l^*}^{\infty} \tilde{w}_{2h} e^{ikx} dk \right). \quad (24)$$

By use of Residual Theorem in mathematics, and after complicated manipulation we can obtain

$$\begin{aligned} w_{1l} &= Re \left\{ \int_0^{l^*} \tilde{w}_{1l} e^{ikx} dk \right\} = g_1(0) [f_2(0) e^{\lambda_1^*(z+h)} + f_3(0) e^{\lambda_2^*(z+h)} \\ &\quad + e^{-\alpha(z+h)}] \int_0^{l^*} e^{-hk} \cos kx dk \\ &= g_1(0) [f_2(0) e^{\lambda_1^*(z+h)} + f_3(0) e^{\lambda_2^*(z+h)} + e^{-\alpha(z+h)}] \\ &\quad [e^{-hl^*} x \sin(l^* x) - b e^{-hl^*} \cos(l^* x) + b] / (b^2 + x^2), \end{aligned} \quad (25)$$

$$\begin{aligned} w_{1h} &= g_2(k_1) \{ f_1(k_1) [e^{\lambda_{11}(z+h)} - e^{\lambda_{21}(z+h)}] - e^{\lambda_{11}(z+h)} + e^{-\alpha(z+h)} \} \sin(k_1 x) \\ &\quad + g_1(k_2) \pi e^{-hk_2} [e^{-ah} (\gamma - \mu_2 + a) - e^{\lambda_{12}h} (\gamma - \mu_2 - \lambda_{12})] [e^{\lambda_{22}(z+h)} \\ &\quad - e^{\lambda_{12}(z+h)}] / \delta_{k_2} \sin(k_2 x), \end{aligned} \quad (26)$$

$$\begin{aligned} w_2 &= g_2(k_1) \{ f_1(k_1) [e^{\lambda_{11}h} - e^{\lambda_{21}h}] - (e^{\lambda_{11}h} - e^{-ah}) \} e^{-\mu_1 z} \sin(k_1 x) + g_1(k_2) \pi e^{-bk_2} \\ &\quad [e^{\lambda_{12}h} (\gamma - \mu_2 - \lambda_{12}) - e^{-ah} (\gamma - \mu_2 + a)] [e^{\lambda_{12}h} - e^{\lambda_{22}h}] / \delta_{k_2} e^{-\mu_2 z} \sin(k_2 x), \end{aligned} \quad (27)$$

where $\lambda_{1,2}^* = \lambda_{1,2}|_{k=0}$, $\lambda_i|_{k=k_j} = \lambda_{ij}$ ($i=1,2, j=1,2$), $\mu|_{k=k_i} = \mu_i$ ($i=1,2$), $g_2(k_1) = Q \frac{\pi b e^{-bk_1}}{2k_1}$, k_1, k_2 , are the two first order singular points of the Fourier integral, and satisfy the following conditions

$$\begin{aligned} k = k_1 \quad & (\alpha + \lambda_1)(\alpha + \lambda_2) = 0 \quad (k = k_1, \lambda_2 = \lambda_{21}, \lambda_1 = \lambda_{11}) \\ & k_1^2 = \alpha^2 - \alpha \tau_n - n^2, \alpha = -\lambda_{21} \\ k = k_2 \quad & [\gamma - \mu(e^{\lambda_{11}h} - e^{\lambda_{21}h}) - (\lambda_1 e^{\lambda_{11}h} - \lambda_2 e^{\lambda_{21}h})] = 0 \quad (\mu = \mu_2, \lambda_1 = \lambda_{12}, \lambda_2 = \lambda_{22}), \end{aligned}$$

namely, k_2 can be obtained by iterative computation according to the following equation

$$(\lambda + \frac{\tau_l}{2} - \sqrt{(\frac{\tau_l}{2})^2 + k^2 - l^2}) sh(h \sqrt{(\frac{\tau_n}{2})^2 + k^2 - n^2}) + \frac{\tau_n}{2} sh(h \sqrt{(\frac{\tau_n}{2})^2 + k^2 + n^2})$$

$$-\sqrt{\left(\frac{\tau_n}{2}\right)^2 + k^2 + n^2} ch(h\sqrt{\left(\frac{\tau_n}{2}\right)^2 + k^2 + n^2}) = 0, \quad (28)$$

and

$$\begin{aligned} \delta_{k_2} &\equiv \left\{ \frac{\partial}{\partial k} \left[(\gamma - \mu)(e^{i_{12}h} - e^{i_{22}h}) - (\lambda_1 e^{i_{12}h} - \lambda_2 e^{i_{22}h}) \right] \right\} \Big|_{k=k_2} \\ &= -\frac{k_2(e^{i_{12}h} - e^{i_{22}h})}{\sqrt{\left(\frac{\tau_l}{2}\right)^2 + k_2^2 - l^2}} + (\gamma - \mu_2)h \frac{k_2}{\sqrt{\left(\frac{\tau_n}{2}\right)^2 + k_2^2 + n^2}} (e^{i_{12}h} + e^{i_{22}h}) \\ &\quad - \frac{k_2}{\sqrt{\left(\frac{\tau_n}{2}\right)^2 + k_2^2 + n^2}} (e^{i_{12}h} + h\lambda_{12}e^{i_{12}h} + e^{i_{22}h} + h\lambda_{22}e^{i_{22}h}). \end{aligned}$$

From solutions (22), (25), (26) and (27) it can be found that the vertical speed perturbations in the unstable heat island boundary layer and the upper stable layer arise from the urban heating when the air flows across the city by day. It consists of both high wave number and relatively low wave number elements, the latter w_H formula and $w_H = 0$ (i.e. (22), (25)) mean that the phase of perturbation is same as that of heating function V . In the downwind of the city, the perturbation reaches the maximum, and decreases with the increase of $|x|$ value by the form of $(e^{-hl^*} x \sin(l^* x) - be^{-hl^*} \cos(l^* x) + b) / (b^2 + x^2)$. This kind of low wave number perturbation varies with height, in the boundary layer by complex exponential law, and it must be pointed out that the perturbation exists only in the heat island boundary layer rather than in the upper stable layer, this may be because of the hypothesis of the two-layer model, which is different from the result of Sang (1988). From above analyzing, it is concluded that the low wave number perturbation is just the cross-hill wave induced by heat island, which is only limited within the unstable boundary layer, not propagating to downwind area of the city.

As to the high wavenumber element perturbations w_{1h} and w_{2h} , they reach the maxima at the top of that by the forms of $e^{-\mu_1 z}$ and $e^{-\mu_2 z}$, which are reflected at the bottom of the upper stable layer and consequent lee waves, which furthermore can propagate to the lower boundary layer and can form the similar waves travelling to the surrounding areas. It is further found that this kind of wave is dominantly made of the two harmonic modes whose wave numbers are $2\pi/k_1$ and $2\pi/k_2$ respectively. Because of the $w_{2h}|_{x=0} = 0$, so it is logically deduced that the wave peak will exist in the downwind district, where convective activities easily occur when the air humidity is appropriate (See Fig. 2). If some special parameters are given, then the numerical results can be carried out and be drawn as Fig. 2, which are consistent with the above analyses mostly.

Suppose the potential temperature lapse rates in lower and upper layers are -1K/km and 3K/km respectively, and the city scale is 10 km . So the other parameters are

$$\begin{aligned} n &= 1.18 \times 10^{-3} \text{ m}^{-1}; \quad \bar{u} = 5 \text{ ms}^{-1} \\ l &= 2.04 \times 10^{-3} \text{ m}^{-1}; \quad \alpha = 2.5 \times 10^{-3} \\ \gamma &= 0.005 \text{ m}^{-1}; \quad b = 2500 \text{ m} \\ Q &= 3.3 \times 10^{-6} \text{ m}^{-1} \text{ s}^{-1} \end{aligned}$$

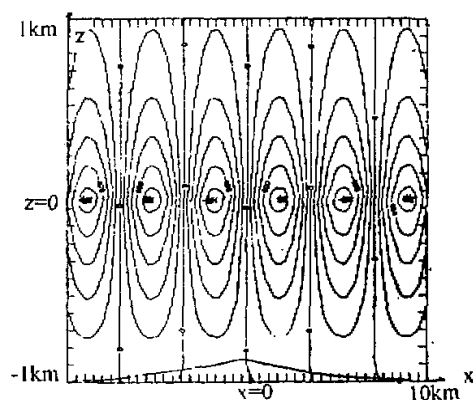


Fig. 2. Vertical wind perturbation field.

$$k_1 = 2.20 \times 10^{-3} \text{ m}^{-1}$$

$$k_2 = 7.07 \times 10^{-2} \text{ m}^{-1}$$

Being valuable to point out, the discontinuity of the densities between the two sides of the interface ($\gamma \neq 0$) can not be the necessary condition, which has also to do with the other factors (seeing from (26) and (27)).

In order to have a broader knowledge of the urban heat island circulation, besides the above vertical wind speed perturbation fields, it is necessary for us to grasp the horizontal wind speed and potential temperature perturbation fields. Substituting (23) to (27) into (3) and (4), we can obtain

$$\begin{aligned} u_1 = & -g_1(0) \left[\lambda_1^* \cdot f_2(0) e^{\lambda_1^* (z+h)} + \lambda_2^* \cdot f_3(0) e^{\lambda_2^* (z+h)} - \alpha e^{-\alpha(z+h)} \right] \arctg\left(\frac{x}{b}\right) \\ & - g_2(k_1) / k_1 \left\{ f_1(k_1) \left[\lambda_{11} e^{\lambda_{11}(z+h)} - \lambda_{21} e^{\lambda_{21}(z+h)} \right] - \lambda_{11} e^{\lambda_{11}(z+h)} - \alpha e^{-\alpha(z+h)} \right\} \cos(k_1 x) \\ & + g_1(k_2) \pi e^{-hk_2} / (\delta_{k_2} k_2) \left[e^{-\alpha h} (\gamma - \mu_2 + \alpha) - e^{\lambda_{12}h} (\lambda - \mu_2 - \lambda_{12}) \right] \\ & \left[\lambda_{12} e^{\lambda_{12}(z+h)} - \lambda_{22} e^{\lambda_{22}(z+h)} \right] \cos(k_2 x) + C_1, \end{aligned} \quad (29)$$

the following approximation is adopted in deduction, i.e. $\int_0^{\infty} e^{-bk} \cos(kx) dk \approx \int_0^{\infty} e^{-bk}$

$\cos(kx) dk = \frac{b}{b^2 + x^2}$, by which the formula $\frac{1}{b^2 + x^2} [e^{-bl^*} x \sin(l^* x) - b e^{-bl^*} \cos(l^* x) + b]$ may be substituted, same as above manipulation, then there are

$$\begin{aligned} u_2 = & -g_2(k_1) \mu_1 / k_1 \left[f_1(k_1) (e^{\lambda_{11}h} - e^{\lambda_{21}h}) - (e^{\lambda_{11}h} - e^{-\alpha h}) \right] e^{-\mu_1 z} \cos(k_1 x) \\ & - g_1(k_2) \pi \mu_2 e^{-hk_2} / (\delta_{k_2} k_2) \left[e^{\lambda_{12}h} (\gamma - \mu_2 - \lambda_{12}) - e^{-\alpha h} (\gamma - \mu_2 + \alpha) \right] \\ & (e^{\lambda_{12}h} - e^{\lambda_{22}h}) e^{-\mu_2 z} \cos(k_2 x) + C_2, \end{aligned} \quad (30)$$

$$\begin{aligned} \theta_1 = & \left(-\frac{1}{u} \frac{\partial \bar{\theta}}{\partial z} \left\{ g_1(0) / b \left[f_2(0) e^{i_1^*(z+h)} + f_3(0) e^{i_2^*(z+h)} + e^{-\alpha(z+h)} \right] \arctg\left(\frac{x}{b}\right) \right. \right. \\ & \left. \left. - g_2(k_1) / k_1 \left[f_1(k_1) \cdot (e^{i_{11}(z+h)} - e^{i_{21}(z+h)}) - e^{i_{11}(z+h)} + e^{-\alpha(z+h)} \right] \cos(k_1 x) \right\} \right. \\ & \left. + Q \bar{\theta}_u / g e^{-\alpha(z+h)} \arctg\left(\frac{x}{b}\right) + C_3 \right. \end{aligned} \quad (31)$$

$$\begin{aligned} \theta_2 = & \left(-\frac{1}{u} \frac{\partial \bar{\theta}}{\partial z} \right) \left\{ -g_2(k_1) / k_1 \left[f_1(k_1) (e^{i_{11}h} - e^{i_{21}h}) - (e^{i_{11}h} - e^{-\alpha h}) \right] e^{-\mu_1 z} \cos(k_1 x) \right. \\ & \left. - g_1(k_2) \pi e^{-hk_2} / (\delta_h, k_2) \left[e^{i_{12}h} (\gamma - \mu_2 - \lambda_{12}) - e^{-\alpha h} (\gamma - \mu_2 + \alpha) \right] \left[e^{i_{12}h} - e^{i_{22}h} \right] \right\} \\ & e^{-\mu_2 z} \cos(k_2 x) + C_4, \end{aligned} \quad (32)$$

where u_2, θ_2 and u_1, θ_1 are the horizontal wind speed and the potential temperature perturbations of the upper stable layer and the lower unstable boundary layer respectively, C_1, C_2, C_3, C_4 are the integral constants waiting to be determined.

By analyzing above solutions, it is known that u_1 and θ_1 are the combinations of both cosine and counter tangent functions horizontally, while u_2 and θ_2 of the cosine functions along the x -direction. Furthermore, they vary with height by the exponential function, and u_1, θ_1 reach the maxima at the top of the boundary layer and then decrease by the form of $e^{-\mu_1 z}$ and $e^{-\mu_2 z}$ above the top of that, these are similar as the results of Sang (1988).

Finally, Fig. 3 is given to combine the vertical and the horizontal speed perturbations.

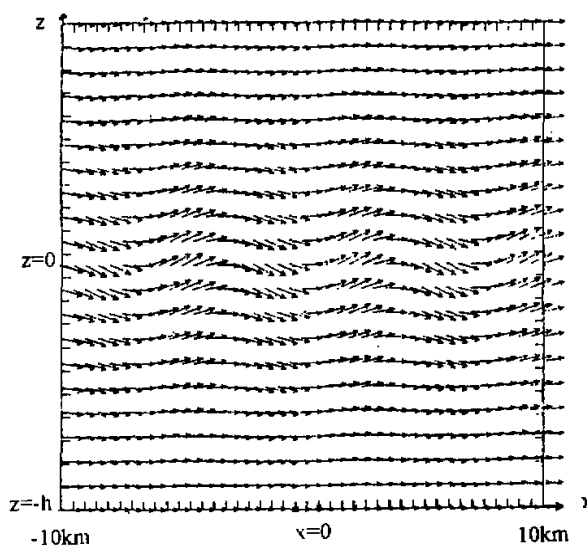


Fig. 3. The whole speed $(\sqrt{(\bar{u} + u_{1,2})^2 + u_{1,2}^2})$ vector.

III. HEIGHT OF THE UNSTABLE BOUNDARY LAYER ABOVE THE URBAN HEAT ISLAND

One of the focuses has been put on the estimation of the urban boundary layer height by the atmospheric researchers and environmental workers, which is very important to the atmospheric pollution control and the planning of the city development, but also the improvement of the climate numerical simulation. Therefore it is necessary to have a further study as follows.

Supposing that the perturbation at the top of the BL is expressed in the vertical displacement at the interface of the model, because of $w = \frac{\partial \eta}{\partial x}$ we obtain

$$\begin{aligned} \eta|_{z=0} &= \int w_1 \frac{dx}{u} = \int (w_{1l} + w_{1h})|_{z=0} \frac{dx}{u} = \int w_{1h}|_{z=0} \frac{dx}{u} \\ &= \frac{g_2(k_1)}{uk_1} \{ f_1(k_1) [e^{\lambda_{21}h} - e^{\lambda_{11}h}] + e^{\lambda_{11}h} - e^{-\alpha h} \} \cos(k_1 x) + \frac{g_1(k_2)}{uk_2 \delta_{k_2}} \\ &\quad \pi e^{-\delta_{k_2} z} [e^{-\alpha h} (\gamma - \mu_2 + \alpha) - e^{\lambda_{12}h} (\gamma - \mu_2 - \lambda_{12})] (e^{\lambda_{12}h} - e^{\lambda_{22}h}) \cos(k_2 x) + C. \end{aligned} \quad (33)$$

From (33) it is known that the height of the boundary layer varies horizontally as a combination of cosine modes, in the downwind of the city or the countryside there exist the wave peaks, this is somewhat consistent with the previous observation analyses (Ochs, 1975, Dirks 1974). Of course, whether the magnitude of it is correct or not will be further verified by the field experiment data.

IV. CONCLUSION

All of above, the vertical and the horizontal wind speed and the potential temperature perturbation fields have been deduced from a simplified two-layer model in the unstable boundary layer, the conclusions from which are mostly in consistence with the observation facts. From Eqs. (5), (6), (7) and the process of deduction of the partial equations, whether the atmosphere is stable or not is the dominant factor to determine the forms of the solutions of the model. If the more layers of model are adopted the better results closer to the real atmosphere will be obtained.

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