The Solitary Waves of the Barotropic Quasi–Geostrophic Model with the Large–scale Orography①

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ABSTRACT

Starting from a modified barotropic quasi–geostrophic model equation, considering the actual situation of the large–orography of the Tibetan Plateau, neglecting its slope in x direction, and using the reductive perturbation method, then the solitary waves are obtained. The results show that the orography is essential factor exciting solitary Rossby waves in a flow without shear.

Key words: Barotrophic quasi–geostrophic model, Reductive perturbation method, Solitary wave

I. INTRODUCTION

The Tibetan Plateau occupies 1 / 4 of China mainland area, its mean sea level elevation is over 4000 meters. It is the highest and most precipitous mountain in the world. Many researches show that the Tibetan Plateau influences both the climate in China and global circulation. Lu (1987) discussed how the large–scale orography has great effect on the Rossby solitary waves. Chen and Liu (1997) modified traditional barotropic quasi–geostrophic model equation with the large–scale orography friction and heating. In the present paper, the solitary solutions of this equation are obtained by using the reductive perturbation method with the friction and heating disregarded.

II. BASIC EQUATION

If friction and heating are disregarded, then the barotropic quasi–geostrophic model equation with the large–scale orography can be given as:

$$\left(\frac{\partial}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)(f+\nabla_h^2\psi)$$
$$=\frac{f_0^2}{C_0^2-f_0\psi_s}\left(\frac{\partial}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)(\psi-\psi_s)\ , \qquad (1)$$

where f is the Coriolis parameter (f_0 is its characteristic value taken as a constant), $C_0=\sqrt{gH}$ (H is the scale height, g is the gravity), ψ is the quasi–geostrophic stream

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function, and $\psi_s = \dfrac{gh_s}{f_0}$ is the orographic stream function (where h_s is the orography height).

$$\nabla_h^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \ , \tag{2}$$

is the horizontal Laplace operator. Setting:

$$\psi = \overline{\psi}(y) + \psi', \quad \overline{u} = -\frac{\partial \overline{\psi}}{\partial y} \ , \tag{3}$$

and substituting (3) into (1), we have:

$$\begin{aligned}&\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)\nabla_h^2 \psi' + \left(\frac{\partial \psi'}{\partial x}\frac{\partial}{\partial y} - \frac{\partial \psi'}{\partial y}\frac{\partial}{\partial x}\right)\left(f + \frac{\partial^2 \overline{\psi}}{\partial y^2} + \nabla_h^2 \psi'\right) \\ &= \frac{f_0^2}{C_0^2 - f_0\psi_s}\left[\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)(\psi' - \psi_s) + \left(\frac{\partial \psi'}{\partial x}\frac{\partial}{\partial y} - \frac{\partial \psi'}{\partial y}\frac{\partial}{\partial x}\right)(\overline{\psi} + \psi' - \psi_s)\right] \ ,\end{aligned} \tag{4}$$

which can be reduced as:

$$\begin{aligned}&\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)\nabla_h^2 \psi' - \frac{f_0^2}{C_0^2 - f_0\psi_s}\frac{\partial \psi'}{\partial t} + \left(\beta_0 - \frac{\partial^2 \overline{u}}{\partial y^2}\right)\frac{\partial \psi'}{\partial x} + J(\psi', \nabla_h^2 \psi') \\ &= \frac{f_0^2}{C_0^2 - f_0\psi_s}\left[-\overline{u}\frac{\partial \psi_s}{\partial x} - J(\psi', \psi_s)\right] \ ,\end{aligned} \tag{5}$$

where β_0 is the Rossby parameter which is taken as a constant, J is the Jacobi operator which is defined as:

$$J(A,B) = \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial B}{\partial x} \ . \tag{6}$$

Considering the orographic character of the Tibetan Plateau whose slope in x direction can be omitted, i.e., $\dfrac{\partial \psi_s}{\partial x} = 0$, (5) can be rewritten as:

$$\begin{aligned}&\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)\nabla_h^2 \psi' - \frac{f_0^2}{C_0^2 - f_0\psi_s}\frac{\partial \psi'}{\partial t} + \left(\beta_0 - \frac{\partial^2 \overline{u}}{\partial y^2}\right)\frac{\partial \psi'}{\partial x} + J(\psi', \nabla_h^2 \psi') \\ &= \frac{f_0^2}{C_0^2 - f_0\psi_s}\frac{\partial \psi'}{\partial x}\frac{\partial \psi_s}{\partial y} \ .\end{aligned} \tag{7}$$

From (7) we can obtain the KdV equation by using the reductive perturbation method and find the solitary wave solutions of the KdV equation.

III. KdV EQUATION

The reductive perturbation method turns the complicated nonlinear equation into the simple one by using the Gardenr–Morikawa transformation, so that we can find its exact solutions.

1. *Gardner–Morikawa Transformation*

Introducing the multiple–scale variables:

$$\xi=\varepsilon^{1/2}(x-ct),\quad \tau=\varepsilon^{3/2}t,\quad y=y, \tag{8}$$

where the small parameter ε is a measure of the wave amplitude which can be seen in the following, c is the travelling velocity. It follows that

$$\begin{cases}\frac{\partial}{\partial t}=-\frac{\partial}{\partial\xi}c\varepsilon^{1/2}+\frac{\partial}{\partial\tau}\varepsilon^{3/2},\\ \frac{\partial}{\partial x}=\frac{\partial}{\partial\xi}\varepsilon^{1/2},\\ \frac{\partial}{\partial y}=\frac{\partial}{\partial y}\ .\end{cases} \tag{9}$$

Substituting (9) into (7), we obtain:

$$\begin{aligned}&[\varepsilon\frac{\partial}{\partial\tau}+(\bar{u}-c)\frac{\partial}{\partial\xi}](\varepsilon\frac{\partial^2\psi'}{\partial\xi^2}+\frac{\partial^2\psi'}{\partial y^2})-\frac{f_0^2}{C_0^2-f_0\psi_s}(\varepsilon\frac{\partial\psi'}{\partial\tau}-c\frac{\partial\psi'}{\partial y})\\ &+(\beta_0-\frac{\partial^2\bar{u}}{\partial y^2})\frac{\partial\psi'}{\partial\xi}+\frac{\partial\psi'}{\partial\xi}\frac{\partial}{\partial y}(\varepsilon\frac{\partial^2\psi'}{\partial\xi^2}+\frac{\partial^2\psi'}{\partial y^2})\\ &-\frac{\partial\psi'}{\partial y}\frac{\partial}{\partial\xi}(\varepsilon\frac{\partial^2\psi'}{\partial\xi^2}+\frac{\partial^2\psi'}{\partial y^2})=\frac{f_0^2}{C_0^2-f_0\psi_s}\frac{\partial\psi_s}{\partial y}\frac{\partial\psi'}{\partial\xi}\ ,\end{aligned} \tag{10}$$

and it can be rewritten as:

$$\begin{aligned}&(\bar{u}-c)\frac{\partial}{\partial\xi}\frac{\partial^2\psi'}{\partial y^2}+[\frac{f_0^2}{C_0^2-f_0\psi_s}(c+\frac{\partial\psi_s}{\partial y})+\beta_0-\frac{\partial^2\bar{u}}{\partial y^2}]\frac{\partial\psi'}{\partial\xi}\\ &+\frac{\partial\psi'}{\partial\xi}\frac{\partial^3\psi'}{\partial y^3}-\frac{\partial\psi'}{\partial y}\frac{\partial^3\psi'}{\partial\xi\partial y^2}+\varepsilon[\frac{\partial}{\partial\tau}\frac{\partial^2\psi'}{\partial y^2}+(u-c)\frac{\partial^3\psi'}{\partial\xi^3}\\ &-\frac{f_0^2}{C_0^2-f_0\psi_s}\frac{\partial\psi'}{\partial\tau}+\frac{\partial\psi'}{\partial\xi}\frac{\partial^3\psi'}{\partial y^2\partial\xi}-\frac{\partial\psi'}{\partial y}\frac{\partial^3\psi'}{\partial\xi^3}]+\varepsilon^2\frac{\partial}{\partial\tau}\frac{\partial^2\psi'}{\partial\xi^2}=0\ ,\end{aligned} \tag{11}$$

then the homogeneous boundary conditions of the Eq. (11) are given by:

$$\psi'|_{y=y_1}=0\ ,\qquad \psi'|_{y=y_2}=0\ . \tag{12}$$

2. *Perturbation Method*

We seek a steady solution in the form:

$$\psi'=\varepsilon\psi_1+\varepsilon^2\psi_2+\cdots, \tag{13}$$

inserting (13) into (9) gives an ordered set of equations, the first one is:

$$(\bar{u}-c)\frac{\partial}{\partial\xi}\frac{\partial^2\psi_1}{\partial y^2}+[\frac{f_0^2}{C_0^2-f_0\psi_s}(c+\frac{\partial\psi_s}{\partial y})+\beta_0-\frac{\partial^2\bar{u}}{\partial y^2}]\frac{\partial\psi_1}{\partial\xi}=0\ , \tag{14}$$

and the second one is:

$$\begin{aligned}&(\bar{u}-c)\frac{\partial}{\partial\xi}\frac{\partial^2\psi_2}{\partial y^2}+[\frac{f_0^2}{C_0^2-f_0\psi_s}(c+\frac{\partial\psi_s}{\partial y})+\beta_0-\frac{\partial^2\bar{u}}{\partial y^2}]\frac{\partial\psi_2}{\partial\xi}\\ &=-\frac{\partial}{\partial\tau}\frac{\partial^2\psi_1}{\partial y^2}+\frac{f_0^2}{C_0^2-f_0\psi_s}\frac{\partial\psi_1}{\partial\tau}-(u-c)\frac{\partial^3\psi_1}{\partial\xi^3}\end{aligned}$$

$$-\frac{\partial\psi_1}{\partial\xi}\frac{\partial^3\psi_1}{\partial y^3}-\frac{\partial\psi_1}{\partial y}\frac{\partial^3\psi_1}{\partial\xi\partial y^2} \quad . \tag{15}$$

In (14), the variable ψ_1 is separated in the form:

$$\psi_1 = A(\xi,\tau)G(y) \ . \tag{16}$$

Substituting (16) into (14), we get:

$$\frac{\partial A}{\partial\xi}\{(\bar{u}-c)\frac{\partial^2 G}{\partial y^2}+[\frac{f_0^2}{C_0^2-f_0\psi_s}(c+\frac{\partial\psi_s}{\partial y})+\beta_0-\frac{\partial^2\bar{u}}{\partial y^2}]G\}=0 \ , \tag{17}$$

$\frac{\partial A}{\partial\xi}\neq 0$, so we obtain:

$$\frac{\partial^2 G}{\partial y^2}+Q(y)G=0 \ , \tag{18}$$

where

$$Q(y)=\frac{\frac{f_0^2}{C_0^2-f_0\psi_s}(c+\frac{\partial\psi_s}{\partial y})+\beta_0-\frac{\partial^2\bar{u}}{\partial y^2}}{\bar{u}-c} \ . \tag{19}$$

Combining Eq.(18) and boundary conditions (12) yields the $O(\varepsilon)$ eigenvalue problem of $G(y)$:

$$\begin{cases}\frac{\partial^2 G}{\partial y^2}+Q(y)G=0 \ , \\ G|_{y=y_1}=0 \ , \quad G|_{y=y_2}=0 \ ,\end{cases} \tag{20}$$

if $Q(y)>0$, then ψ_1 expresses oscillation. For example:

(i) Disregarding the orography, and $\bar{u}=$const., then (19) is transformed into:

$$Q=\frac{\lambda_0^2 c+\beta_0}{\bar{u}-c}=l^2 \ , \tag{21}$$

where l is the Rossby wave number in y direction, $\lambda_0=\frac{f_0}{C_0}$, λ_0^{-1} is the barotropic Rossby radius of deformation. We obtain:

$$c=\frac{-\beta_0+l^2\bar{u}}{\lambda_0^2+l^2} \ , \tag{22}$$

where c is the Rossby velocity in x direction as $k\to 0$ (k is the Rossby wave number in x direction).

(ii) Considering the orography, but $\frac{\partial\psi_s}{\partial y}=$const., and $\bar{u}=$const., defining $\beta_s=\lambda_0^2\frac{\partial\psi_s}{\partial y}$ (β_s is called the orography Rossby parameter which denotes the slope in y direction), then Eq.(19) can be rewritten as:

$$Q \approx \frac{\lambda_0^2 c + \beta_s + \beta_0}{\bar{u} - c} = l^2 \ , \tag{23}$$

and

$$c = \frac{-(\beta_0 + \beta_s) + l^2 \bar{u}}{\lambda_0^2 + l^2} \tag{24}$$

is the Rossby velocity in x direction with the slope in y direction as $k \to 0$.

Considering the second-order equation (15), and substituting (16) and (19) into (15) yields:

$$\begin{aligned}(\bar{u} - c)\frac{\partial}{\partial \xi}[\frac{\partial^2 \psi_2}{\partial y^2} + Q(y)\psi_2] = &-\left(\frac{\partial A}{\partial \tau}\frac{\partial^2 G}{\partial y^2} - \frac{f_0^2}{C_0^2 - f_0 \psi_s} G \frac{\partial A}{\partial \tau}\right) \\ &- (\bar{u} - c)\frac{\partial^3 A}{\partial \xi^3} G - \left(G \frac{\partial A}{\partial \xi} A \frac{\partial^3 G}{\partial y^3} - A \frac{\partial G}{\partial y}\frac{\partial A}{\partial \xi}\frac{\partial^2 G}{\partial y^2}\right) ,\end{aligned} \tag{25}$$

dividing (25) by $\bar{u} - c$, we obtain:

$$\begin{aligned}\frac{\partial}{\partial \xi}[\frac{\partial^2 \psi_2}{\partial y^2} + Q(y)\psi_2] = &\frac{Q(y) + \dfrac{f_0^2}{C_0^2 - f_0 \psi_s}}{\bar{u} - c} G \frac{\partial A}{\partial \tau} - G \frac{\partial^3 A}{\partial \xi^3} \\ &- \frac{1}{\bar{u} - c}\left(G \frac{\partial^3 G}{\partial y^3} - \frac{\partial G}{\partial y}\frac{\partial^2 G}{\partial y^2}\right) A \frac{\partial A}{\partial \xi} \ ,\end{aligned} \tag{26}$$

integrating (26) by y after multiplied by G, and using (20), noticing that:

$$\begin{cases} \dfrac{\partial}{\partial \xi}\displaystyle\int_{y_1}^{y_2} G(\frac{\partial^2 \psi_2}{\partial y^2} + Q\psi_2) dy \\ = \dfrac{\partial}{\partial \xi}[\displaystyle\int_{y_1}^{y_2} \frac{\partial}{\partial y} G(G\frac{\partial \psi_2}{\partial y} - \psi_2 \frac{\partial G}{\partial y}) dy + \int_{y_1}^{y_2} \psi_2 \left(\frac{\partial^2 G}{\partial y^2} + QG\right) dy] \\ = 0 \ , \\ \displaystyle\int_{y_1}^{y_2} \{\frac{Q(y) + \dfrac{f_0^2}{C_0^2 - f_0 \psi_s}}{\bar{u} - c} G^2 \frac{\partial A}{\partial \tau} - G^2 \frac{\partial^3 A}{\partial \xi^3} \\ - \dfrac{G}{\bar{u} - c}\left(G \dfrac{\partial^3 G}{\partial y^3} - \dfrac{\partial G}{\partial y}\dfrac{\partial^2 G}{\partial y^2}\right) A \dfrac{\partial A}{\partial \xi}\} dy \\ = I_1 \dfrac{\partial A}{\partial \tau} + I_2 A \dfrac{\partial A}{\partial \xi} + I_3 \dfrac{\partial^3 A}{\partial \xi^3} \ , \end{cases} \tag{27}$$

we get:

$$\frac{\partial A}{\partial \tau} + \alpha A \frac{\partial A}{\partial \xi} + \gamma \frac{\partial^3 A}{\partial \xi^3} = 0 \ , \tag{28}$$

where

$$\alpha = \frac{I_2}{I_1} , \qquad \gamma = \frac{I_3}{I_1} , \tag{29}$$

and

$$\begin{cases} I_1 = \displaystyle\int_{y_1}^{y_2} \frac{Q(y) + \dfrac{f_0^2}{C_0^2 - f_0 \psi_s}}{\bar{u} - c} G^2 dy , \\ I_2 = \displaystyle\int_{y_1}^{y_2} - \frac{G}{\bar{u} - c} \left(G \frac{\partial^3 G}{\partial y^3} - \frac{\partial G}{\partial y} \frac{\partial^2 G}{\partial y^2} \right) dy , \\ I_3 = \displaystyle\int_{y_1}^{y_2} - G^2 dy . \end{cases} \tag{30}$$

(28) is the KdV equation derived from (1) by using the reductive perturbation method. A separable solution for ψ_2 also can be obtained because $A(\xi, \tau)$ can be found according to the KdV equation (28).

IV. SOLUTIONS OF THE KdV EQUATION

In the following we solve (28) by using the travelling wave method. Setting:

$$A = A(\eta), \quad \eta = \xi - c_1 \tau = \varepsilon^{1/2} [x - (c + \varepsilon c_1) t] . \tag{31}$$

Here c_1 is the speed of the phase $\xi - c_1 \tau$. (29) can be transformed into:

$$- c_1 \frac{dA}{d\eta} + \alpha A \frac{dA}{d\eta} + \gamma \frac{d^3 A}{d\eta^3} = 0 , \tag{32}$$

integrating (32) gets:

$$- c_1 A + \alpha A^2 + \gamma \frac{d^2 A}{d\eta^2} = B_0 , \tag{33}$$

where B_0 is a constant of the first integration. The second integration may be affected after multiplying both sides of (33) by $\frac{dA}{d\eta}$. Then we have

$$\left(\frac{dA}{d\eta} \right)^2 = - \frac{\alpha}{3\gamma} \left(A^3 - \frac{3c_1}{\alpha} A^2 - \frac{6B_0}{\alpha} A - \frac{6B_1}{\alpha} \right) , \tag{34}$$

where B_1 is an integral constant. Defining:

$$\begin{aligned} P(A) &= A^3 - \frac{3c_1}{\alpha} A^2 - \frac{6B_0}{\alpha} A - \frac{6B_1}{\alpha} \\ &= (A - A_1)(A - A_2)(A - A_3) , \end{aligned} \tag{35}$$

in order to obtain the wave solution, we suppose A_1, A_2, A_3 being the three real roots of $P(A) = 0$, and $A_1 \geqslant A_2 \geqslant A_3$. From (33) we get:

$$\begin{cases} c_1 = \frac{\alpha}{3}(A_1 + A_2 + A_3) \ , \\ B_0 = -\frac{\alpha}{6}(A_1A_2 + A_2A_3 + A_1A_3) \ , \\ B_1 = \frac{\alpha}{6}A_1A_2A_3 \ . \end{cases} \tag{36}$$

Now we discuss the problem in two cases:

(1) $\alpha\gamma > 0$

$$\begin{aligned} A(\xi,\tau) = A(\eta) &= A_2 + (A_1 - A_2)cn^2\sqrt{\frac{\alpha}{12\gamma}(A_1 - A_3)}\,\eta \\ &= A_2 + (A_1 - A_2)cn^2\sqrt{\frac{\alpha}{12\gamma}(A_1 - A_3)}\,\varepsilon^{1/2}[x - (c + \varepsilon c_1)t] \ , \end{aligned} \tag{37}$$

where modulus k is given by

$$k^2 = \frac{A_1 - A_2}{A_1 - A_3} \ . \tag{38}$$

If $A_2 \to A_3$, $k \to 1$, then (37) is reduced to:

$$\begin{aligned} A(\xi,\tau) = A(\eta) &= A_2 + (A_1 - A_2)\mathrm{sech}^2\sqrt{\frac{\alpha}{12\gamma}(A_1 - A_3)}\,\eta \\ &= A_2 + (A_1 - A_2)\mathrm{sech}^2\sqrt{\frac{\alpha}{12\gamma}(A_1 - A_3)}\,\varepsilon^{1/2}[x - (c + \varepsilon c_1)t] \ , \end{aligned} \tag{39}$$

where

$$\begin{cases} c_1 = \frac{\alpha}{3}(A_1 + 2A_2) = \alpha[A_2 + \frac{1}{3}(A_1 - A_2)] = \alpha(A_2 + \frac{1}{3}a) \ , \\ a = A_1 - A_2 \ . \end{cases} \tag{40}$$

(2) $\alpha\gamma < 0$

$$\begin{aligned} A(\xi,\tau) = A(\eta) &= A_2 - (A_2 - A_3)cn^2\sqrt{\left|\frac{\alpha}{12\gamma}\right|(A_1 - A_3)}\,\eta \\ &= A_2 - (A_2 - A_3)cn^2\sqrt{\left|\frac{\alpha}{12\gamma}\right|(A_1 - A_3)}\,\varepsilon^{1/2}[x - (c + \varepsilon c_1)t] \ , \end{aligned} \tag{41}$$

where

$$k^2 = \frac{A_2 - A_3}{A_1 - A_3} \ , \tag{42}$$

if $A_1 \to A_2$, $k \to 1$, then (41) can be reduced to:

$$\begin{aligned} A(\xi,\tau) = A(\eta) &= A_2 - (A_2 - A_3)\mathrm{sech}^2\sqrt{\left|\frac{\alpha}{12\gamma}\right|(A_1 - A_3)}\,\eta \\ &= A_2 - (A_2 - A_3)\mathrm{sech}^2\sqrt{\left|\frac{\alpha}{12\gamma}\right|(A_1 - A_3)}\,\varepsilon^{1/2}[x - (c + \varepsilon c_1)t] \ , \end{aligned} \tag{43}$$

where

$$\begin{cases} c_1 = \frac{\alpha}{3}(2A_2 + A_3) = \alpha[A_2 - \frac{1}{3}(A_2 - A_3)] = \alpha(A_2 - \frac{1}{3}a) \ , \\ a = A_2 - A_3 \ . \end{cases} \quad (44)$$

Eqs.(37) and (41) are the cnoidal wave solutions of the KdV equation (28), while Eqs.(39) and (43) represent the solitary waves, and their amplitudes are given as a in (40) and (44) respectively. The velocity in x axis is $c + \varepsilon c_1$.

V. CONCLUSIONS

The above results can be summarized as:

1. (24) expresses the Rossby velocity in x direction with the slope in y direction of the Tibetan Plateau orography after a large-scale approximation. Disregarding the slope in y direction of large-scale orography, Eq.(24) is turned into Eq.(22) which expresses the general Rossby velocity in x direction after a long-wave approximation. Comparing (22) with (24) shows that the orography makes the Rossby waves tend to move eastward.
2. The cnoidal waves of the barotropic quasi-geostrophic model equation with the large-scale orography are (37) and (41), while the solitary wave solutions are (39) and (43).
3. When the orography is neglected and $\bar{u}$ = const., then Q is independent of y. From (20) we know that: $G\frac{\partial^3 G}{\partial y^3} - \frac{\partial G}{\partial y}\frac{\partial^2 G}{\partial y^2} = 0$, which gives rise to $I_2 = 0$ in (30), and we can see from (29) that $\alpha = 0$, then there is no solitary wave solution. On the other hand, considering that the orography causes $\alpha \neq 0$, there exists the solitary wave even under the condition of $\bar{u}$ = const..

REFERENCES

Chen Jiong and Liu Shikuo (1997), A barotropic quasi-geostrophic model with large-scale orography, friction and heating, *Acta Scientiarum Naturalium Universitatis Pekinensis*, in press (in Chinese with English abstract).

Foruberg, B. and G. Whitham (1978), A numerical and theoretical study of certain nonlinear wave phenomena, *Phil. Trans. Roy. Soc.*, London A., **289**: 373–404.

Hukada, H. (1979), Solitary Rossby waves in a two-layer system, *Tulles*, **2**: 161–169.

Long, R. (1964), Solitary waves in the westerlies, *J. Atmos. Sci.*, **21**: 197–200.

Lu Keli (1987), The effects of orography on the solitary Rossby waves in a barotropic atmosphere, *Acta Meteorologica Sinica*, **45**: 267–273 (in Chinese with English abstract).

Lu Keli and Jiang Houshuo (1996), Force solitary Rossby waves in a near-resonant flow in the presence of topography, *Acta Meteorologica Sinica*, **54**: 142–153 (in Chinese with English abstract).

Patoine, A. and T. Warn (1982), The interaction of long quasistationary baroclinic waves with topography, *J. Atmos. Sci.*, **39**: 1018–1025.

Redekopp, L. G. (1977), On the theory of solitary Rossby waves, *J. Fluid Mech.*, **82**: 725–745.

Scott, A. C., F. Chu and D. W. McLaughlin (1973), The soliton: a new concept in applied science, Proc. IEEE, **61**: 1443–1483.

Weidman, P. D. (1980), Solitary Rossby waves in the presence of vertical shear, *J. Atoms. Sci.*, **37**: 2243–2247.