Reconstruction of the Rainfall in Rainy Season Based on Historical Drought / Flood Grades

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ABSTRACT

On the basis of historical yearly drought and flood grades from A. D. 1470 to A. D. 1949, the ten-year mean precipitation in the rainy season in Wuhan district was reconstructed by means of statistical method. The reconstructing method was testified to be simple and effective.

Key words: Drought / Flood grades, Reconstruction, Rainfall

1. Introduction

The premise of quantitatively studying historical climate is to transform the historical wordy record about climate into quantitative data. So far, the most systematic quantitative data are the drought and flood grades in last 500 years around China (CCMB, 1981), in which the drought and flood are divided into 5 grades according to their occurring time, scope and degree recorded in the historical materials, and grades 1, 2, 3, 4 and 5 symbolize extreme flood, flood, normal, drought and extreme drought respectively. Since then, some other quantitative series of drought and flood have been obtained from the grade data such as the series of drought and flood index (Zhang, 1983). Besides the grades, there are some other quantitative methods, in which the statistics are worked out to describe the historical drought and flood, e.g., the series of dry and wet relative degree which are composed of the ratio and the difference of drought and flood, and the curve of dry and wet variation, which is obtained from the frequency ratio of drought and flood years for every 50 years in provinces of China by Nanjing University (Nanjing University, 1977).

Compared with the above, the more detailed data are the rainfall reconstructed from Clear and Rain Record (CRR), which is the official record about weather phenomena in the Qing dynasty in Beijing, Nanjing, Suzhou and Hangzhou of China. The record is good for its integrity and continuity. So far, the major series of rainfall reconstructed from the record are: yearly rainfall from 1704 to 1904 of Beijing (Fong, 1980); May–August monthly rainfall from 1723 to 1798 of Nanjing, from 1736 to 1816 of Suzhou and from 1723 to 1773 of Hangzhou (Zhang, 1990). All those series are obtained from the records about rainy days, intensity and duration in CRR combined with the instrumental data by using the statistical correlation method.

Compared with the results obtained from CRR, the data of the drought and flood grades in last 500 years, obtained from the local chronicles around China, have the merit of covering more extensive areas and lasting longer time. The authors suggest that the grade data will still be the important data resource for the historical climate research. Therefore, the reconstruction of the historic rainfall in this paper is based on the grade data.

2. Data and method

The historical grade data are derived from the China Central Meteorology Bureau (CCMB, 1981) from A. D. 1470 to A. D. 1949, 470 years in total. According to the local historical records about the drought and flood, the grades are ascertained, since the records generally concern the time, scope and damage degree of drought and flood only in spring, summer and autumn, the grades reflect mainly the rainfall in rainy season, that is, from May to September in the eastern China.

Assume that the rainfall in rainy season follows the normal distribution $\sim N(m, \sigma)$, where m is the mean value; σ is the mean square deviation. When the rainfall is within the interval $[-\infty, R]$, the cumulative frequency will be

$$p(R) = \Phi\left(\frac{R-m}{\sigma}\right) .$$

where $\Phi(t)$ is the standard normal distribution function, $\Phi(t) = \int_{-\infty}^{t} \varphi(\xi) d\xi$, $\varphi(\xi)$ is the standard normal distribution density function $\varphi(\xi) = \frac{1}{\sqrt{2}} e^{-\xi^2/2}$.

Let: $t = \frac{R-m}{\sigma}$, $p(R) = \Phi(t)$

Since the relationship between t and $\Phi(t)$ is single value function, that between R and t is linear function, in such coordinates system as R is abscissa, $P(R)$ is the ordinate of standard normal probability, the curve of cumulative frequency distribution R is a straight line.

$$R = m + \sigma t \tag{1}$$

When some observational samples of R and t are known, the m and σ can be evaluated by means of least square. The calculating steps are as follows.

2.1 *Defining corresponding rainfall intervals of drought and flood grades*

To grade the drought and flood is to divide the amount of rainfall into several grades. The instrumental data can be divided according to the frequencies of drought and flood. The division standard is given by CCMB (CCMB, 1981) as follows:

$$\begin{aligned}
&\text{grade 1: } R > m + 1.17\sigma , \\
&\text{grade 2: } m + 0.33\sigma < R \leqslant m + 1.17\sigma , \\
&\text{grade 3: } m - 0.03\sigma < R \leqslant m + 0.33\sigma , \\
&\text{grade 4: } m - 1.17\sigma < R \leqslant m - 0.33\sigma , \\
&\text{grade 5: } R \leqslant m - 1.17\sigma .
\end{aligned} \tag{2}$$

When rainfall follows normal distribution, the corresponding frequencies of grades 1, 2, 3, 4 and 5 are 12%, 25.1%, 25.8%, 25.1% and 12% respectively. In this paper Wuhan area is taken as the example. Because the frequencies of grades 1 and 5 are higher and that of grade 3 is lower in Wuhan area, the frequencies of grades 1–5 are adjusted as 10.9%, 25.4%, 27.4%, 25.4% and 10.9% respectively. The corresponding grade standard is as follows:

$$\begin{aligned}
&\text{grade 1: } R > R_1 (R_1 = m + 1.23\sigma) \ , \\
&\text{grade 2: } R_2 < R \leqslant R_1 (R_2 = m + 0.35\sigma) \ , \\
&\text{grade 3: } R_3 < R \leqslant R_2 (R_3 = m - 0.35\sigma) \ , \\
&\text{grade 4: } R_4 < R \leqslant R_3 (R_4 = m - 1.23\sigma) \ , \\
&\text{grade 5: } R \leqslant R_4 \ .
\end{aligned} \tag{3}$$

The division values of rainfall $R_i (i = 1 \cdots\cdots 4)$ are obtained from the data of May–September rainfall in 1951–1990.

2.2 *Calculating cumulative frequency of each rainfall grade for any certain time interval during historical period*

Based on the frequency of each drought and flood grade for any certain time interval (N years) during historical period and the above division values of rainfall, the cumulative frequency $P(R_i)$ of each rainfall grade for that period can be simply evaluated. According to $p(R_i) = \Phi(t_i) = \int_{-\infty}^{t_i} \varphi(\xi) d\xi$, each t_i can be evaluated by using the numerical integration and the successive approximation.

2.3 *Calculating the mean value and mean square deviation of rainfall for any certain time interval during historical period*

Using Eq.(1), according to the least square:

$$\sigma = \frac{\sum_{i=1}^{4} (R_i - \bar{R})(t_i - \bar{t})}{\sum_{i=1}^{4} (t_i - \bar{t})^2} \ , \quad m = \bar{R} - \sigma \bar{t} \tag{4}$$

Where $\bar{R}$ is the mean value of R_i, $\bar{R}$ is a constant value in certain district. $\bar{t}$ is the mean value of $t_i (i = 1, \cdots\cdots, 4)$.

m and σ are the estimate value of mean value and mean square deviation for the given period.

3. Statistical test of the normal distribution of may–september rainfall

Generally speaking, the short time rainfall (e. g. hourly or daily rainfall etc.) follows the partial normal distribution, while the longer time rainfall can be fitted by the normal distribution. The basic assumption of reconstructing rainfall with grade data is that the May–September rainfall follows the normal distribution. In order to testify the assumption, the method of x^2 statistical test and the data of the May–September rainfall in 1951–1990 are used. The test process is as follows:

$x^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i}$ follows the x^2 distribution with the freedom degree $k - n - 1$, where f_i is the frequency of observation, e_i is the frequency of theory and k is the number of sample groups.

The data in 1951–1990 are divided into three period samples: i. e. 1951–1990, 1951–1980 and 1961–1990. The test results are listed in Table. 1. All of the three period samples pass the statistical test, which proves that the May–September rainfall follows the normal distribution. The degree of confidence is 95%.

Table 1. The statistical testing results of may–september rainfall

Period	1951–1990	1951–1980	1961–1990
Number of sample group	25	17	18
χ^2	27.55	17.92	16.92
Value of passing test	33.92	23.69	25.0

4. Testing the effect of regression equation in above calculation

Since only 5 grades show more or less rainfall in the drought / flood grade in the last 500 years, there are merely 4 samples to evaluate m and σ by Eq.(1). In theory, only if the rainfall strictly follows the normal distribution during the given time interval, the relationship between R and t is always linear. Thus, It needs only two points that the parameters m and σ of the linear function can be evaluated. Actually, that is impossible. So it is necessary to evaluate m and σ by using statistical regression, but a great many samples are not needed.

In order to prove the regression effect of the 4 samples, the period from 1470 to 1989 is divided into 49 intervals (each interval has 40 years), and Eq.(4) is used to evaluate m and σ for each interval. The regression effect is testified by the statistic.

$$F = \frac{U / m}{Q / (n - m - 1)} ,$$

where U and Q are the regression variance and the residual variance respectively, $n = 4$, $m = 1$.

The results show that the regression effect of all the 49 intervals passes the significance test with the degree of confidence 0.10 and, among them, 37 intervals with the degree of confidence 0.05. Therefore, it is suitable to evaluate m and σ with Eq.(4).

5. Testing the effect of reconstructed rainfall

5.1 *Testing with non–independent samples*

The instrumental data of May–September rainfall in 1951–1990, firstly, are divided into 5 grades according to the above standard. And then, the mean rainfall and the mean square deviation of the intervals for 40, 30, 20 and 10 years respectively are reconstructed. Table 2 shows the observational results (OR) and the calculated results (CR).

Table 2. OR and CR for mean value and mean square deviation of rainfall

Period		51–90	51–80	56–85	61–90	51–70	61–80	71–90	51–60	61–70	71–80	81–90
Years of Interval		40	30	30	30	20	20	20	10	10	10	10
Mean Rainfall (mm)	OR	751.6	723.0	713.9	731.1	772.0	678.0	731.2	813.1	730.9	625.0	837.3
	CR	746.3	713.9	710.6	729.1	762.3	672.6	730.8	655.5	725.0	620.8	698.4
Mean Square Deviation (mm)	OR	271.0	278.8	268.5	277.4	282.3	285.6	257.5	240.6	313.3	243.7	225.2
	CR	252.7	255.7	250.1	271.1	231.8	271.4	271.6	299.3	264.2	264.6	371.2

It can be seen from Table 2 that the longer the interval is, the less the error between OR and CR is. The reconstructed effects for the intervals of 40, 30 and 20 years are more convincing and the calculated effect of mean rainfall is better than that of mean square deviation.

5.2 *Testing with independent samples*

The more confident method for testing reconstructed effect is the test of independent samples. There are both the data of yearly grades and instrumental data of rainfall during the period of 1880–1939 in Wuhan. The calculated and actual results of mean rainfall and mean square deviation for the intervals of 40 and 30 years are listed in Table 3. The years of the samples are shorter than the interval because of breaking observation. It can be seen from Table 3 that the mean rainfall of both 40 years and 30 years intervals can be reconstructed very well, but the results of mean square deviation are not so good.

6. Reconstructing the mean rainfall for each 10–year since 1470

It can be seen from Table 2 that the mean rainfall for 10 years reconstructed by the grade data for 10 years has some larger error. It is due to the number of samples being too less. So, another calculation scheme is worked out as follows: Firstly, the mean rainfalls for 40 and 30 years are reconstructed according to the corresponding grade data respectively. Then, the mean rainfall of 10 years is obtained by using Eq.(5).

Table 3. OR and CR for mean value and mean square deviation of rainfall

Period		1880–1919	1890–1929	1900–1939	1880–1909	1890–1919	1900–1929	1910–1939
Years of Interval		38	39	38	28	29	30	28
Mean Rainfall (mm)	OR	798.2	757.6	761.5	789.6	785.0	756.9	758.4
	CR	803.7	751.4	766.1	787.7	801.6	744.8	764.0
Mean Square Deviation (mm)	OR	256.0	228.9	234.8	252.6	241.8	246.2	218.8
	CR	250.5	266.3	286.7	220.1	260.1	305.0	288.9

Supposing the arbitrary rainfall series of an interval for 40 years is $\chi_1, \chi_2 \cdots \chi_{40}$. Thus, the mean rainfall of the first 10 years in that interval can be evaluated from the equation:

$$\bar{\chi}_{10} = \frac{1}{10}\sum_{i=1}^{10}\chi_i = \frac{1}{10}\left(\sum_{i=1}^{40}\chi_i - \sum_{i=11}^{40}\chi_i\right) = 4\bar{\chi}_{40} - 3\bar{\chi}_{30} \tag{5}$$

Where $\bar{\chi}_{40}$ and $\bar{\chi}_{30}$ are the reconstructed mean rainfall, $\bar{\chi}_{30}$ refers to the last 30 years in the 40 years.

The reconstructed 10–year mean rainfall of Wuhan is shown in Fig. 1.

Table 4 shows the reconstructed rainfall and the instrumental data for five 10–year intervals. The mean error of the reconstructed rainfall of 5 intervals is 5.3%, in which the absolute error is much less than the interannual rainfall variation in rainy season. The result of the reconstructed rainfall also shows that when the actual rainfall is around the mean value (760.2 mm), the reconstructed effect is the best, and when the difference between the actual rainfall and mean value of multi years is larger, the reconstructed effect is not so good.

In order to further confirm the reconstructed effect, Table 5 lists the number of drought

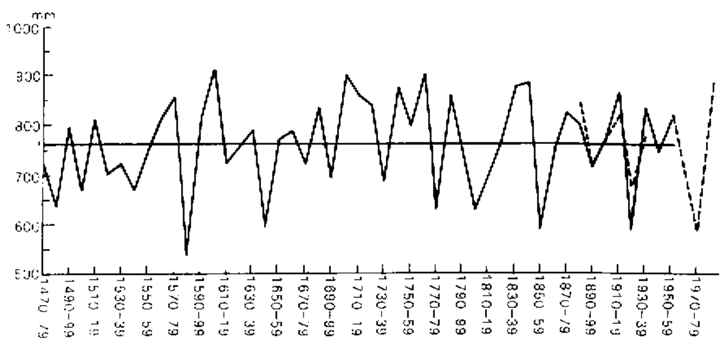

Fig. 1. The curve of the reconstructed mean rainfall of 10 years (during 1470–1959) in rainy season of Wuhan. The solid line is CR, the dashed is OR.

and flood years during several typical rainy and less rainy periods from Fig. 1. The numbers of the drought years are ones of the years which the grades are 4 or 5 for each 10 years, the flood years are of 1 or 2, the normal years are of the rest.

Obviously, the reconstructed curve in Fig. 1 reflects the basic characters: the rainy periods correspond to the flood periods, the less rainy periods correspond to the drought periods.

Table 4. The reconstructed value and the actual value of 10-year mean rainfall

Period	1890–1899	1900–1909	1910–1919	1920–1929	1930–1939
Actual value (mm)	722.8	770.2	822.3	678.2	778.7*
Calculated value (mm)	716.5	770.3	866.1	589.7	835.3

* There are only 8 years instrumental records from 1930 to 1939.

Table 5. Actual condition of historical drought and flood during each reconstructed period

	Rainy period					Less rainy period				
	1600–1609	1700–1709	1760–1769	1830–1839	1840–1849	1580–1589	1640–1649	1770–1779	1850–1859	1920–1929
Numbers of drought years	0	0	0	1	0	5	5	3	4	6
Numbers of flood years	4	4	3	6	3	0	0	0	0	2
Numbers of normal years	6	6	7	3	7	5	5	7	6	2

Based on analyzing the variation of historical drought / flood in the Yangtze–Huaihe River basin, Qiao (1992) pointed out that there are three less flood periods (1470–1559, 1620–1659, 1770–1819), three more flood periods (1560–1619, 1660–1769, 1820–1889), a more drought period (1470–1729) and a less drought period (1730–1899), and no large-range drought disaster hit the Yangtze–Huaihe River basin during the successive 40 years of 1730–1769. Those cases are presented well in Fig. 1.

7. Conclusions

By means of the statistical method, the quantitative reconstruction of rainfall is testified to be simple and effective based on the drought and flood grades in the historical period. Only if the yearly variation of rainfall in rainy season has the character of the normal distribution, the data of drought and flood grades in the last 500 years can be transformed into the rainfall amount in rainy season which, therefore, can be compared with the instrumental rainfall record.

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