Retrieval of Tropospheric CO Profiles Using Correlation Radiometer. II: Effects of Other Gases and the Retrieval in Cloudy Atmosphere

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ABSTRACT

The effects of methane, ozone, water vapor and nitrous oxide on the retrieval of tropospheric CO profiles using correlation radiometer have been assessed. The scheme of the retrieval in the presence of solid clouds have been proposed. The effect of methane and nitrous oxide can be well accounted by their mean profile, and that of ozone can be represented by a typical middle latitude ozone profile, while for water vapor, less than 50% uncertainty is required. With the assumption of blackbody for cloud surface, the CO profile may be retrieved for low and middle solid clouds. However, the retrieval of CO profile will lose quality for high clouds.

Key words: CO, Retrieval, Correlation radiometer

1. Introduction

A scheme for retrieving CO profiles in the troposphere has been discussed in the previous part I (Wu and Gille, part I) for a gas correlation radiometer Measurements of Pollution in the Troposphere (MOPITT) working at the 4.6 μm waveband. The basic equations have been discussed. The use of the wideband signal for providing the surface temperature and an additional channel for the columnar CO amount to improve the retrieval in the near surface layer has been tested. It is found that errors in the temperature profile may increase errors in the retrieved profiles at latitude lower than 70° are generally less than 20% with the first guess of 100 ppbv. (If a better first guess was used, the errors may decrease). An extreme case was calculated with temperature profile at 75° plus typical winter errors. In this case the error was about 35%, much greater than other cases. Retrieval experiments with 10 typical profiles representing the global CO profiles have shown that the r.m.s. error of the retrieved CO profiles is about 10% or 15–20 ppbv.

Beside CO lines, there are lines of methane, ozone, water vapor, nitrous oxide and carbon dioxide in the spectral interval 2140–2190 cm^{-1}. Although the gas–correlation technique can filter out most of lines of gases other than carbon monoxide, some of lines which overlap the CO lines may still interfere with the measurement. Therefore it is necessary to study their effects on the retrieval. As the concentration of carbon dioxide is relatively predictable, its effect will not be a serious problem. In this part, the effects of methane, ozone, water vapor and nitrous oxide are studied.

Comparing with gas absorption, the effect of clouds is more complicated to assess because their great temporal–spatial variability. Usually, as a quite accurate approximation, clouds at lower height can be considered to be blackbodies. In most studies, clouds are treated as a solid horizontally homogeneous parallel plane for mathematical simplicity. Although this assumption is not accurate in some cases like in strong convective cloud field or cirrus clouds, it is still valid for most first order estimations. This part of the paper will also describe a retrieval scheme in the presence of solid clouds.

2. Effects of methane, ozone, water vapor and nitrous oxide

2.1 *The effect of methane*

2.1.1 *The effect on signal*

The global mean CO profile calculated by McConnell (1989), the temperature profile for 40°N in June and three CH_4 profiles (The maximum, minimum and mean, see Fig. 1) from (Atmospheric Ozone, WMO Global Ozone Research and Monitoring Project, Report No. 16, Vol. II, 1985, P478 and P490., NASA FAA, NOAA, UNEP, WMO, CEC, BMFT) were used to calculate the wideband and modulated signals. The results were compared with the results for the same case but without CH_4 in Tables 1–4. Clearly the effect on the modulated signals is very small. The change in the modulated signal caused by adding a CH_4 mean profile into the calculation is about 1×10^{-5} W $m^{-2}sr^{-1}$which is less than the NER of the radiometer (1.8×10^{-5} W $m^{-2}sr^{-1}$). However, the change in the wideband signal is slightly larger. The change is about 1.0×10^{-4} W $m^{-2}sr^{-1}$. The changes in signals for the whole range of CH_4 change are about 2×10^{-6} W $m^{-2}sr^{-1}$ for the modulated signal and 3×10^{-5} W $m^{-2}sr^{-1}$ for the wideband signal.

Table 1. Radiance (W $m^{-2}sr^{-1}$) for temperature profile of 40°N, June, 10 mm PMR, target temperature =290.8 K, without CH_4

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-1.3798×10^{-3}	1.2263×10^{-1}	1.2125×10^{-1}	-8.1766×10^{-4}	1.1057×10^{-2}	1.0239×10^{-2}
mean	100–200	-2.3215×10^{-3}	1.3102×10^{-1}	1.2869×10^{-1}	-1.0659×10^{-3}	5.7178×10^{-3}	4.6519×10^{-3}
mean	50–100	-3.3793×10^{-3}	1.3532×10^{-1}	1.3194×10^{-1}	-1.0496×10^{-3}	2.8996×10^{-3}	1.8500×10^{-3}
mean	25–50	-4.3022×10^{-3}	1.3749×10^{-1}	1.3319×10^{-1}	-7.9612×10^{-4}	1.4488×10^{-3}	6.5264×10^{-4}

Table 2. Radiance (W m^{-2} sr^{-1}) same as Table 1 except for CH_4 mean profile

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-1.4809×10^{-3}	1.2263×10^{-1}	1.2114×10^{-1}	-8.2715×10^{-4}	1.1057×10^{-2}	1.0230×10^{-2}
mean	100–200	-2.4291×10^{-3}	1.3102×10^{-1}	1.2858×10^{-1}	-1.0692×10^{-3}	5.7178×10^{-3}	4.6486×10^{-3}
mean	50–100	-3.4890×10^{-3}	1.3532×10^{-1}	1.3183×10^{-1}	-1.0506×10^{-3}	2.8996×10^{-3}	1.8490×10^{-3}
mean	25–50	-4.4125×10^{-3}	1.3749×10^{-1}	1.3308×10^{-1}	-7.9640×10^{-4}	1.4488×10^{-3}	6.5237×10^{-4}

Table 3. Radiance (W m^{-2} sr^{-1}) same as Table 1 except for CH_4 minimum profile

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-1.4677×10^{-3}	1.2263×10^{-1}	1.2116×10^{-1}	-8.2591×10^{-4}	1.1057×10^{-2}	1.0231×10^{-2}
mean	100–200	-2.4151×10^{-3}	1.3102×10^{-1}	1.2860×10^{-1}	-1.0688×10^{-3}	5.7178×10^{-3}	4.6490×10^{-3}
mean	50–100	-3.4747×10^{-3}	1.3532×10^{-1}	1.3185×10^{-1}	-1.0505×10^{-3}	2.8996×10^{-3}	1.8491×10^{-3}
mean	25–50	-4.3981×10^{-3}	1.3749×10^{-1}	1.3310×10^{-1}	-7.9636×10^{-4}	1.4488×10^{-3}	6.5241×10^{-4}

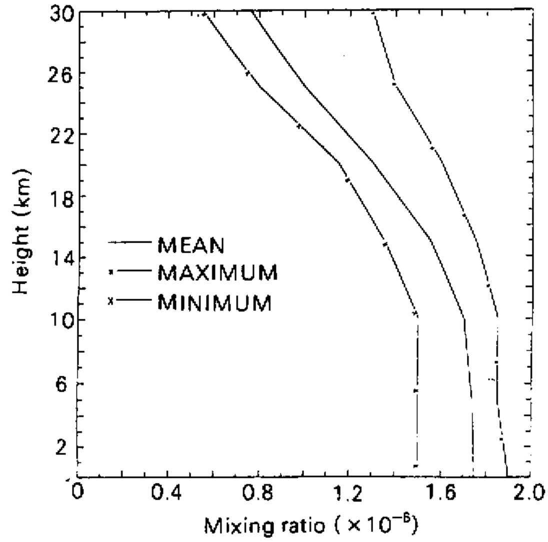


Fig. 1. The methane profiles used in the experiment.

Table 4. Radiance (W m^{-2} sr^{-1}) same as Table 1 except for CH_4 maximum profile

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-1.4915×10^{-3}	1.2263×10^{-1}	1.2113×10^{-1}	-8.2815×10^{-4}	1.1057×10^{-2}	1.0229×10^{-2}
mean	100–200	-2.4403×10^{-3}	1.3102×10^{-1}	1.2857×10^{-1}	-1.0695×10^{-3}	5.7178×10^{-3}	4.6482×10^{-3}
mean	50–100	-3.5004×10^{-3}	1.3532×10^{-1}	1.3182×10^{-1}	-1.0507×10^{-3}	2.8996×10^{-3}	1.8489×10^{-3}
mean	25–50	-4.4240×10^{-3}	1.3749×10^{-1}	1.3307×10^{-1}	-7.9642×10^{-4}	1.4488×10^{-3}	6.5235×10^{-4}

Table 5. The r.m.s. error in the retrieved profile and the residual (Chahine method 3 with the ground temperature T_g being iterated, k is the number of iteration, First guess: mean profile, the values below 4.5 km for the last iteration are derived from the regression relationship.
Measurement error = 1.8×10^{-2} W m^{-5} sr^{-1}, T profile: 40°N without errors, for CO profiles: 64°N)

			Profile				Signal	
k	$\Delta T_g(K)$	ΔS_{md}(max)	With respect to $q^{(k)}(p)$		With respect to $q^{(0)}$		With resepct to S_0	
			ab	rela	ab	rela	ab	rela
0	0.40	$4.24\times^{-5}$	/	/	3.38×10^{-8}	0.196	1.69×10^{-4}	0.1380
1	0.45	2.96×10^{-5}	1.26×10^{-8}	0.1219	2.55×10^{-8}	0.137	7.48×10^{-5}	0.0616
2	0.47	1.90×10^{-5}	6.57×10^{-9}	0.0530	2.29×10^{-8}	0.125	3.01×10^{-5}	0.0244
3	0.48	1.57×10^{-5}	2.03×10^{-9}	0.0166	2.23×10^{-8}	0.125	2.79×10^{-5}	0.0211
4	0.48	1.62×10^{-5}	2.03×10^{-9}	0.0166	2.23×10^{-8}	0.129	1.59×10^{-5}	0.0126

2.1.2 *The effects on the retrieval results*

In this test, the same temperature profile was used. The CO profile observed at 64°N was used as the true CO profile. In order to estimate the extension to which CH_4 affects the retrieval, this test was carried out in the following way: The minimum CH_4 profile was used in the retrieval. The comparison of errors in the retrieved CO profiles is shown in Tables 5, 6 and 7. It can be seen that the effect of the CH_4 can be well accounted by a mean profile of CH_4.

Table 6. The r.m.s. error in the retrieved profile and the residual (same as Table 5 except for minimum CH_4 profile 1st guess: maximum)

			Profile				Signal	
k	$\Delta T_g(K)$	ΔS_{md}(max)	With respect to $q^{(k)}(p)$		With respect to $q^{(0)}$		With resepct to S_0	
			ab	rela	ab	rela	ab	rela
0	0.40	3.98×10^{-5}	/	/	3.38×10^{-8}	0.196	1.75×10^{-4}	0.1426
1	0.45	2.60×10^{-5}	1.34×10^{-8}	0.1289	2.52×10^{-8}	0.135	7.66×10^{-5}	0.0628
2	0.47	1.43×10^{-5}	7.04×10^{-9}	0.0558	2.29×10^{-8}	0.125	2.93×10^{-5}	0.0238
3	0.48	1.99×10^{-5}	2.26×10^{-9}	0.0174	2.25×10^{-8}	0.127	2.69×10^{-5}	0.0206
4	0.49	1.94×10^{-5}	2.51×10^{-9}	0.0175	2.26×10^{-8}	0.129	1.28×10^{-5}	0.0102

Table 7. The r.m.s. error in the retrieved profile and the residual (same as Table 5 except for maximum CH_4 profile 1st guess: minimum)

			Profile				Signal	
k	$\Delta T_g(K)$	ΔS_{md}(max)	With respect to $q^{(k)}(p)$		With respect to $q^{(0)}$		With resepct to S_0	
			ab	rela	ab	rela	ab	rela
0	0.40	4.09×10^{-5}	/	/	3.38×10^{-8}	0.196	1.74×10^{-4}	0.1420
1	0.45	2.73×10^{-5}	1.32×10^{-8}	0.1279	2.52×10^{-8}	0.136	7.63×10^{-5}	0.0626
2	0.47	1.41×10^{-5}	6.98×10^{-9}	0.0554	2.29×10^{-8}	0.125	2.93×10^{-5}	0.0238
3	0.48	1.85×10^{-5}	2.22×10^{-9}	0.0173	2.24×10^{-8}	0.127	2.71×10^{-5}	0.0207
4	0.48	1.80×10^{-5}	2.50×10^{-9}	0.0175	2.22×10^{-8}	0.128	1.31×10^{-5}	0.0104

2.2 *The effect of ozone*

2.2.1 *The effect on signal*

The global mean CO profile calculated by J. McConnell (1989), the temperature profile for 40°N in June and one O_3 profile of mid-latitude (US Standard Atmosphere, 1976) (see Fig. 2) were used to calculate the wideband and modulated signals. The results were compared with the results for the same case but without O_3 in Table 5, 8 and 1. Clearly the effect on the modulated signals is very small. The difference of the modulated signal is less than 2×10^{-5} W $m^{-2}sr^{-1}$ which is about the NER of the radiometer. However, the difference in the wideband signal is slightly larger. The change is about 2.3×10^{-4} W $m^{-2}sr^{-1}$.

Table 8. Radiance (W $m^{-2}sr^{-1}$) same as Table 1 except for O_3 mid-latitude profile

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-1.6092×10^{-3}	1.226×10^{-1}	1.2102×10^{-1}	-8.3793×10^{-4}	1.1057×10^{-2}	1.0219×10^{-2}
mean	100–200	-2.5655×10^{-3}	1.3102×10^{-1}	1.2845×10^{-1}	-1.0747×10^{-3}	5.7178×10^{-3}	4.6430×10^{-3}
mean	50–100	-3.6293×10^{-3}	1.3532×10^{-1}	1.3169×10^{-1}	-1.0528×10^{-3}	2.8996×10^{-3}	1.8467×10^{-3}
mean	25–50	-4.5543×10^{-3}	1.3749×10^{-1}	1.3294×10^{-1}	-7.9714×10^{-4}	1.4488×10^{-3}	6.5163×10^{-4}

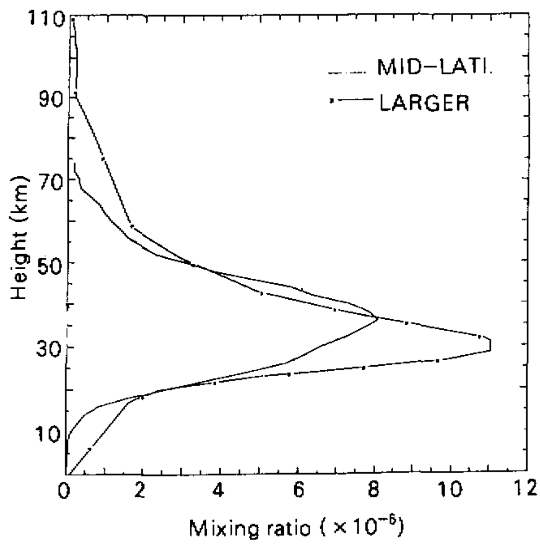


Fig. 2. The ozone profiles used in the experiment.

Table 9. The r.m.s. error in the retrieved profile and the residual (same as Table 5 except for maximum O_3 profile 1st guess: midlatitude)

k	$\Delta T_g(K)$	ΔS_{md}(max)	Profile				Signal	
			With respect to $q^{(k)}(p)$		With respect to $q^{(0)}$		With resepct to S_0	
			ab	rela	ab	rela	ab	rela
0	0.34	5.34×10^{-5}	/	/	3.38×10^{-8}	0.196	1.81×10^{-4}	0.1470
1	0.39	3.89×10^{-5}	1.40×10^{-8}	0.1351	2.49×10^{-8}	0.134	7.85×10^{-5}	0.0643
2	0.42	3.58×10^{-5}	7.46×10^{-9}	0.0582	2.29×10^{-8}	0.126	2.91×10^{-5}	0.0237
3	0.42	7.60×10^{-6}	2.45×10^{-9}	0.0183	2.26×10^{-8}	0.128	2.67×10^{-5}	0.0206
4	0.43	3.22×10^{-5}	2.66×10^{-9}	0.0181	2.26×10^{-8}	0.129	1.07×10^{-5}	0.0086

2.2.2 *The effects on the retrieval results*

In this test, the same temperature profile was used. The CO profile observed at 64°N was used as the true CO profile. A typical O_3 profile with larger O_3 amount (typical profiles selected by Bailey from LIMS sounding) was used in the retrieval. Error in the retrieved CO profiles is shown in Table 9. By comparing the results in Table 9 with that in Table 5, it can be seen that the effect of O_3 may also be represented by a typical O_3 profile.

In the retrieval with O_3 or CH_4, the contribution of these two gases to the modulated signals and the weighting function can be neglected. One only needs to consider the effects of them on the wideband signals. Therefore, it is possible to use a priori CH_4 and O_3 profiles in the calculation of wideband signals for determining the surface temperature then carry out the retrieval without modifying the profiles of those two gases. But the situation with water vapor is quite different as will be discussed in the following section.

2.3 *The effect of water vapor*

2.3.1 *The effect on signal*

The global mean CO profile calculated by McConnell (1989), the temperature profile for 40°N in June and three H_2O profiles (typical profiles selected by Bailey from LIMS sounding) (see Fig. 3) were used to calculate the wideband and modulated signals. The results were compared with the results for the same case but without H_2O in Tables 10–12 and Table 1. The effect of water vapor on signals is on the order of 3×10^{-4} W $m^{-2}sr^{-1}$ for the modulated signals and 4×10^{-3} W $m^{-2}sr^{-1}$ for the wideband signal which is greater than the effects of O_3 and CH_4. It is impossible to neglect water vapor either in calculating the wideband signal or calculating the modulated signal. Therefore, it is necessary to know the information on the water vapor profiles.

Table 10. Radiance (W m^{-2} sr^{-1}) same as Table 1 except for H_2O mean profile

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-5.1510×10^{-3}	1.2263×10^{-1}	1.1747×10^{-1}	-1.1438×10^{-3}	1.1057×10^{-2}	9.9134×10^{-3}
mean	100–200	-6.3215×10^{-3}	1.3102×10^{-1}	1.2469×10^{-1}	-1.1979×10^{-3}	5.7178×10^{-3}	4.5199×10^{-3}
mean	50–100	-7.4675×10^{-3}	1.3532×10^{-1}	1.2785×10^{-1}	-1.0937×10^{-3}	2.8996×10^{-3}	1.8058×10^{-3}
mean	25–50	-8.4190×10^{-3}	1.3749×10^{-1}	1.2908×10^{-1}	-8.0946×10^{-4}	1.4488×10^{-3}	6.3931×10^{-4}

Table 11. Radiance (W m^{-2} sr^{-1}) same as Table 1 except for H_2O minimum profile

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-1.4809×10^{-3}	1.2263×10^{-1}	1.2114×10^{-1}	-8.2971×10^{-4}	1.1057×10^{-2}	1.0227×10^{-2}
mean	100–200	-2.4310×10^{-3}	1.3102×10^{-1}	1.2858×10^{-1}	-1.0705×10^{-3}	5.7178×10^{-3}	4.6473×10^{-3}
mean	50–100	-3.4917×10^{-3}	1.3532×10^{-1}	1.3183×10^{-1}	-1.0510×10^{-3}	2.8996×10^{-3}	1.8486×10^{-3}
mean	25–50	-4.4154×10^{-3}	1.3749×10^{-1}	1.3308×10^{-1}	-7.9651×10^{-4}	1.4488×10^{-3}	6.5226×10^{-4}

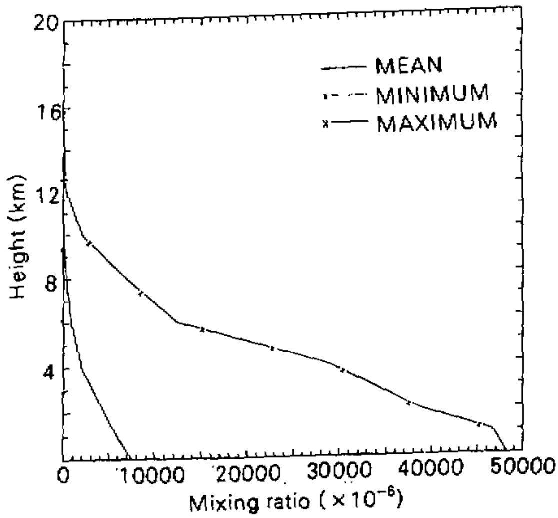


Fig. 3. The water vapor profiles used in the experiment.

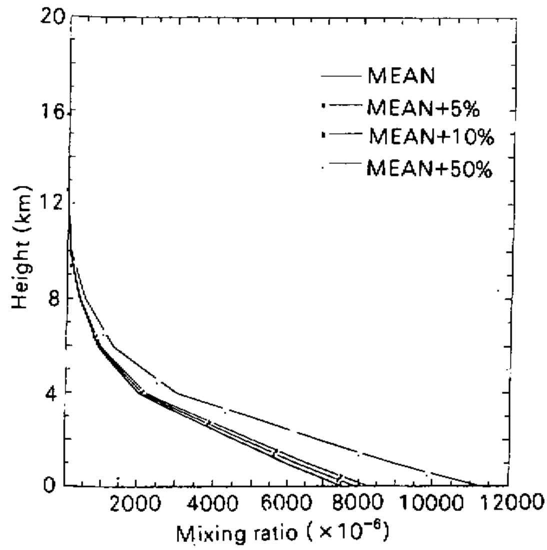


Fig. 4. The water vapor profiles with errors used in the experiment.

Table 12. Radiance ($W\ m^{-2}\ sr^{-1}$) same as Table 1 except for H_2O maximum profile

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-1.8256×10^{-2}	1.2263×10^{-1}	1.0437×10^{-1}	-2.1325×10^{-3}	1.1057×10^{-2}	8.9247×10^{-3}
mean	100–200	-2.0136×10^{-2}	1.3102×10^{-1}	1.1088×10^{-1}	-1.6268×10^{-3}	5.7178×10^{-3}	4.0910×10^{-3}
mean	50–100	-2.1575×10^{-2}	1.3532×10^{-1}	1.1375×10^{-1}	-1.2522×10^{-3}	2.8996×10^{-3}	1.6474×10^{-3}
mean	25–50	-2.2632×10^{-2}	1.3749×10^{-1}	1.1486×10^{-1}	-8.6130×10^{-4}	1.4488×10^{-3}	5.8747×10^{-4}

Table 13. The r.m.s. error in the retrieved profile and the residual (same as Table 5 except for H_2O profile: mean +5% error (1st guess: mean))

			Profile				Signal	
k	$\Delta T_g(K)$	ΔS_{md}(max)	With respect to $q^{(k)}(p)$		With respect to $q^{(0)}$		With resepct to S_0	
			ab	rela	ab	rela	ab	rela
0	−0.73	5.12×10^{-5}	/	/	3.38×10^{-8}	0.196	1.81e−4	0.1507
1	−0.65	3.52×10^{-5}	1.39×10^{-8}	0.1348	2.47×10^{-8}	0.133	9.06×10^{-5}	0.0754
2	−0.62	2.56×10^{-5}	8.06×10^{-9}	0.0644	2.22×10^{-8}	0.125	4.32×10^{-5}	0.0356
3	−0.60	1.44×10^{-5}	3.15×10^{-9}	0.0247	2.17e−8	0.131	3.54×10^{-5}	0.0272
4	−0.59	1.88×10^{-5}	3.07×10^{-9}	0.0215	2.16×10^{-8}	0.136	2.11×10^{-5}	0.0172
5	−0.59	1.95×10^{-5}	1.28×10^{-9}	0.0102	2.02×10^{-8}	0.129	1.55×10^{-5}	0.0135

2.3.2 *The effects on the retrieval results*

In this test, the same temperature profile was used. The CO profile observed at 64°N was used as the true CO profile. The "real signal" was calculated with the true CO profile and the water vapor profiles obtained by adding 5%, 10% and 50% error to the mean profile of water vapor mentioned above (Fig. 4). The mean water vapor profile was then used in the retrieval simulation. The results are shown in Tables 13–15. They show that the retrieval results are still very good as long as one knows the water vapor profile with 50% accuracy which is not an over critical demand.

Table 14. The r.m.s. error in the retrieved profile and the residual (same as Table 5 except for H_2O profile: mean +10% error (1st guess: mean))

			Profile				Signal	
k	$\Delta T_g(K)$	ΔS_{md}(max)	With respect to $q^{(k)}(p)$		With respect to $q^{(0)}$		With resepct to S_0	
			ab	rela	ab	rela	ab	rela
0	−0.76	5.82×10^{-5}	/	/	3.38×10^{-8}	0.196	1.78×10^{-4}	0.1487
1	−0.69	4.26×10^{-5}	1.36×10^{-8}	0.1324	2.48×10^{-8}	0.134	8.91×10^{-5}	0.0744
2	−0.66	3.94×10^{-5}	7.92×10^{-9}	0.0635	2.22×10^{-8}	0.125	4.25×10^{-5}	0.0351
3	−0.64	9.48×10^{-6}	3.08×10^{-9}	0.0242	2.17×10^{-8}	0.130	3.50×10^{-5}	0.0270
4	−0.63	3.31×10^{-5}	3.02×10^{-9}	0.0213	2.15×10^{-8}	0.134	2.09×10^{-5}	0.0171
5	−0.62	3.37×10^{-5}	1.30×10^{-9}	0.0103	2.03×10^{-8}	0.128	1.55×10^{-5}	0.0136

2.4 *The effect of nitrous oxide*

2.4.1 *The effect on signal*

The global mean CO profile calculated by McConnell (1989), the temperature profile for 40°N in June and three N_2O profiles obtained by adopting the value measured by Scripps Institute for Oceanography (in WMO Global Ozone Research and Monitoring Project–Report No. 20, Scientific Assessment of Stratospheric Ozone: 1989, Vol. 1, pp251, NASA, UKDOE, NOAA, UNEP, WMO) for the layer below 15 km and the profile from the LIMS sounding for the atmosphere above 15 km (see Fig. 5) were used to calculate the wideband and modulated signals. The results were compared with the results for the same case but without N_2O in Tables 16–18 and Table 1. The effect of nitrous oxide on signals is on the order of 10^{-3} W m^{-2} sr^{-1} for the modulated signals and 1×10^{-2} W m^{-2} sr^{-1} for the wideband signal which is greater than the effects of O_3 and CH_4. It is impossible to neglect nitrous oxide either in calculating the wideband signal or calculating the modulated signal.

Table 15. The r.m.s. error in the retrieved profile and the residual (same as Table 5 except for H_2O profile: mean +50% error (1st guess: mean))

			Profile				Signal	
k	$\Delta T_g(K)$	ΔS_{md}(max)	With respect to $q^{(k)}(p)$		With respect to $q^{(0)}$		With resepct to S_0	
			ab	rela	ab	rela	ab	rela
0	−1.02	1.47×10^{-4}	/	/	3.38×10^{-8}	0.196	1.57×10^{-4}	0.1333
1	−0.95	1.42×10^{-4}	1.17×10^{-8}	0.1142	2.58×10^{-8}	0.140	7.88×10^{-5}	0.0669
2	−0.92	1.45×10^{-4}	6.71×10^{-9}	0.0555	2.28×10^{-8}	0.126	3.80×10^{-5}	0.0315
3	−0.91	1.16×10^{-4}	2.47×10^{-9}	0.0207	2.19×10^{-8}	0.125	3.37×10^{-5}	0.0260
4	−0.90	1.40×10^{-4}	2.68×10^{-9}	0.0198	2.11×10^{-8}	0.125	2.14×10^{-5}	0.0173
5	−0.90	1.41×10^{-4}	1.47×10^{-9}	0.0113	2.09×10^{-8}	0.126	1.69×10^{-5}	0.0156

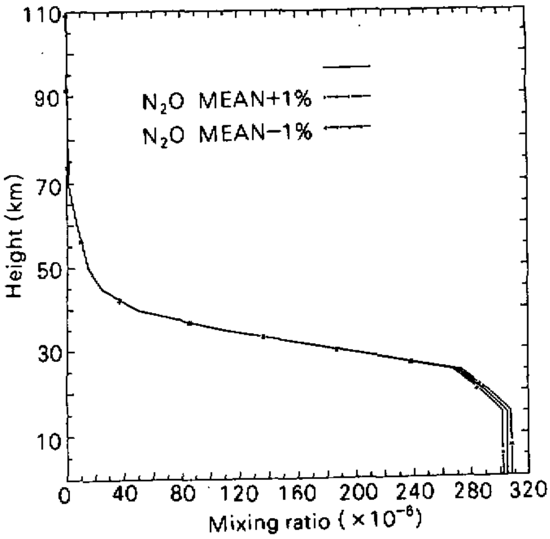


Fig. 5. The N_2O profiles used in the experiment.

Table 16. Radiance ($W\ m^{-2}\ sr^{-1}$) same as Table 1 except for N_2O mean profile

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-1.3040×10^{-2}	$1.2263E^{-1}$	$1.0959E^{-1}$	-1.8307×10^{-3}	$1.1057E^{-2}$	$9.2264E^{-3}$
mean	100–200	-1.4684×10^{-2}	1.3102×10^{-1}	1.1633×10^{-1}	-1.4567×10^{-3}	5.7178×10^{-3}	4.6211×10^{-3}
mean	50–100	-1.6002×10^{-2}	$1.3532E^{-1}$	1.1932×10^{-1}	-1.1801×10^{-3}	2.8996×10^{-3}	1.7195×10^{-3}
mean	25–50	-1.7010×10^{-2}	1.3749×10^{-1}	1.2049×10^{-1}	-8.3505×10^{-4}	1.4488×10^{-3}	6.1372×10^{-4}

Table 17. Radiance ($W\ m^{-2}\ sr^{-1}$) same as Table 1 except for N_2O mean profile−1%

CO profile	Pressure (hPa)	Sw *(diff)*	Sw *(surface)*	Sw *(total)*	Sm *(diff)*	Sm *(surface)*	Sm *(total)*
mean	200–400	-1.3296×10^{-2}	1.2263×10^{-1}	1.0966×10^{-1}	-1.8235×10^{-3}	1.1057×10^{-2}	9.2337×10^{-3}
mean	100–200	-1.4600×10^{-2}	1.3102×10^{-1}	1.1641×10^{-1}	-1.4537×10^{-3}	5.7178×10^{-3}	4.2641×10^{-3}
mean	50–100	-1.5916×10^{-2}	1.3532×10^{-1}	1.1941×10^{-1}	-1.1790×10^{-3}	2.8996×10^{-3}	1.7205×10^{-3}
mean	25–50	-1.6923×10^{-2}	1.3749×10^{-1}	1.2057×10^{-1}	-8.3473×10^{-4}	1.4488×10^{-3}	6.1404×10^{-4}

Table 18. Radiance ($W\ m^{-2}\ sr^{-1}$) same as Table 1 except for N_2O mean profile+1%

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	$-1.3119E^{-2}$	$1.2263E^{-1}$	$1.0951E^{-1}$	-1.8379×10^{-3}	$1.1057E^{-2}$	$9.2192E^{-3}$
mean	100–200	-1.4767×10^{-2}	1.3102×10^{-1}	1.1625×10^{-1}	-1.4596×10^{-3}	5.7178×10^{-3}	4.2582×10^{-3}
mean	50–100	-1.6087×10^{-2}	1.3532×10^{-1}	1.1924×10^{-1}	-1.1811×10^{-3}	2.8996×10^{-3}	1.7185×10^{-3}
mean	25–50	-1.7096×10^{-2}	1.3749×10^{-1}	1.2040×10^{-1}	-8.3536×10^{-4}	1.4488×10^{-3}	6.1341×10^{-4}

2.4.2 *The effects on the retrieval results*

In this test, the same temperature profile was used. The CO profile observed at 64°N was used as the true CO profile. The "real signal" was calculated with the true CO profile and the nitrous oxide profiles obtained by adding 1% error to the mean profile of nitrous oxide mentioned above (Fig. 5). The results are shown in Table 19. It shows that the retrieval results are still very good as long as one knows the mean profile of nitrous oxide. Since the global variation of nitrous oxide is only 1%, the mean profile is accurate enough for the retrieval.

Table 19. The r.m.s. error in the retrieved profile and the residual (same as Table 5 except for N_2O profile: mean −1% error (1st guess: mean +1%)

			Profile				Signal	
k	$\Delta T_g(K)$	ΔS_{md}(max)	With respect to $q^{(k)}(p)$		With respect to $q^{(0)}$		With resepct to S_0	
			ab	rela	ab	rela	ab	rela
0	−0.57	3.17×10^{-5}	/	/	3.38×10^{-8}	0.196	1.80×10^{-4}	0.1545
1	−0.50	1.57×10^{-5}	1.45×10^{-8}	0.1404	2.45×10^{-8}	0.132	9.23×10^{-5}	0.0789
2	−0.46	2.19×10^{-5}	8.68×10^{-9}	0.0685	2.22×10^{-8}	0.127	4.49×10^{-5}	0.0380
3	−0.44	4.50×10^{-5}	3.56×10^{-9}	0.0271	2.20×10^{-8}	0.135	3.66×10^{-5}	0.0290
4	−0.43	3.69×10^{-5}	3.45×10^{-9}	0.0236	2.23×10^{-8}	0.143	2.09×10^{-5}	0.0176
5	−0.42	2.61×10^{-5}	1.22×10^{-9}	0.0100	2.17×10^{-8}	0.144	1.48×10^{-5}	0.0129

3. Retrieval experiment in the presence of solid clouds

Clouds influence the outgoing radiance by reflecting, emitting and absorbing the radiation, which results in errors in remote sensing of the atmospheric temperature and composition profiles. Therefore, it is necessary to determine the height of the clouds so as to eliminate its effects. A demonstrative test was carried out to study the effects of clouds on the radiance received by gas–correlation radiometers and the possible way to determine their heights.

3.1 *The wide–band and modulated signals with clouds presented*

In this test, clouds were assumed to be black surfaces at various clouds heights. The cloud temperature was assumed to be the same as the atmospheric temperature at that height. A temperature profile at latitude 40°N in June and a global mean CO profile calculated by McConnell (1989) were adopted. Various terms of the signals (see Part I for definitions) were calculated for 10 mm PMRs channel for 4 cloud heights and also for a case without cloud. The results were shown in Tables 20–23 and Table 1. The wideband and modulated signals as a function of cloud height were shown in Fig. 6.

Table 20. Radiance (W $m^{-2}sr^{-1}$) same as Table 1 except for cloud height = 1.5 km

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-6.8705×10^{-4}	8.8720×10^{-2}	8.8035×10^{-2}	-4.2486×10^{-4}	7.9952×10^{-3}	7.5703×10^{-3}
mean	100–200	-1.1996×10^{-3}	9.4785×10^{-2}	9.3585×10^{-2}	-6.0035×10^{-4}	4.1344×10^{-3}	3.5341×10^{-3}
mean	50–100	-1.8210×10^{-3}	9.7900×10^{-2}	9.6080×10^{-2}	-6.4236×10^{-4}	2.0966×10^{-3}	1.4543×10^{-3}
mean	25–50	-2.4016×10^{-3}	9.9475×10^{-2}	9.7070×10^{-2}	-5.1877×10^{-4}	1.0476×10^{-3}	5.2881×10^{-4}

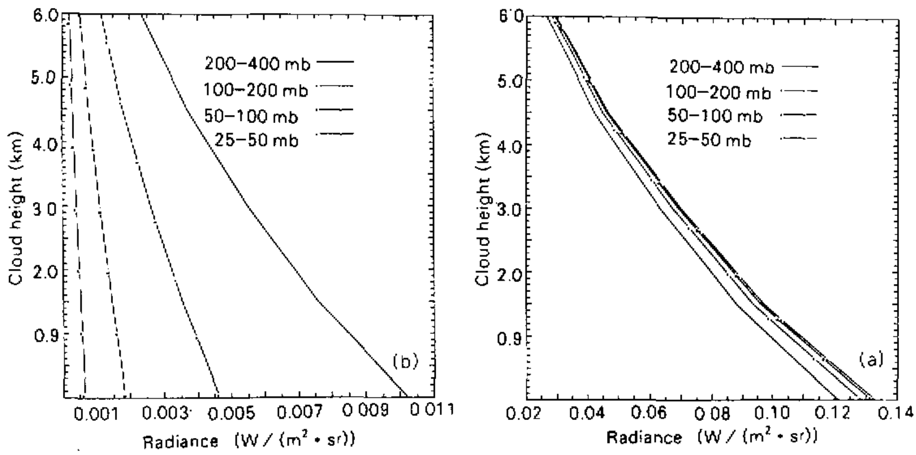


Fig. 6. The wideband and modulated signals as a function of cloud height.

Table 21. Radiance (W $m^{-2}sr^{-1}$) same as Table 1 except for cloud height = 3.0 km

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-3.6069×10^{-4}	6.3485×10^{-2}	6.3125×10^{-2}	-2.2830×10^{-4}	5.7176×10^{-3}	5.4893×10^{-3}
mean	100–200	-6.4530×10^{-4}	6.7820×10^{-2}	6.7175×10^{-2}	-3.4091×10^{-4}	2.9567×10^{-3}	2.6157×10^{-3}
mean	50–100	-1.0106×10^{-3}	7.0050×10^{-2}	6.9040×10^{-2}	-3.8970×10^{-4}	$1.4994E^{-3}$	$1.1097E^{-3}$
mean	25–50	-1.3718×10^{-3}	7.1175×10^{-2}	6.9800×10^{-2}	-3.3279×10^{-4}	7.4916×10^{-4}	4.1636×10^{-4}

Table 22. Radiance (W $m^{-2}sr^{-1}$) same as Table 1 except for cloud height = 4.5 km

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-1.6929×10^{-4}	4.2313×10^{-2}	4.2143×10^{-2}	-1.0920×10^{-4}	3.8081×10^{-3}	3.6989×10^{-3}
mean	100–200	-3.0981×10^{-4}	4.5201×10^{-2}	4.4891×10^{-2}	-1.7185×10^{-4}	1.9692×10^{-3}	1.7974×10^{-3}
mean	50–100	-5.0105×10^{-4}	4.6685×10^{-2}	4.6184×10^{-2}	-2.1066×10^{-4}	9.9862×10^{-4}	7.8795×10^{-4}
mean	25–50	-7.0230×10^{-4}	4.7434×10^{-2}	4.6731×10^{-2}	-1.9181×10^{-4}	9.9896×10^{-4}	3.0715×10^{-4}

Table 23. Radiance (W $m^{-2}sr^{-1}$) same as Table 1 except for cloud height = 6.0 km

CO profile	Pressure (hPa)	Sw (diff)	Sw (surface)	Sw (total)	Sm (diff)	Sm (surface)	Sm (total)
mean	200–400	-7.1335×10^{-5}	2.6514×10^{-2}	2.6443×10^{-2}	-4.6686×10^{-5}	2.3842×10^{-3}	1.1559×10^{-3}
mean	100–200	-1.3319×10^{-4}	2.8323×10^{-2}	2.8190×10^{-2}	-7.7036×10^{-5}	1.2329×10^{-3}	1.3376×10^{-3}
mean	50–100	-2.2237×10^{-4}	2.9252×10^{-2}	2.9030×10^{-2}	-1.0131×10^{-4}	6.2524×10^{-4}	5.2393×10^{-4}
mean	25–50	-3.2251×10^{-4}	2.9721×10^{-2}	2.9398×10^{-2}	-9.8967×10^{-5}	3.1240×10^{-4}	2.1344×10^{-4}

3.2 *The test to determine cloud height with wide–band signals*

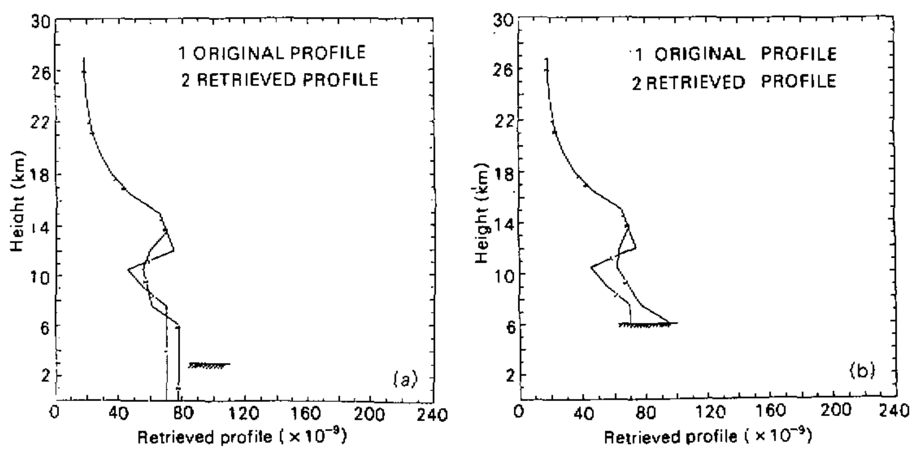
Because the wideband signal changes very much with the cloud height, it is possible to deduce the cloud temperature in the same way as used to determine the surface temperature. In this test, the regression relationship between the wide–band signal S_{w0} and the surface temperature T_g was calculated without taking clouds into account. Then, the wide–band signal calculated by placing a black surface at the cloud height was used to find out the corresponding temperature from the regression relationship, assuming that the CO profile and temperature profile were known exactly. The difference of the deduced temperature and the true cloud temperature is shown in Table 24. It is clear that the difference is very small for clouds lower than 4.5 km, but a little larger for clouds at the height of 6 km. This is caused by the assumption in calculating the regression relationship that the underlying surface is the ground surface thus all levels below the cloud height contribute to the signals. This difference is expected to be less if an estimated cloud height is used to correct the regression relationship.

After the cloud temperature is determined, it is possible to infer the cloud height from an independent measurement of atmospheric temperature profile. The error of the cloud height will be about several hundred meter. As long as the cloud heights are known, it is possible to retrieve the CO profile above the cloud top in the same way as in the non–cloud case. The only change will be to move the ground surface to the cloud height.

It should be mentioned that this is merely a simpified test. Clouds are not completely black especially the higher clouds such as cirrus which can hardly be considered as a black body. It needs further study to assess this effect.

3.3 *The retrieval experiment with clouds at various levels*

The experiment was carried out with solid clouds at 3,6,7.5 and 9 km, respectively. 4 CO profiles were tested for each case. Figs. 7 (a–f) show some retrieved profiles.



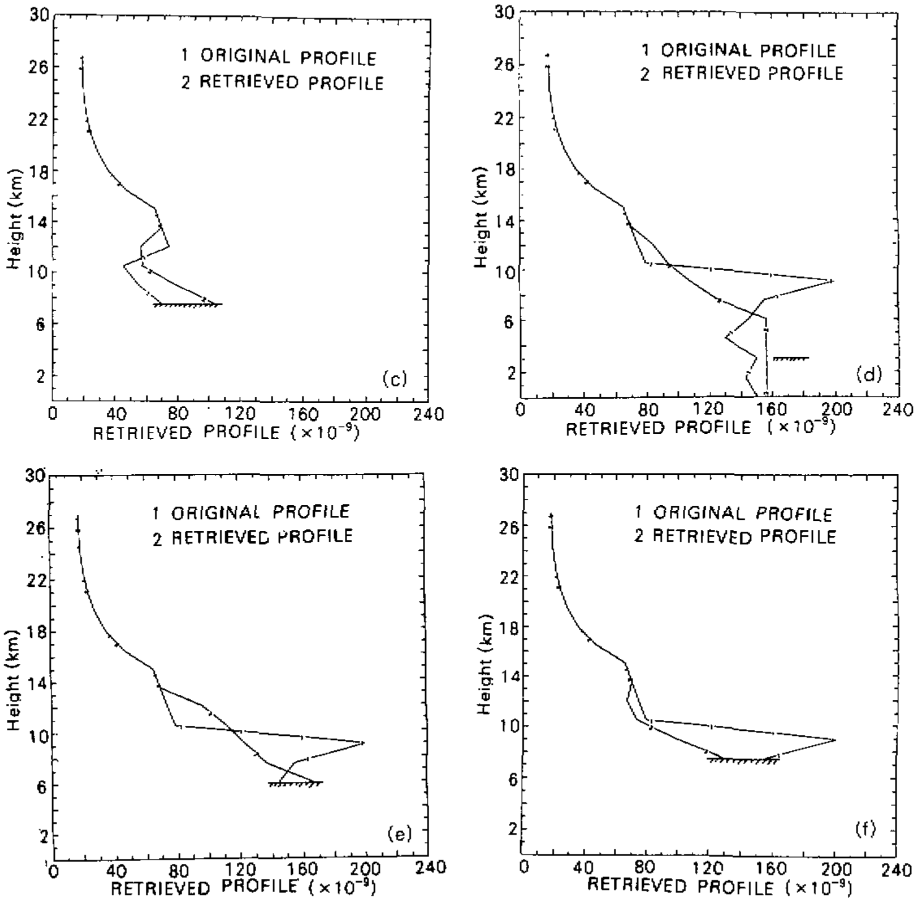


Fig. 7 (a–f). Typical retrieved profiles with cloud height of 3,6,7.5 resepctively, The cloud height is indicated in the figure. The 53s and 64n.2 are the latitudes where the CO profiles used in the present study were measured. (a) 53s, cloud height 3 km; (b) 53s, 6 km; (c) 53s, 7.5 km; (d) 64n.2, 3 km; (e) 64n.2, 6 km; (f) 64n.2, 7.5 km.

Table 24. Comparison of the true cloud temperature and the deduced cloud temperature

Cloud Height (km)	True Temperature (K)	Dedeuced Temperature (K)	Difference (K)
1.5	282.27	282.84	0.57
3.0	273.96	274.07	0.11
4.5	264.52	263.77	−0.75
6.0	254.42	252.36	−2.06

The retrieval errors for cloudy cases compared with the clear cases are shown in Tables 25–28. For the case with clouds at 3 km level, the retrieved profile is almost the same as in the case without clouds, although the retrieval errors are slightly larger.

As shown in Table 23, the modulated signal with clouds at 6 km is about 5×10^{-5} W $m^{-2}sr^{-1}$ which is about 3 times the NER (1.8×10^{-5} W $m^{-2}sr^{-1}$) for 200 K thermoelectric cooler. In this case, the retrieval errors with higher NER (Table 26) are worse than that with lower NER (Table 27). The latter NER can be obtained by using the 77 K Stirling cycle cooler.

Table 25. The r.m.s. Errors in the retrieved profile above 3 km and the residual for cases with solid cloud at 3 km and without clouds (Chahine method 3 with the ground temperature T_g being iterated, first guess: mean profile, measurement error = 1.8×10^{-5} W $m^{-2}sr^{-1}$, T profile 40°N without error)

CO Profile	Case	ΔT_g (K)	ΔS_{md}(max)	Profile With respect to q_0 r.m.s.ab	Profile With respect to q_0 r.m.s.rela	Signal With respect to S_0 r.m.s.ab	Signal With respect to S_0 r.m.s.rela
64n.2	Clear	0.48	1.62×10^{-5}	2.29×10^{-8}	0.131	1.59×10^{-5}	0.0126
64n.2	Cloudy	0.71	1.64×10^{-5}	2.47×10^{-8}	0.142	1.40×10^{-5}	0.0290
58n	Clear	0.48	1.62×10^{-5}	1.18×10^{-8}	0.085	1.59×10^{-5}	0.0126
58n	Cloudy	0.71	1.64×10^{-5}	1.26×10^{-8}	0.091	1.40×10^{-5}	0.0290
42n	Clear	0.48	1.62×10^{-5}	1.50×10^{-8}	0.095	1.59×10^{-5}	0.0126
42n	Cloudy	0.71	1.64×10^{-5}	2.48×10^{-8}	0.133	1.40×10^{-5}	0.0290
53s	Clear	0.48	1.62×10^{-5}	5.14×10^{-9}	0.084	1.59×10^{-5}	0.0126
53s	Cloudy	0.71	1.64×10^{-5}	5.93×10^{-9}	0.092	1.40×10^{-5}	0.0290

Table 26. The r.m.s. Errors in the retrieved profile above 6 km and the residual for cases with solid cloud at 6 km and without clouds (same as Table 25)

CO Profile	Case	ΔT_g (K)	ΔS_{md}(max)	Profile With respect to q_0 r.m.s.ab	Profile With respect to q_0 r.m.s.rela	Signal With respect to S_0 r.m.s.ab	Signal With respect to S_0 r.m.s.rela
64n.2	Clear	0.48	1.62×10^{-5}	2.39×10^{-8}	0.137	1.59×10^{-5}	0.0126
64n.2	Cloudy	−0.49	1.73×10^{-5}	3.55×10^{-8}	0.242	1.38×10^{-5}	0.1133
58n	Clear	0.48	1.62×10^{-5}	1.25×10^{-8}	0.090	1.59×10^{-5}	0.0126
58n	Cloudy	0.71	2.07×10^{-5}	1.44×10^{-8}	0.105	1.43×10^{-5}	0.1867
42n	Clear	0.48	1.62×10^{-5}	8.11×10^{-9}	0.072	1.59×10^{-5}	0.0126
42n	Cloudy	0.71	2.15×10^{-5}	7.39×10^{-9}	0.061	1.35×10^{-5}	0.1816
53s	Clear	0.48	1.62×10^{-5}	5.28×10^{-9}	0.087	1.59×10^{-5}	0.0126
53s	Cloudy	0.71	2.33×10^{-5}	1.69×10^{-8}	0.246	1.33×10^{-5}	0.1886

Table 27. The r.m.s. errors in the retrieved profile above 6 km and the residual for cases with cloud at 6 km and without clouds (same as Table 25 except for measurement error = 0.6×10^{-5} W $m^{-2}sr^{-1}$)

CO Profile	Case	ΔT_g (K)	ΔS_{md}(max)	Profile With respect to q_0 r.m.s.ab	Profile With respect to q_0 r.m.s.rela	Signal With respect to S_0 r.m.s.ab	Signal With respect to S_0 r.m.s.rela
64n.2	Clear	0.48	1.62×10^{-5}	2.39×10^{-8}	0.137	1.59×10^{-5}	0.0126
64n.2	Cloudy	−0.29	5.39×10^{-6}	2.33×10^{-8}	0.171	5.32×10^{-6}	0.0565
58n	Clear	0.48	1.62×10^{-5}	1.25×10^{-8}	0.090	1.59×10^{-5}	0.0126
58n	Cloudy	−0.29	6.38×10^{-6}	1.29×10^{-8}	0.106	5.21×10^{-6}	0.0555
42n	Clear	0.48	1.62×10^{-5}	8.11×10^{-9}	0.072	1.59×10^{-5}	0.0126
42n	Cloudy	−0.29	4.92×10^{-6}	7.39×10^{-9}	0.061	3.62×10^{-6}	0.0631
53s	Clear	0.48	1.62×10^{-5}	5.28×10^{-9}	0.087	1.59×10^{-5}	0.0126
53s	Cloudy	−0.27	5.98×10^{-6}	9.27×10^{-9}	0.155	5.73×10^{-6}	0.1066

For the case with clouds at 7.5 km, the received modulated signal is even smaller. In this case, the retrieved profile can still be reasonable, if the radiometer useds 77 K Stirling cycle cooler to obtain lower noise (Table 28). But the improvement is limited by the instrument noise to certain extent.

For the case with clouds at 9 km, the modulated signal is too small to obtain sufficient information for improving the first guess of CO profile.

Table 28. The r.m.s. errors in the retrieved profile above 7.5 km and the residual for cases with solid cloud at 7.5 km and without clouds. (same as Table 25 except for measurement error = 0.6×10^{-5} W $m^{-2}sr^{-1}$)

CO Profile	Case	ΔT_g (K)	ΔS_{md}(max)	Profile With respect to q_0		Signal With respect to S_0	
				r.m.s.ab	r.m.s.rela	r.m.s.ab	r.m.s.rela
64n.2	Clear	0.48	1.62×10^{-5}	2.47×10^{-8}	0.139	1.59×10^{-5}	0.0126
64n.2	Cloudy	0.94	5.09×10^{-6}	2.81×10^{-8}	0.146	4.05×10^{-6}	0.0937
58n	Clear	0.48	1.62×10^{-5}	1.29×10^{-8}	0.093	1.59×10^{-5}	0.0126
58n	Cloudy	0.95	5.15×10^{-6}	1.30×10^{-8}	0.101	3.84×10^{-6}	0.0925
42n	Clear	0.48	1.62×10^{-5}	5.54×10^{-9}	0.057	1.59×10^{-5}	0.0126
42n	Cloudy	0.95	5.18×10^{-6}	6.85×10^{-9}	0.077	3.85×10^{-6}	0.0970
53s	Clear	0.48	1.62×10^{-5}	5.36×10^{-9}	0.089	1.59×10^{-5}	0.0126
53s	Cloudy	0.97	5.31×10^{-5}	1.26×10^{-8}	0.200	4.95×10^{-6}	0.1526

4. Summary

In this paper, the effects of CH_4, O_3, H_2O and N_2O on the retrieval results have been assessed. The scheme of retrieval in the presence of solid clouds has been proposed. The following conclusion may be drawn from the above study:

a. The effect of CH_4 and N_2O may be well accounted by their mean profile. The effect of O_3 may be represented by a typical middle–latitude O_3 profile. But for H_2O, a knowledge of H_2O profile with less than 50% uncertainty is needed.

b. Assuming that clouds are a black surface, the CO profile above the clouds may be retrieved for clouds at 3 km and at 6 km with measurement errors of 1.8×10^{-5}W $m^{-2}sr^{-1}$ and for clouds at 7.5 km with measurement errors of 0.6×10^{-5}W $m^{-2}sr^{-1}$. But for the case with clouds higher than 9 km, the modulated signal is too small to obtain sufficient information.

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Errata of the paper "A Diagnostic Study of Moist Potential Vorticity Generation in an Extratropical Cyclone" by Zuohao Cao and G.W.K. Moore published in No. 2, 1998, AAS page 152–page 166

Wrong	Correct
1. P. 157, line 2 "The vector $\nabla\theta\times p$ is called ……"	"The vector $\nabla\theta\times\nabla p$ is called ……"
2. P. 160, Eq.(5), line 2 $-\frac{\partial\theta}{\partial y}(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x})+\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y})-\frac{\partial\theta}{\partial x}(\cdots\cdots$	$-\frac{\partial\theta}{\partial y}(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y})-\frac{\partial\theta}{\partial x}(\cdots\cdots$
3. P. 162, Eq.(6) $\frac{d}{dt}(\frac{\zeta_a}{\rho}\cdot\nabla\theta_e)\mathrm{A}\rho T(\frac{\partial\overline{V}_g}{\partial n}\cdot\nabla_p q)$	$\frac{d}{dt}(\frac{\zeta_a}{\rho}\cdot\nabla\theta_e)=\mathrm{A}\rho T(\frac{\partial\overline{V}_g}{\partial n}\cdot\nabla_p q)$
4. P. 163, Eq.(7) $MPV\vert_{t_2}-MPV\vert_{t_1}+\overline{V}_h\cdot\nabla_h(MPV)\Delta t$ $+w\overline{\frac{\partial(MPV)}{\partial p}}\Delta t=\overline{A(\nabla\theta\times\nabla p\cdot\nabla q}\Delta t$	$MPV\vert_{t_2}-MPV\vert_{t_1}+\overline{\overline{V}_h\cdot\nabla_h(MPV)}\Delta t$ $+w\overline{\frac{\partial(MPV)}{\partial p}}\Delta t=\overline{A(\nabla\theta\times\nabla p)\cdot\nabla q}\Delta t$