Nonlinear Stability Analysis of the Zonal Flows at Middle and High Latitudes①

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ABSTRACT

An attempt has been made to apply Arnol'd type nonlinear stability criteria to the diagnostic study of the persistence (stability) or breakdown (instability) of the atmospheric flows. In the case of the blocking high, the cut–off low and the zonal flow, the relationships of the geostrophic stream function versus the potential vorticity of the observed atmosphere are analyzed, which indicates that Arnol'd second type nonlinear stability theorem is more relevant to the observed atmosphere than the first one. For both the stable and unstable zonal flows, Arnol'd second type nonlinear stability criteria are applied to the diagnosis. The primary results show that our analyses correspond well to the evolution of the atmospheric motions. The synoptically stable zonal flows satisfy Arnol'd second type nonlinear stability criteria; while the synoptically unstable ones violate the nonlinear stability criteria.

Key words: Zonal flow, Quasi–geostrophic, Nonlinear stability, Persistence

1. Introduction

In atmospheric dynamics, the stable flows correspond to those ones which persist the configuration well in a given region, while the unstable ones are those whose patterns break down in a short period of time. It becomes crucial to decide if a basic flow is stable or unstable. Most of the previous works were concentrated upon the linear or weakly nonlinear stability analyses for a given basic state. The linear stability analysis is applicable to the case that the amplitude of the perturbation is sufficiently small; while the weakly nonlinear stability analysis deals with the finite amplitude perturbations, with the restriction that the supercritical number is small enough, and the validity of its results should be checked by other approaches, such as the numerical method, etc.

On the basis of Arnol'd's work in the 1960's, the nonlinear stability theorems are fully developed for the quasi–geostrophic models. Holm et al. (1985), Zeng (1989) and Ripa (1992) set up a series of Arnol'd first type nonlinear stability criteria, which correspond to the cases in which the functionals are positive definite, while McIntyre and Shepherd (1987), Mu (1991, 1992), Ripa (1992) and Mu and Wang (1992) established some Arnol'd second type nonlinear stability criteria in which the functionals are negatively definite.

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In the application of Arnol'd type nonlinear stability criteria to the observed atmosphere, it is necessary to determine the relationship of the stream function versus the potential vorticity (referred as RSP). Dowling (1993, 1995) discovered that the RSP is approximately linear in the atmosphere of Jupiter, and by virtue of Arnol'd second nonlinear stability theorem he succeeded in establishing the neutral stability of the zonal flows in the midlatitude atmosphere of Jupiter. Some others also considered the RSP from different points of view, such as the blocking cases, see Ek et al. (1994). Tang (1997) attacked the problem of applying the nonlinear stability criteria to the persistence or breakdown of the zonal flows, and compared his results with those obtained by the Rayleigh–Kuo theorem, which shows that the Arnol'd type nonlinear stability theorem is much more applicable than Rayleigh–Kuo's linear stability criterion.

Within the framework of two–dimensional quasi–geostrophic dynamics, the aim of this paper is two–fold: First, the RSP for some characteristic synoptical patterns is further studied, i.e. the block high, the cut–off low and the zonal flow. Secondly, Arnol'd 's nonlinear stability theorems are applied to the study of the maintenance or breakdown of the simplest case, i.e. the zonal flows at middle and high latitudes. The paper is arranged as follows: The nonlinear stability criteria of the two–dimensional quasi–geostrophic model are reviewed in Section 2; then the RSP in the atmosphere for the above three cases is analyzed in Section 3; in Section 4 the nonlinear stability criteria are applied to the diagnosis of the persistence or breakdown of the zonal flows at middle and high latitudes; the conclusions and their potential applications are presented and some problems are discussed in the final section.

2. Arnol'd type nonlinear stability criteria of the two–dimensional quasi–geostrophic flows

The two–dimensional quasi–geostrophic motions in the atmosphere can be described by the potential equation on the β–plane:

$$\frac{\partial P}{\partial t} + J(\Phi, P) = 0 \ , \qquad (2.1)$$

$$P = \nabla^2 \Phi - F\Phi + f(x,y) \ , \qquad (2.2)$$

where $J(\cdot\ ,\ \cdot)$ is the Jacobian, ∇^2 is the Laplacian. x and y are the eastward and northward directed coordinates respectively, t is time. Φ is the stream function, P is the potential vorticity. $F = f_0^2 / gd, f(x,y) = f_0 h(x,y) / H + f_0 + \beta y$ represents the combined influence of the topography and the Coriolis force to the potential vorticity, H is the characteristic height of the atmosphere, f_0 is the constant Coriolis parameter.

The horizontal domain D under consideration is a bounded, simply (or multiply) connected domain on the β plane with smooth boundary. The boundary conditions are the usual ones of no normal flow and conservation of circulation, namely

$$\frac{\partial \Phi}{\partial s} = 0 \quad \text{on} \quad \partial D, \quad \frac{d}{dt}\int_{\partial D} \nabla \Phi \cdot \vec{n} ds = 0 \ , \qquad (2.3)$$

where s is arc length along the boundary ∂D, and $\vec{n}$ is the outward unit normal.

Now suppose that $(\Phi, P) = (\Psi(x,y), Q(x,y))$ is the basic flow to (2.1), (2.2) which satisfies the boundary condition (2.3). It follows that $J(\Psi, Q) = 0$, and consequently the isoline of the stream function Ψ coincides to that of the potential vorticity Q, and we can further assume that there exists a continuous function $\Psi(\cdot)$ such that

$$\Psi(x,y)=\Psi(Q(x,y)), \quad (x,y)\in D . \tag{2.4}$$

The finite amplitude disturbance (ψ,q) to this basic flow is defined according to

$$\Phi=\Psi+\psi , \quad P=Q+q ,$$

with

$$q(x,y,t)=\nabla^2\psi-F\psi .$$

Arnol'd (1965, 1966), McIntyre and Shepherd (1987), Mu (1992) and Mu and Shepherd et al. (1994) *established the following nonlinear stability criteria.*

Arnol'd's First Theorem Assume the stream function Ψ is a smooth function of potential vorticity Q, and there exist two positive constants C_1 and C_2 such that

$$0< C_1=\min\frac{d\Psi}{dQ}\leqslant \max\frac{d\Psi}{dQ}=C_2<+\infty . \tag{2.5}$$

Then the basic flow (Ψ,Q) is nonlinearly stable.

Arnol'd's Second Theorem Assume the stream function Ψ is a smooth function of potential vorticity Q, and there exist two positive constants C_1 and C_2 such that

$$0< C_1=\min\left(-\frac{d\Psi}{dQ}\right)\leqslant \max\left(-\frac{d\Psi}{dQ}\right)=C_2<+\infty , \tag{2.6}$$

and

$$C_1(\lambda_2+F)>1 . \tag{2.7}$$

Then the basic flow (Ψ,Q) is nonlinearly stable. Here λ_2 is the least positive eigenvalue of the operator ∇^2 with the following homogeneous boundary conditions:

$$\left.\frac{\partial\psi}{\partial s}\right|_{\partial D}=0 , \quad \int_{\partial D}\nabla\Phi\cdot\vec{n}ds=0 .$$

Therefore, we see that Arnol'd's first theorem corresponds to the case that the stream function is monotonic increasing with the potential vorticity, while the second one corresponds to the case that the stream function is monotonic decreasing with the potential vorticity.

3. The relation of the streamfunction versus the potential vorticity at 500 hPa

Since Arnol'd type nonlinear stability criteria are presented in terms of RSP, and the RSP of an atmospheric or oceanographic system can provide crucial insight into the dynamical study of the systems, especially for the energy transport (Haines et al., 1993). Therefore, it is necessary to investigate the RSP of the observed atmosphere at first, and then diagnose which of Arnol'd's two stability criteria is more relevant to the observed atmosphere. In this section, the ECMWF analyzed data with a resolution 2.5°×2.5° at 500 hPa are utilized in an attempt to obtain the RSP in the framework of quasi-geostrophic barotropic fluid dynamics, and to investigate which kind of Arnol'd two nonlinear stability theorems is more relevant and applicable.

For simplicity, the effect of the topography is omitted. The domain D is a periodic channel

$$D=\{-X\leqslant x\leqslant X,-L\leqslant y\leqslant L\},$$

where $X=\pi(\theta_2-\theta_1)a/360$, and $L=\pi(\varphi_2-\varphi_1)a/360$, θ_1,θ_2 are the start and end longitudes, and φ_1 and φ_2 are the latitudinal boundaries, a is the radius of the earth. Within the two–dimensional geostrophic fluid dynamics, the geostrophic streamfunction Φ_{ij}, the geopotential height Z_{ij} and the potential vorticity P_{ij} at the point (i,j) satisfy

$$\Phi_{ij}=gZ_{ij}/f_0\ ,$$

and

$$P_{ij}=\nabla^2\Phi_{ij}-F\Phi_{ij}+f_0+\beta y(j)\ ,$$

in which the Laplace term is calculated by the centered finite difference scheme

$$\nabla^2\Phi_{ij}=\frac{\Phi_{i+1,j}+\Phi_{i-1,j}-2\Phi_{ij}}{\delta x_j^2}+\frac{\Phi_{i,j+1}+\Phi_{i,j-1}-2\Phi_{ij}}{\delta y^2}\ ,$$

and

$$\delta y=\pi a/72,\quad \delta x_j=\pi a\cos[\varphi_1+2.5(j-1)\pi/180]/72\ .$$

Here the zonal grid–spacing decreases northward due to the use of the latitude longitude projection. Denote

$$y(j)=\pi a(j-(N+1)/2)/72\ ,$$

$$f_0=2\Omega\sin((\varphi_1+\varphi_2)/2),\quad \beta=2\Omega\cos((\varphi_1+\varphi_2)/2)/a,\quad F=f_0^2/gH\ .$$

The parameters are taken as

$$a=6.371\times10^6\ \text{m},\ H=1\times10^4\ \text{m},\ \Omega=7.292\times10^{-5}\ \text{s}^{-1},$$
$$g=9.81\ \text{ms}^{-2},\ \pi=3.1416.$$

Due to the complexity of the atmospheric motion, it is beyond our power to establish RSP for all kinds of flow patterns of the atmospheric motions. Here, we only choose three characteristic atmospheric flow patterns, i.e., the blocking high, the cut–off low, and the zonal flow, and study their RSP by the scatter diagram approach. The experiments are as follows:

Blocking High

1. Feb. 09–12, 1989, 35–60°N, 15°W–60°E;
2. May 29–31, 1989, 40–70°N, 50–20°W;
3. Feb. 03–07, 1989, 35–75°N, 177.5–100°W.

Cut–off Low

1. Sep. 01–04, 1988, 32.5–67.5°N, 130–90°W;
2. May 03–06, 1989, 27.5–47.5°N, 52.5–87.5°E;
3. Sep. 09–12, 1988, 30–52.5°N, 122.5–97.5°W;
4. Feb. 26–28, 1989, 40–65°N, 20°W–40°E.

Zonal Flow

1. Sep. 05–08, 1988, 37.5–47.5°N, 27.5°W–27.5°E;
2. Feb. 02–05, 1989, 27.5–37.5°N, 72.5–107.5°E;
3. May 10–15, 1989, 37.5–47.5°N, 92.5–147.5°E;
4. Jun. 16–20, 1989, 37.5–47.5°N, 117.5–72.5°W.

Scatter diagrams are plotted for the above experiments, since the scatter diagrams of each pattern show the similar configuration, only one scatter diagram of each pattern is presented here.

From Figs. 1–3 we see that for the three atmospheric flows, the streamfunction is negatively correlated to the potential vorticity, which indicates that Arnol'd's second nonlinear stability criteria are more relevant to the observed atmospheric flows. Further study shows that in the case of the blocking high and cut–off low the RSP is very complicated. Usually they are nonlinear and in some cases they are even multivalued. For the zonal flow case, the situation is comparatively simpler, the stream function is almost proportional to the potential vorticity. This provides us the possibility to make use of Arnol'd's second stability criteria to study the persistence or breakdown of the zonal flows.

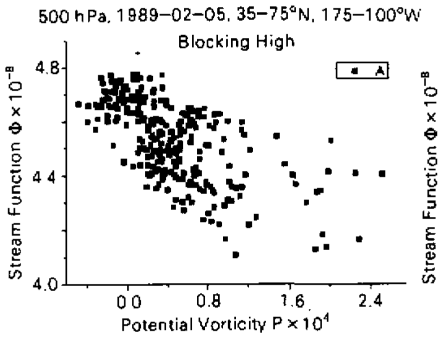

Fig. 1. The scatter diagram of the RSP for the blocking high on Feb. 05, 1989.

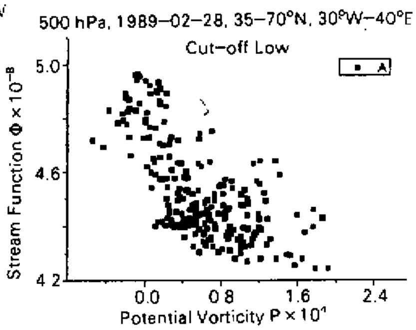

Fig. 2. The scatter diagram of the RSP for the cut–off low on Feb. 28, 1989.

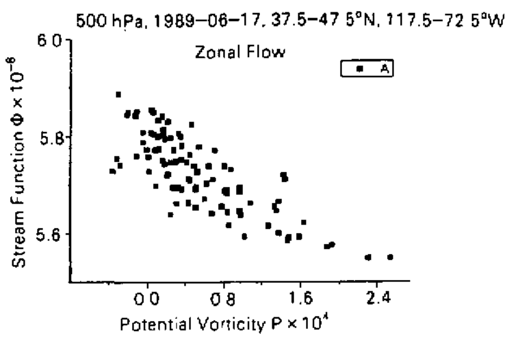

Fig. 3. The scatter diagram of the RSP for the zonal flow on Jun. 17, 1989.

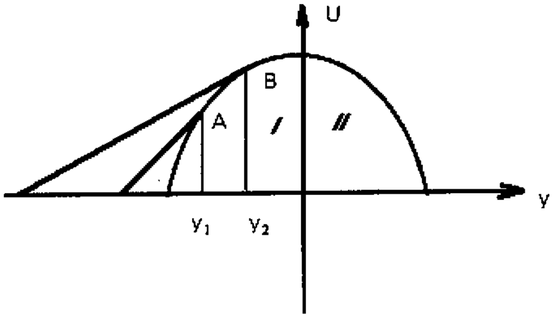

Fig. 4. The general velocity profile in the jet flow region.

Remark 1: The reason why the potential vorticity is negatively correlated to the stream function in the jet flow region of the midlatitudes can be illustrated as follows: The general velocity profile can be expressed as Fig. 4.

In the domain I, $\frac{\partial U}{\partial y} > 0$, for any two points A, B on the velocity profile, with $A = (y_1, U_1)$, $B(y_2, U_2)$ and $y_1 < y_2$, since $\alpha_1 > \alpha_2$, we have that $\frac{\partial U}{\partial y}(y_1) > \frac{\partial U}{\partial y}(y_2)$. It follows that

$$\frac{\partial^2 U}{\partial y^2} \leqslant 0 ,$$

and

$$Q_y = -\frac{\partial^2 U}{\partial y^2} + \beta > 0 \quad \text{in the domain I} .$$

In quite the same way, we have that $Q_y > 0$ in the domain II, and therefore $c = -\frac{U}{Q_y} < 0$, which verifies our assertion.

Another reason might be that if the generalized energy of the atmospheric motion is dominated by a wave whose streamfunction can be expressed approximately as $\Psi \approx \psi_0 + A\sin ky + B\cos ky$, then

$$P = \nabla^2 \Psi - F\Psi + f_0 + \beta y \approx -(k^2 + F)\Psi + f_0 + \beta y .$$

Notice that the streamfunction decreases as the latitude increasing, βy is negatively correlated to Ψ, so does the potential vorticity P.

4. Nonlinear stability analyses of zonal flows

We see from the above scatter diagrams that the ratio $\frac{d\Phi}{dP}$ is negative for the chosen three characteristic atmospheric flow patterns. Therefore, it is reasonable to employ Arnol'd's second nonlinear stability criteria in the explanation of the persistence or breakdown of some atmospheric flows at middle and high latitudes. The zonal flow case, whose RSP is approximately linear, is much simpler than the other two cases, hence we concentrate upon it and restrict the basic flows to those whose streamfunction belongs to the function set E in which

$$E=\{\Phi|\Phi=c(\nabla^2\Phi-F\Phi+f_0+\beta y)\ d \quad \text{for some } c \text{ and } d\}\ ,$$

where c and d are parameters. To get the optimal basic flows to the observed data, we assume that there are M eastward horizontal grid points and N northward grid points, and the cost function $I(c,d)$ is defined as

$$I(c,d)=\sum_{i=1}^{M}\sum_{j=1}^{N}(\Phi_{ij}-cP_{ij}-d)^2\ .$$

Then for the optimal basic flow to the observed data, c and d are determined in such a way that at which the cost function $I(c,d)$ attains its minimum. Denote

$$S_\Phi=\sum_{i=1}^{M}\sum_{j=1}^{N}\Phi_{ij}\ ,\quad S_P=\sum_{i=1}^{M}\sum_{j=1}^{N}P_{ij}\ ,$$

$$S_{P^2}=\sum_{i=1}^{M}\sum_{j=1}^{N}P_{ij}^2\ ,\quad S_{\Phi P}=\sum_{i=1}^{M}\sum_{j=1}^{N}\Phi_{ij}P_{ij}\ .$$

By the least square approach, it is easy to obtain that

$$\begin{aligned} c&=(MNS_{\Phi P}-S_\Phi\cdot S_P)/(MNS_{P^2}-S_P^2)\ ,\\ d&=(S_{\Phi P}\cdot S_P-S_\Phi\cdot S_{P^2})/(S_P^2-MNS_{P^2})\ . \end{aligned} \tag{4.1}$$

Since the optimal basic flow is the nearest one to the observed in the linear REP function set, it possesses the essential features of the observed atmospheric motion. Therefore, after the determination of the optimal basic flow to the observed data, we can then apply Arnol'd type nonlinear stability criteria to analyze it and then use its stabilily or instability to explain the persistence or breakdown of the zonal flows. In the periodic channel, we have (c.f. Liu, 1998)

$$\lambda_2=(\pi/(2L))^2+(\pi/(2X))^2\ .$$

The stability index is defined as

$$I_s=|c(\lambda_2+F)|\ .$$

According to Arnol'd stability cirteria, we know that if $c>0$ or $c<0$ and $I_s>1$, then the basic flow is nonlinearly stable. Otherwise, the basic flow might be unstable. Since the stability index is sensitive to the width of the channel considered, we choose the width of the channel in such a way that I_s is around 1.0. Numerical experiments show that c is of the order 1.0×10^{11}, by scaling analysis we know that cF contributes little to the index, and it is reasonable to take L at about 1.0×10^6 m. So we choose 10 degrees of latitudes as the meridional width. Fourteen individual experiments are implemented, in which seven cases are synoptically stable, the rest are unstable. Here, for synoptical stable we mean that on the practical synoptical maps the flow keeps its essential flow configuration for more than three days, while the unstable one distorts its flow pattern obviously in one or two days. The cases we studied are as follows:

Synoptically stable cases

1. Sep. 05–08, 1988, 37.5–47.5°N, 27.5°W–27.5°E;
2. Feb. 11–14, 1989, 42.5–52.5°N, 122.5–157.5°E;
3. May 22–26, 1989, 32.5–42.5°N, 112.5–77.5°W;
4. May 10–13, 1989, 37.5–47.5°N, 92.5–147.5°E;
5. Feb. 16–20, 1989, 40–50°N, 80–40°W;
6. Apr. 01–04, 1989, 35–45°N, 120–70°W;
7. Mar. 28–31, 1989, 45–55°N, 100–60°W;
8. Jun. 17–20, 1989, 37.5–47.5°N, 117.5–72.5°W.

Synoptically unstable cases

1. Sep. 27–28, 1988, 40–50°N, 130–80°W;
2. Feb. 03–04, 1989, 27.5–37.5°N, 72.5–107.5°E;
3. May 17–19, 1989, 37.5–47.5°N, 92.5–147.5°E;
4. Jun. 04–05, 1989, 40–50°N, 35–75°E;
5. Jun. 27–29, 1989, 35–45°N, 30–90°E;
6. Jan. 16–17, 1989, 32.5–42.5°N, 110–80°W;
7. Jan. 29–30, 1989, 35–45°N, 80–120°E.

4.1 *Numerical Results*

The ECMWF 500 hPa analysis geopotential height dataset is used in the numerical experiments. The generalized disturbance energy is defined as

$$E_{rr} = \|\Phi - cP - d\|^2 / \|\Phi\|^2$$

to estimate the relative magnitude of the disturbance to the observed data in the linear RSP function set. Meanwhile, it is also an index to judge if the RSP reflects well the essential relationship between the streamfunction and the potential vorticity of the observed atmospheric motion. Seven cases of the above eigth synoptically stable cases are nonlinearly stable according to Arnol'd's second nonlinear stability theorem, that is, the stability index $I_s > 1.0$. Only the case in June, 17–20, 1989 failed. Five cases of the seven synoptically unstable ones destroy the Arnol'd's type nonlinear stability theorem. Since all the stable cases possess a general behavior, so do the unstable flows, we choose one characteristic example for each pattern and plot its scatter diagram of the RSP. For the synoptically stable case, it is obvious that the scatter diagram keeps its form well from Feb. 16 to Feb. 20 (see Figs. 5–8), while in the unstable case, the scatter diagram distorts greatly just one day after Feb. 03 (see Figs. 8–9).

Table 1. The generalized distarbance energy and stability index, stable case

1989.2.16—1989.2.20, 40°–50°N, 80°–40°W

Date	I_s	E_{rr}
16	2.80	3.22×10^{-4}
17	2.53	3.40×10^{-4}
18	1.71	6.82×10^{-4}
19	0.69	6.44×10^{-4}
20	2.93	3.67×10^{-4}

Table 2. The generalized disturbance energy and stability index, unstable case

1989.2.3—1989.2.4, 27.5°–37.5°N, 72.5°–107.5°E

Date	I_s	E_{rr}
3	0.77	1.28×10^{-4}
4	0.80	1.07×10^{-4}

Generally speaking, the synoptical persistence or breakdown of the zonal flows can be diagnosed by Arnol'd's second nonlinear stability criterion. In most of the cases, the theoretical stability results correspond well to the evolution of the chosen zonal flows.

It should be mentioned that Arnol'd type nonlinear stability criteria are only sufficient conditions to ensure the nonlinear stability of the basic flows, those which do not satisfy them might be stable as well. Meanwhile, at present only the general Arnol'd's nonlinear stability theorems are applied, a more accurate nonlinear stability theorem for the special periodic case is not implemented. This might be one of the reasons that a few of the cases do not satisfy the nonlinear stability analysis. Some numerical experiments show that the improved Arnol'd's nonlinear stability criterion makes the results obtained here much better, especially in the synoptically unstable cases. However, the boundary conditions there are relatively more strict, and should be checked carefully.

Meanwhile, the linear Rayleigh–Kuo stability criterion (RKC) is also utilized and the comparison of the results between the Arnold's and the Rayleigh–Kuo's is analyzed. Generally speaking, they correspond well in most of the cases. However, the results obtained by RKC are somewhat uncertain in the sense that in a complete synoptically stable or unstable process, it is sometimes stable in one day, and then unstable in the next day. The results obtained by Arnold's criteria are relatively in accordance with each other. The reason might be that the Rayleigh–Kuo's criterion needs only one point to break the stability criterion, so it is very sensitive to the data and the algorithm of the computation.

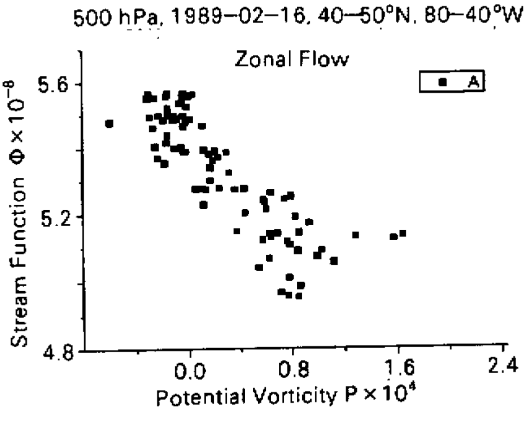

Fig. 5. The scatter diagram of the RSP at 500 hPa for Feb.16, 1989.

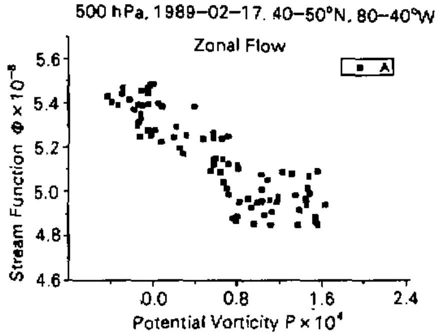

Fig. 6. The scatter diagram of the RSP at 500 hPa for Feb. 17, 1989.

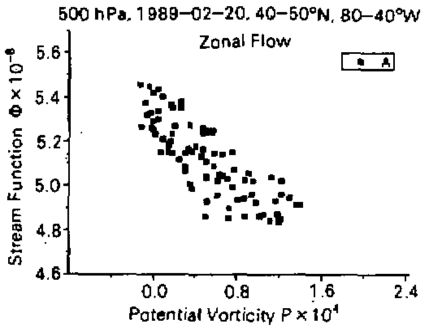

Fig. 7. The scatter diagram of the RSP at 500 hPa for Feb. 20, 1989.

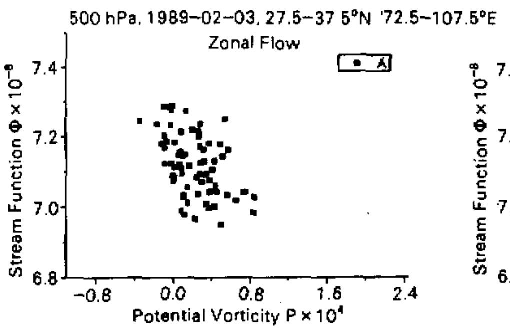

Fig. 8. The scatter diagram of the RSP at 500 hPa for Feb. 03, 1989.

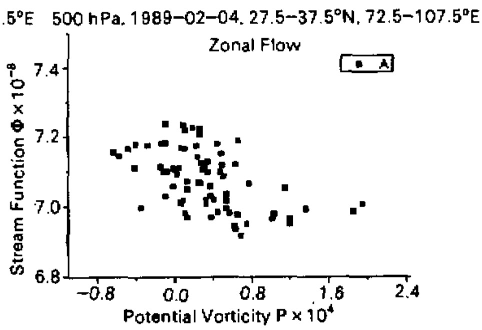

Fig. 9. The scatter diagram of the RSP at 500 hPa for Feb. 04, 1989.

In our numerical experiments, we notice that though the term $F\varphi$ is not important in the potential vorticity, it dominates the generalized energy

$$\frac{1}{2}[\|\nabla\varphi\|^2 + F\|\varphi\|^2] + \frac{1}{2}c\|P\|^2 \quad .$$

In fact, the scaling analayses show that $\|\nabla\varphi\|^2$ is of the order of 10^{14}—10^{15}, $F\|\varphi\|^2$ is of the order of 10^{17} and $c\|P\|^2$ is of the order of 10^{14}. Therefore, from the energy point of view, the term $F\varphi$ is not negligible in the potential energy in the quasi-geostrophic dynamics.

Another fact is that the boundary energy flux

$$\int_{\partial D} \Psi\frac{\partial\Psi}{\partial n}ds$$

is of the order of 1×10^{10} and is negligible comparing with the kinetic energy and potential energy in the perturbation conservation law. So the boundary conditions (2.3) are approximately satisfied for the flows we studied here.

5. Discussions

A qualitative RSP for the chosen three characteristic atmospheric flow patterns is investigated by the scatter diagram approach. Arnol'd type nonlinear stability criteria for two–dimensional quasi–geostrophic flows have been utilized to study the persistence or breakdown of the zonal flows at middle and high latitudes, which set up a bridge to connect the theoretical stability research with the application to the observed atmospheric motions. Experiments show that the geostrophic streamfunction and the potential vorticity are negatively correlated for the chosen three characteristic atmospheric flow patterns, which makes it possible to apply Arnol'd's second type nonlinear stability to the explanation of the persistence or breakdown of some atmospheric flows.

For both the synoptically stable zonal flows and synoptically unstable ones, Arnol'd's second nonlinear stability criteria are applied in the diagnosis. The primary results show that the theoretical analyses correspond well to the evolution of the real atmospheric motions. Most of the results are consistent with Arnol'd's stability criteria. The synoptically stable atmospheric zonal flows satisfy Arnol'd second type nonlinear stability criteria; while the unstable zonal flows destroy the nonlinear stability criteria.

Comparing our results to those of Dowling (1993, 1995), we find that Dowling presented evidence that Jupiter zonal winds are neutrally stable with respect to Arnol'd's second stability theorem. We show here that the theorem is applicable to the diagnosis of both stable and unstable zonal flows in the atmosphere of the earth.

It should be emphasized that the work in this paper is merely at its early stage. Only the zonal flow is treated by the nonlinear stability analysis, and in this special case, there are still a few examples in which the stability analyses do not coincide with the observed atmospheric motions. There are many important aspects of this subject, which have not been attacked here, expected to be investigated in the future. Some of them are mentioned as follows: First, due to the complexity of the atmospheric motions, the two–dimensional quasi–geostrophic model, which is the first order approximation of the primitive equations, might be too simple to include all the physical processes. More accurate models, such as the two–layer or multi–layer models, or the three–dimensional continuously stratified model should be considered. Secondly, we only investigated the stability and the instability of zonal flows, of which the RSP can be approximated by a linear relationship. The RSP of more interesting problems such as the blocking high is usually nonlinear, which should be attacked in the future in spite of its difficulties and complexity. Finally, Arnol'd's nonlinear stability criteria are established on the basis of the conservative system, the corresponding nonlinear stability theorems for the system which includes the viscosity, the dissipation, and the external forcing should be studied for sake of both our better understanding of the complicated nonlinear phenomena in the atmospheric motion and the prediction of the weather.

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