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Wave–Mean Flow Interaction: the Role of Continuous–Spectrum Disturbances①

Zeng Qingcun (曾庆存)
Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100080,
Zhang Minghua (张明华)
Institute for Terrestrial and Planetary Atmospheres, State University of New York at Stony Brook, Stony Brook, NY 11794–5000

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ABSTRACT

Traditionally, "eddy feeds zonal flow" in the atmosphere is considered as a result of decaying unstable waves. We show that disturbances made of non–modal solutions —the continuous–spectrum disturbances—can also effectively transport zonal angular momentum and interact with the zonal basic flow. These disturbances, though stable, eventually decay, losing their energy to strengthen the westerly jets in the atmosphere.

Calculations with observational data illustrate that the atmospheric zonal flow is maintained primarily by continuous–spectrum disturbances rather than by unstable waves. Angular momentum transport by continuous–spectrum disturbances is one order of magnitude larger than that by all kinds of normal modes (referred as discrete–spectrum disturbances) including unstable waves.

Key words: Maintenance of zonal flow, Continuous–spectrum, Angular momentum, Transport

1. Introduction

Westerlies are one of the most important components of the general circulation of the atmosphere. Their movement and variability affect the characteristics of planetary waves, which further control blocking events and other synoptic scale weather systems. Study of the maintenance of westerly and its variability has attracted many interests in the past because of its theoretical and practical significances.

Investigation of the maintenance of zonal flow in the atmosphere dated back to Jeffreys (1926) who attributed it to the meridional transport of angular momentum by eddies. Many years later, Rossby (1947) reported that this process was accomplished by the lateral mixing of large scale atmospheric disturbances. After that, Starr (1948) and Yeh (1949) showed that tilted troughs and ridges in the atmosphere could cause the angular momentum transport. Starr (1948), Mintz (1954), and Yeh and Yang (1955) calculated this transport by using observed atmospheric data. At the same time, Kuo (1951) reported the counter–gradient transport of energy by eddies, concluding that energy of the zonal flow in the atmosphere is maintained by conversion of energy from eddies. Later on, Lorenz (1967) systematically described

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this energy conversion process, and "eddy feeds zonal flow" became a well accepted notion in the literature. Because the direction of this energy conversion differs from that in the classical turbulence theory, Starr referred to this phenomenon as "negative viscosity". On the other hand, calculations by Palmen and Vuorela (1963) showed that angular momentum transport by the mean meridional circulation is important only in low latitudes. Subsequent laboratory studies have supported these findings (e.g. Pfeffer et al., 1980).

Up to now, most of the studies consider the disturbances that carry out the angular momentum transport as composed of unstable decaying waves. This is simply because unstable modes have tilted troughs and ridges, and neutral modes do not exhibit tilted structures. However, observations show that tilted troughs and ridges always prevail in the atmosphere regardless of the stability of the atmospheric basic flow. The wave packet theory (e.g., Zeng, 1983a; 1983b) has also shown that transfer of energy from disturbances to zonal flow depends on the structures of disturbances and their positions relative to the zonal flow regardless of its stability. These pose doubts to the pre–requisite of instability of the basic flow as a condition of the formation of titled structures of disturbances.

The problem is elucidated by results of several recent studies which are reported that most previous researches have only taken the discrete–spectrum disturbances into account and missed the continuous–spectrum disturbances (e.g., Zeng, 1979; Farrell, 1984; Farrel et al., 1994; Zhang and Zeng, 1997). That is, only one part of the atmospheric disturbances, composed of normal modes (or discrete spectra) is typically considered; while the other part, composed of non–modal solutions (or continuous spectrum), is often missed. And the continuous–spectrum disturbances, in terms of both disturbance energy and enstrophy, constitute the major part of atmospheric disturbances (Zhang, 1987; Zhang and Zeng, 1999).

In this paper, using atmospheric observations at 300 hPa, we present calculations of angular momentum transport by discrete–spectrum disturbances (normal modes including unstable waves) and continuous–spectrum disturbances (non–modal solutions). The objectives of the study are to clarify the mechanism of angular momentum transport by eddies, and to provide a quantitative description of the relative role of the modal and non–modal forms of disturbances in the maintenance of the atmospheric zonal flow. This work builds on the work of Lu et al. (1986) and Lu (1987) who calculated the angular momentum transport by continuous–spectrum disturbances using idealized models. A related work was done by Zeng et al. (1986; 1987) who argued that the maintenance of the real atmospheric zonal flow is primarily due to continuous–spectrum disturbances. This paper focuses on the angular momentum transport of disturbances, and therefore it only treats one part of the wave–mean flow interaction problem. Eddy–induced meridional circulation and the net eddy acceleration of zonal flow (Edmon et al., 1980) in the framework of discrete–spectrum and continuous–spectrum disturbances are subjects of future studies.

The paper is organized as follow. The second section briefly describes a weakly nonlinear wave–mean flow interaction model and puts the angular momentum transport of disturbances in perspective. Section 3 presents a theoretical formulation of angular momentum transport by classical normal modes (discrete–spectrum disturbances). Section 4 discusses the angular momentum transport of non–modal disturbances (continuous–spectrum disturbances) using results of the wave packet theory. Section 5 shows calculations of the angular momentum transports by continuous–spectrum disturbances and discrete–spectrum disturbances in the real atmosphere. Section 6 discusses the role of continuous–spectrum disturbances in atmospheric stationary waves and transient waves. The last section contains a brief summary and discussion.

2. Interaction model

The governing equation of the non–divergent barotropic quasi–geostrophic model of the atmosphere is the conservation of absolute vorticity. In this model, the interaction between disturbance and zonal flow can be written as:

$$\frac{\partial \bar{\zeta}}{\partial t} = -\overline{J(\psi', \zeta')} + \bar{F}, \tag{1}$$

$$\frac{\partial \zeta'}{\partial t} = -J(\bar{\psi}, \zeta') - J(\psi', \bar{\zeta}) - J(\psi', \zeta') + \overline{J(\psi', \zeta')} + F'. \tag{2}$$

Here, an overbar represents the zonal mean. J is the Jacobi operator, $\bar{\zeta}$ and ζ' are the absolute vorticity of the zonal flow and the disturbance vorticity with $\bar{\zeta} = 2\omega\cos\theta + \partial \bar{v}_\lambda \sin\theta / a\partial\theta$ and $\zeta' = a^{-2}\nabla^2\psi'$, where θ is the co–latitude, λ is the longitude, and a is the radius of the Earth. $\bar{F}$ and F' are the zonal forcing and disturbance forcing. Other symbols are as commonly used.

Neglecting the forcing and considering

$$v'_\lambda = \frac{\partial \psi'}{a\partial\theta}, \tag{3}$$

$$v'_\theta = -\frac{\partial \psi'}{a\sin\theta\partial\lambda}, \tag{4}$$

integrating (1) with respect to θ, we can write (1) as:

$$\frac{\partial a\bar{v}_\lambda \sin\theta}{\partial t} = -\frac{\partial(\overline{v'_\lambda v'_\theta}\, a\sin^2\theta)}{a\sin\theta\partial\theta}, \tag{5}$$

(5) can be also obtained by taking the zonal average of the momentum equation in the λ direction.

Note that when horizontal divergence and baroclinicity are taken into account, the variation of zonal flow is controlled by the convergence of meridional and vertical angular momentum transports of disturbances and by the mean meridional circulation. The vertical transport is typically one order of magnitude smaller than the horizontal transport, and the mean meridional circulation can be determined by the horizontal transport of angular momentum (e.g., the non–acceleration theorem). Therefore, the horizontal transport of angular momentum is essential in the wave–mean interaction problem, and in this study we focus on the influence of the convergence of horizontal angular momentum transport of disturbances on the variation of zonal flow.

Taking $\bar{F} = F' = 0$ (no forcing and no dissipation) in (1) and (2), we have the energy and enstrophy equations as follows:

$$\frac{\partial}{\partial t}\int_0^\pi\int_0^{2\pi}\frac{1}{2}|\nabla\psi'|^2 a^2\sin\theta d\lambda d\theta = -\int_0^\pi\int_0^{2\pi} v'_\lambda v'_\theta a\sin\theta \;\frac{\partial\bar{\lambda}}{a\partial\theta} a^2\sin\theta d\lambda d\theta$$

$$= -\frac{\partial}{\partial t}\int_0^\pi\int_0^{2\pi}\frac{1}{2}|\nabla\bar{\psi}|^2 a^2\sin\theta d\lambda d\theta. \tag{6}$$

$$\frac{\partial}{\partial t}\int_0^{\pi}\int_0^{2\pi}\frac{1}{2}\zeta'^2 a^2\sin\theta d\lambda d\theta$$
$$=-\int_0^{\pi}\int_0^{2\pi}\zeta' v'_\theta\frac{\partial\bar{\zeta}}{a\partial\theta}a^2\sin\theta d\lambda d\theta$$
$$=-\frac{\partial}{\partial t}\int_0^{\pi}\int_0^{2\pi}\frac{1}{2}\bar{\zeta}^2 a^2\sin\theta d\lambda d\theta. \tag{7}$$

where $\bar{\dot{\lambda}}\equiv\bar{v}_\lambda/(a\sin\theta)$. Thus, an increase of the disturbance energy or enstrophy, is accompanied by a decrease of the zonal flow energy or its enstrophy; and vise versa. The exchange of energy or enstrophy between disturbances and zonal current is determined by the structures of westerly and disturbances. In the wave–mean flow interaction process, there are further constraints due to the conservation of angular momentum in addition to (6) and (7):

$$\frac{\partial}{\partial t}\int_0^{\pi}\int_0^{2\pi}av'_\lambda\sin^2\theta d\lambda d\theta=-\frac{\partial}{\partial t}\int_0^{\pi}\int_0^{2\pi}a\bar{v}\lambda\sin^2\theta d\lambda d\theta=0. \tag{8}$$

Zeng, (1979, 1983a) pointed out that in most cases, especially when the zonal flow is stable, the disturbance energy may completely convert into the energy of the zonal flow; but only a small fraction of the westerly energy can convert into the disturbance energy.

Ideally, (1) and (2) should be solved simultaneously to evaluate the energy conversions and wave–mean flow interactions. However, in the case of small disturbances with amplitude on the order of $O(\varepsilon)$, the nonlinear term in (2) is on the order of $O(\varepsilon^2)$, and $\frac{\partial\bar{v}_\lambda}{\partial t}\sim O(\varepsilon^2)$. We can therefore use the linearized version of (2) and take the basic flow as steady to study the evolutionary process of a disturbance and its interaction with the basic flow. That is, (2) can be linearized as

$$\left(\frac{\partial}{\partial t}+\bar{\dot{\lambda}}\frac{\partial}{\partial\lambda}\right)\left[\frac{1}{a^2\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin\theta}\frac{\partial^2}{\partial\lambda^2}\right)\psi'\right]-\frac{\partial\psi'}{a\sin\theta\partial\lambda}\frac{\partial\bar{\zeta}}{a\partial\theta}=0, \tag{9}$$

where $\bar{\dot{\lambda}}$ and $\bar{\zeta}$ are considered as time independent. Note that the linearized equation is also valid if the disturbances can be represented as wave packets with finite amplitudes, since the nonlinear term then becomes much smaller than the other terms although $\bar{\dot{\lambda}}$ and $\bar{\zeta}$ in (9) may be taken as time dependent, and $\partial\bar{v}_\lambda/\partial t$ may not be small.

The energy and enstrophy relations described by (6) and (7) are still satisfied in the linearized model. In the following discussions, we use (9) to study the evolution of disturbances under a given basic flow, and use (5) to study the effect of disturbances on the basic flow. This kind of wave–mean flow interaction model is commonly referred as the weakly nonlinear interaction model which has been widely used in the literature (Andrews and McIntyre, 1978; Zeng, 1979; Edmon et al., 1980; Zeng, 1983a; Zeng et al., 1986a).

3. Transport of angular momentum by discrete–spectrum disturbances (normal modes)

Adding boundary conditions

$$\psi'|_{\theta=0,\pi}=0 \tag{10}$$

and initial condition

$$\psi'|_{t=0} = \psi'_0 \tag{11}$$

to (9), we form an initial–value problem for disturbances in a given basic flow. The spectrum and spectral functions, as well as the forms of solution expressed in terms of spectral functions of the discrete spectra and continuous spectrum of this problem, have been discussed in Zhang and Zeng (1997). It was shown that the evolution of an initial disturbance on a basic flow is described by

$$\psi'(\theta,\lambda,t) = \sum_k \Big\{ \sum_l A_l^k G_l^k(\theta) e^{ik(\lambda - c_l^k t)} + \int_{\bar{\lambda}_{\min}}^{\bar{\lambda}_{\max}} B^k(c) G^k(\theta,c) e^{ik(\lambda - ct)} dc \Big\}, \tag{12}$$

where k is the zonal wavenumber, $G_l^k(\theta)$ is the spectral function of the discrete–spectrum of (9) (normal mode), and $G^k(\theta,c)$ is the spectral function of the continuous–spectrum of the model. All spectral functions are dependent on the basic flow. $A_l^k, B^k(c)$ are the expansion coeffcients of the initial disturbance along with the spectral functions of the discrete– and continuous–spectrum.

Consider the discrete–spectrum part of the disturbance in (12) (referred to as the discrete–spectrum disturbance), which is composed of normal modes or characteristic waves:

$$\psi'_d(\theta,\lambda,t) = \sum_k \sum_l A_l^k G_l^k(\theta) e^{ik(\lambda - c_l^k t)}, \tag{13}$$

where $k = \pm 1, \pm 2, \cdots$, and $A_l^k = (A_l^{-k})^*$, $G_l^k = (G_l^{-k})^*$, $c_l^k = (c_l^{-k})^*$, with the asterisk denoting complex conjugate; and

$$c_l^k = c_{l,r}^k + i c_{l,i}^k, \tag{14}$$

$$G_l^k(\theta) = \left| G_l^k(\theta) \right| e^{i\alpha_l^k(\theta)}, \tag{15}$$

we have the transport of angular momentum by discrete–spectrum disturbances as:

$$a\sin^2\theta \overline{v'_\lambda v'_\theta} = \sum_k \sin\theta k \sum_l \{ \left| A_l^k G_l^k(\theta) \right|^2 e^{2kc_{l,i}^k t} \frac{\partial \alpha_l^k(\theta)}{a\partial\theta}$$
$$- \sum_m \sum_{l>m} P_{l,m}^k(\theta) e^{k(c_{l,i}^k + c_{m,i}^k)t} \sin[(\alpha_l^k - \alpha_m^k) - (c_{l,r}^k - c_{m,r}^k)t + \varphi_{l,m}^k(\theta)]\}, \tag{16}$$

where $k = 1, 2, \cdots$, and

$$[P_{l,m}^k(\theta)]^2 = \left| A_l^k \right|^2 \left| A_m^k \right|^2 \{ [\frac{\partial \left| G_l^k \right|}{a\partial\theta} \left| G_m^k \right| - \frac{\partial \left| G_m^k \right|}{a\partial\theta} \left| G_l^k \right|]^2$$
$$+ [\left| G_l^k \right| \left| G_m^k \right| (\frac{\partial \alpha_l^k(\theta}{a\partial\theta} + \frac{\partial \alpha_m^k(\theta)}{a\partial\theta})]^2 \}, \tag{17}$$

$$\mathrm{tg}[\varphi_{l,m}^k(\theta)] = \frac{\left| G_l^k \right| \left| G_m^k \right| (\frac{\partial \alpha_l^k(\theta)}{a\partial\theta} + \frac{\partial \alpha_m^k(\theta)}{a\partial\theta})}{\frac{\partial \left| G_l^k \right|}{a\partial\theta} \left| G_m^k \right| - \frac{\partial \left| G_m^k \right|}{a\partial\theta} \left| G_l^k \right|}. \tag{18}$$

From (16), we see that modes with different zonal wavenumbers do not interact to contribute to the angular momentum transport. We can therefore discuss disturbances with a given zonal wavenumber. The first term on the right hand side of (16) represents the transport of individual waves. The second term describes the cross products of different modes with the same zonal wavenumber.

For an individual neutral wave, $c^k_{l,i} = 0$, and it can be shown that its troughs and ridges exhibit north–soutrh orientation. Therefore, it does not transport angular momentum. Combinations of any two neutral normal modes produce a transport that vanishes when averaged in the time interval of $T = 2\pi / (k|c^k_{l,r} - c^k_{m,r}|)$, thus on an average making no contribution to the maintenance of zonal flow.

The only circumstance in which there is net angular momentum transport by normal modes is with unstable waves. For the developing and decaying unstable waves, $c^k_{l,i} \neq 0$, and $\partial\alpha^k_l(\theta) / \partial\theta \neq 0$, and it can be shown that the troughs and ridges of the waves are tilted; and therefore they transport angular momentum with an exponentially increasing or decreasing rate with respect to time. If the trough line is northeast–southwest oriented, the wave transports angular momentum northward. If the trough line is northwest–southeast oriented, the wave transports angular momentum southward. The magnitude of the transports depends on the amplitude of the wave and the meridional slope of the lines of trough and ridge $-\partial\alpha^k_l(\theta) / \partial\theta$. Therefore, for the discrete–spectrum disturbance to accelerate or decelerate the zonal flow in a climatological sense, it must contain unstable waves.

4. Transport of angular momentum by continuous–spectrum disturbances (non–modal solutions)

The integral part on the right hand side of (12) is called as the continuous–spectrum disturbance

$$\psi'_c(\theta,\lambda,t) = \sum_k \int_{\bar{\lambda}_{min}}^{\bar{\lambda}_{max}} B^k(c) G^k(\theta,c) e^{ik(\lambda - ct)} dc, \tag{19}$$

where $k = \pm 1, \pm 2, \cdots$. In principle, one can calculate the associated v'_λ and v'_θ (denoted as $v'_{c\lambda}$ and $v'_{c\theta}$ respectively) using ψ', and consequently, the transport of angular momentum $a\sin\theta \overline{v'_{c,\lambda} v'_\theta}$. In the case of monotonic basic flow, in which $\partial\bar{\lambda} / \partial\theta$ does not change its sign, we can write

$$a\sin^2\theta \overline{v'_{c\lambda} v'_\theta} = -\sum_{k=\pm 1,\cdots} a^{-1} k \sin\theta$$

$$\mathrm{Im}\left[\int_{\bar{\lambda}_{min}}^{\bar{\lambda}_{max}} \int_{\bar{\lambda}_{min}}^{\bar{\lambda}_{max}} B^k(c') B^{-k}(c'') G^k(\theta,c') \frac{\partial G^{-k}(\theta,c'')}{\partial\theta} e^{-k(c' - c'')t} dc' dc''\right]. \tag{20}$$

In the general case where $\partial\bar{\lambda} / \partial\theta$ changes sign, validity of (20) depends on the singularity of spectral function $G^k(\theta,c)$.

Transport of angular momentum by continuous–spectrum disturbances can be understood using the wave packet theory. A pure continuous–spectrum disturbance can be accurately described by a combination of wave packets (Zeng et al., 1986a):

$$\begin{aligned} \psi'_c &= \sum_k \sum_l \int_{c_l - \delta c/2}^{c_l + \delta c/2} B^k(c) G^k(\theta,c) e^{ik(\lambda - ct)} dc \\ &= \sum_k \sum_l B^k_l(\theta,\varepsilon t) e^{ik(\lambda - c_l t)} = \sum_k \sum_l \varphi'^k_{cl}, \end{aligned} \tag{21}$$

where $k = \pm 1, \pm 2, \cdots, \bar{\lambda}_{max} + \delta c / 2 \leqslant c_l \leqslant \bar{\lambda}_{max} - \delta c / 2$, and

$$B_l^k(\theta,\varepsilon t)=\int_{c_l-\delta c/2}^{c_l+\delta c/2} B^k(c)G^k(\theta,c)e^{ik(c_l-c)t}\,dc. \tag{22}$$

B_l^k is a slowly varying function of t, and ε is a small parameter proportional to δc. Because $B_l^{-k}=B_l^{k*}$, we have

$$\varphi'^{k}_{cl}+\varphi'^{-k}_{cl}=2\mathrm{Re}(B_l^k e^{ik(\lambda-c_l t)})=2\left|B_l^k\right|\cos\Omega_l^k, \tag{23}$$

where $\Omega_l^k=\alpha_l^k(\theta,\varepsilon t)+k\lambda-kc_l t$, and

$$\alpha_l^k=\mathrm{arctg}[\mathrm{Im}(B_l^k)/\mathrm{Re}(B_l^k)] \tag{24}$$

is the phase angle depending on θ and εt. α_l^k describes the tilt of the trough–ridge lines of the wave packet. The disturbance velocity can then be calculated from (23) as

$$v'^{k}_{\theta cl}+v'^{-k}_{\theta cl}=-\frac{\partial}{a\sin\theta\partial\lambda}(\varphi'^{k}_{cl}+\varphi'^{-k}_{cl})=2\frac{k\left|B_l^k\right|}{a\sin\theta}\sin\Omega_l^k,$$
$$v'^{k}_{\lambda cl}+v'^{-k}_{\lambda cl}=\frac{\partial}{a\partial\theta}(\varphi'^{k}_{cl}+\varphi'^{-k}_{cl})=-2\frac{\left|B_l^k\right|}{a}\frac{\partial\alpha_l^k}{\partial\theta}\sin\Omega_l^k+\frac{2}{a}\frac{\partial\left|B_l^k\right|}{\partial\theta}\cos\Omega_l^k.$$

Hence, the angular momentum transport by the continuous–spectrum disturbances can be written as

$$a\sin^2\theta\overline{v'_{c\lambda}v'_{c\theta}}=-2\frac{\sin^2\theta}{a}\sum_k\sum_l\sum_{l'}$$
$$\{\left|B_l^k\right|\left|B_{l'}^k\right|\frac{\partial\alpha_{l'}^k}{\sin\theta\partial\theta}(\omega_l^k-\omega_{l'}^k)+\left|B_l^k\right|\frac{\partial\left|B_{l'}^k\right|}{\partial\theta}\sin(\omega_l^k-\omega_{l'}^k)\}, \tag{25}$$

where $k=1,2,\cdots$, and

$$\omega_l^k=\alpha_l^k-kc_l t. \tag{26}$$

It is seen that transports can result from either $\partial\alpha_l^k/\partial\theta\neq 0$ (tilted trough–ridge lines) or $\partial\left|B_l^k\right|^2/\partial\theta\neq 0$ (meridional dependence of the wave packet amplitude).

The efficiency of wave packet transport of angular momentum and its associated zonal flow acceleration have been discussed in detail in Lu and Zeng (1981), Zeng (1983a, 1983b) and Zeng et al. (1986a, 1986b). It was shown that all wave packets eventually evolve to have tilted structures that are favorable to the transport of angular momentum towards the jets; and as a consequence, the jets are strengthened by the wave packets. Therefore, it is straightforward that continuous–spectrum disturbances play important role in the angular momentum transport and the maintenance of atmospheric zonal flow.

As shown in Zhang and Zeng (1997), in practice, both the discrete–spectrum disturbances and the continuous–spectrum disturbances can be calculated numerically. The continuous spectrum is approximated by numerically distorted discrete spectra of the continuum. Angular momentum transport of the continuous–spectrum disturbances is numerically represented in the same way as that described in the previous section. However, since there are numerious c_l^k and $c_{l'}^k$, $\left|c_l^k - c_{l'}^k\right|$ could be very small to ensure actual transport of angular momentum by the disturbances.

5. Computation of angular momentum transport in the real atmosphere

When the observed atmospheric disturbances are decomposed as discrete–spectrum and continuous–spectrum disturbances:

$$\psi'(\theta,\lambda,t)= \psi'_{\mathrm{d}}(\theta,\lambda,t)+ \psi'_{\mathrm{c}}(\theta,\lambda,t), \tag{27}$$

the disturbance velocities can be written as:

$$v'_\theta = v'_{\theta\mathrm{d}} + v'_{\theta\mathrm{c}}, \tag{28}$$

$$v'_\lambda = v'_{\lambda\mathrm{d}} + v'_{\lambda\mathrm{c}}. \tag{29}$$

Since $\vec{v}'_{\mathrm{d}}$ and $\vec{v}'_{\mathrm{c}}$ are not orthogonal, we have cross products in the angular momentum transport:

$$\frac{\partial a\bar{v}_\lambda \sin\theta}{\partial t} = -\frac{\partial \overline{v'_\lambda v'_\theta} a\sin^2\theta}{a\sin\theta\partial\theta} = M = M_{\mathrm{c}} + M_{\mathrm{d}} + M_{\mathrm{cd}} + M_{\mathrm{dc}}, \tag{30}$$

where

$$M_{\mathrm{c}} = -\frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}(\overline{v'_{\lambda\mathrm{c}} v'_{\theta\mathrm{c}}} a\sin^2\theta), \tag{31}$$

is the convergence of angular momentum transport purely by continuous–spectrum disturbances;

$$M_{\mathrm{d}} = -\frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}(\overline{v'_{\lambda\mathrm{d}} v'_{\theta\mathrm{d}}} a\sin^2\theta), \tag{32}$$

is purely by discrete–spectrum disturbances;

$$M_{\mathrm{cd}} = -\frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}(\overline{v'_{\lambda\mathrm{c}} v'_{\theta\mathrm{d}}} a\sin^2\theta), \tag{33}$$

and

$$M_{\mathrm{dc}} = -\frac{1}{a\sin\theta}\frac{\partial}{\partial\theta}(\overline{v'_{\lambda\mathrm{d}} v'_{\theta\mathrm{c}}} a\sin^2\theta), \tag{34}$$

are the transports from the cross products of discrete– and continuous–spectrum disturbances.

To evaluate each term in (30), a convenient way is to first decompose the stream function ψ' into ψ'_{c} and ψ'_{d} for every day as in Zhang and Zeng (1999), and then calculate $\vec{v}'_{\mathrm{c}}$ and $\vec{v}'_{\mathrm{d}}$. We have used the 300 hPa atmospheric winds in 1982 from the ECMWF analysis.

Figure 1 gives the mean zonal flow in January of 1982. The dotted line is the distribution of absolute vorticity gradient. It can be seen that, from the North Pole to the South Pole, $-\partial\bar{\zeta}/\partial\theta > 0$, so the basic flow is barotropically stable.

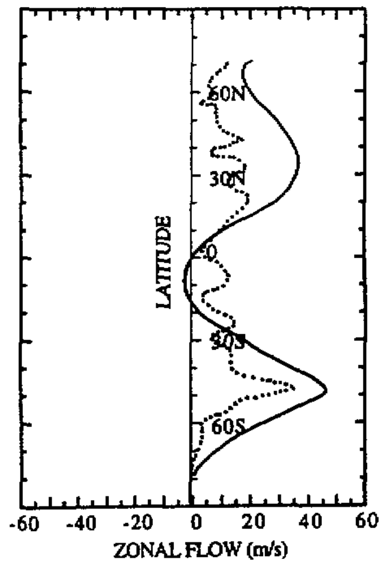

Fig. 1. Zonal mean basic westerly $\bar{v}_\lambda$ (m / s) (solid) at 300 hPa in January 1982, and the non-dimensional gradient of absolute vorticity $-d\bar{\zeta}/d\theta$ (dotted).

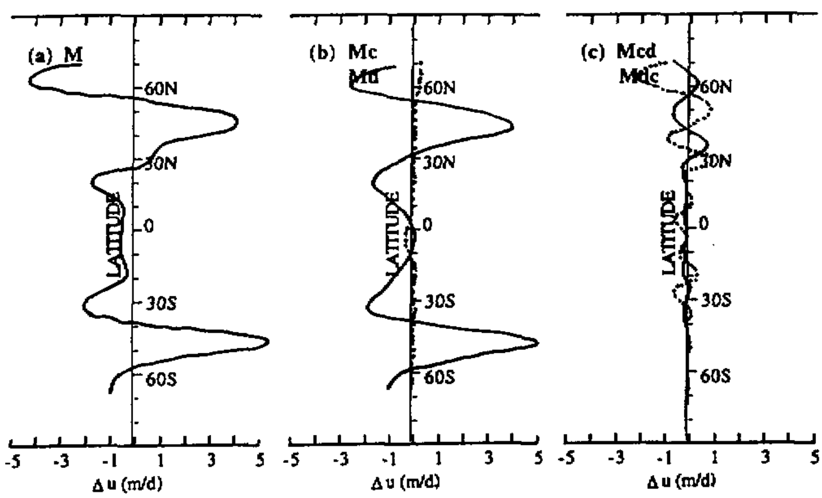

Fig. 2. Variation of zonal flow (m / d) due to disturbance angular momentum transport in January 1982. (a) total transport M; (b) transport by continuous-spectrum disturbances M_c (solid) and by discrete-spectrum disturbances M_d (dotted); (c) Cross products M_{cd} (solid) and M_{cd} (dotted).

Figure 2a shows the monthly mean acceleration of zonal wind by disturbance transports of angular momentum (M) in January 1982. Maximum zonal flow acceleration occurs near 45°N in the Northern Hemisphere and 50°S in the Southern Hemisphere. Figures 2b and 2c show the transport components of (31–34). It is seen that transport by discrete–spectrum disturbances is very small, and transport by continuous–spectrum disturbances dominates the total transport in Fig. 2a. M_{cd} and M_{dc} are also relatively small and are typically out of phase with each other, and they only make a small contribution to the total transport. It is therefore the angular momentum transport by continuous–spectrum disturbances that maintain the westerly current against dissipation and friction.

Each transport term can be further examined in the zonal wavenumber space since waves with different zonal wavenumbers do not interact to accelerate the zonal flow. Figure 3a shows the total transport term M in the latitude–wavenumber k space. In the Northern Hemisphere, maximum convergences of disturbance angular momentum transport occur near wavenumbers 3 and 7–8, corresponding to the winter planetary waves and synoptic transient waves; in the Southern Hemisphere, the maximum center is near wavenumbers 5 to 6.

Figure 3b shows the corresponding transport purely by continuous–spectrum disturbances. In all respects, the distribution is very similar to that of the total transport M. Figure 4 shows the other two components M_{cd} and M_{dc}. They are relatively small. These are consistent with Zhang and Zeng (1999) who showed that observed atmospheric disturbances are primarily composed of continuous–spectrum disturbances.

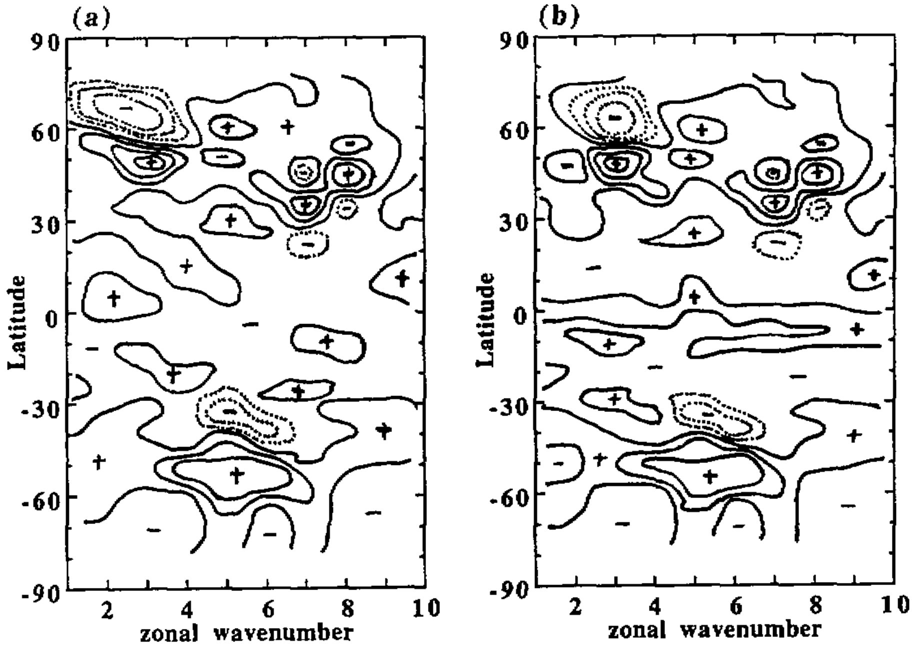

Fig. 3. Zonal flow variation represented in the latitude–wavenumber k space in January 1982. Negative lines are dotted. Positive lines are 0 m / d, 0.5 m / d, 1 m / d, and 2 m / d. Negative lines are −0.5 m / d, −1 m / d, and −2 m / d. (a) M; (b) M_c.

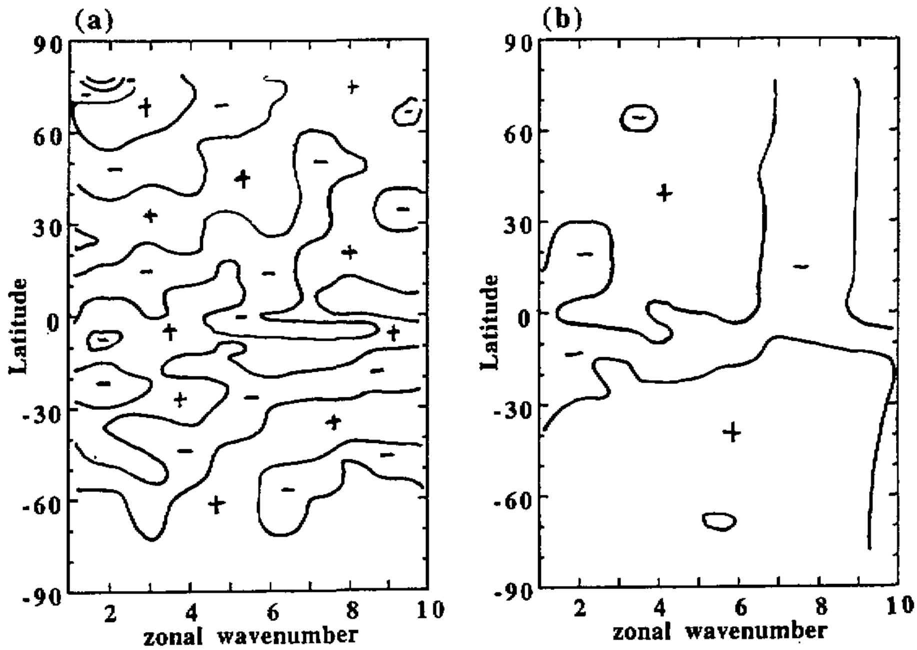

Fig. 4. Same as Fig. 3, except for M_{cd} in (a), and M_{dc} in (b).

In July 1982, the 300 hPa mean zonal flow is barotropically unstable. Figure 5 shows the mean zonal flow and the corresponding absolute vorticity gradient $-\partial\bar{\zeta}/\partial\theta > 0$ which changes sign in the Northern Hemisphere. Several unstable normal modes are found. Together with neutral normal modes, they constitute the set of spectral functions of the discrete spectra; and the discrete–spectrum disturbances were calculated by projecting the observed atmospheric disturbances on them (Zhang and Zeng, 1997).

Figures 6a–c show the total zonal flow acceleration M and its decomposed components M_c, M_d, M_{cd}, and M_{dc}, for July 1982. Although the acceleration in summer is weaker than that in winter (maximum is about 2.5 m/d), transport from continuous–spectrum disturbances still dominates the total transport; and transport from discrete–spectrum disturbances, which include contributions from unstable waves, is still relatively unimportant.

Figure 7 shows the latitude–zonal wavenumber distribution of the convergences of angular momentum for July (M and M_c). The dominance of contribution from continuous–spectrum disturbances remains the same, except with the maximum acceleration occurring at zonal wavenumbers 3 to 6 in both hemispheres.

6. Decomposed angular momentum transport for stationary and transient waves

We now treat the time–averaged disturbances as stationary waves, and the deviations from the stationary waves as transient waves. The stationary and transient waves are then

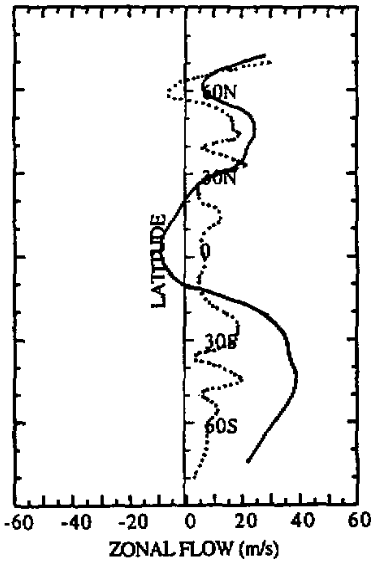

Fig. 5. Same as Fig. 1, except for July 1982.

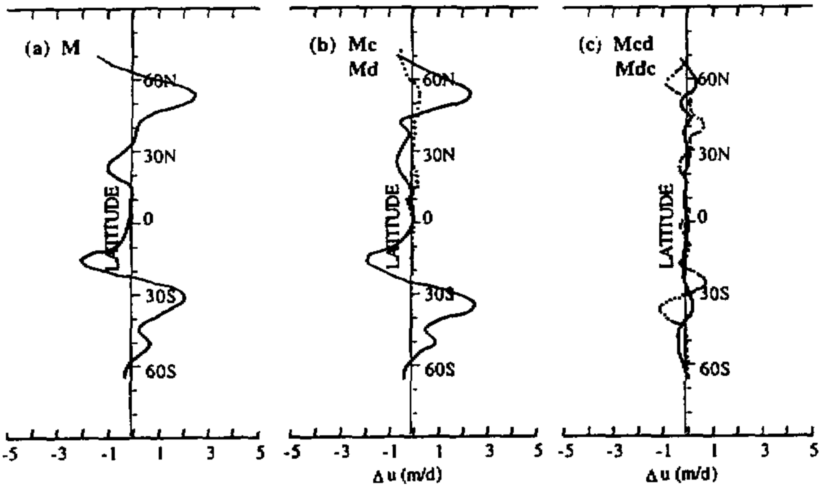

Fig. 6. Same as Fig. 2, except for July 1982.

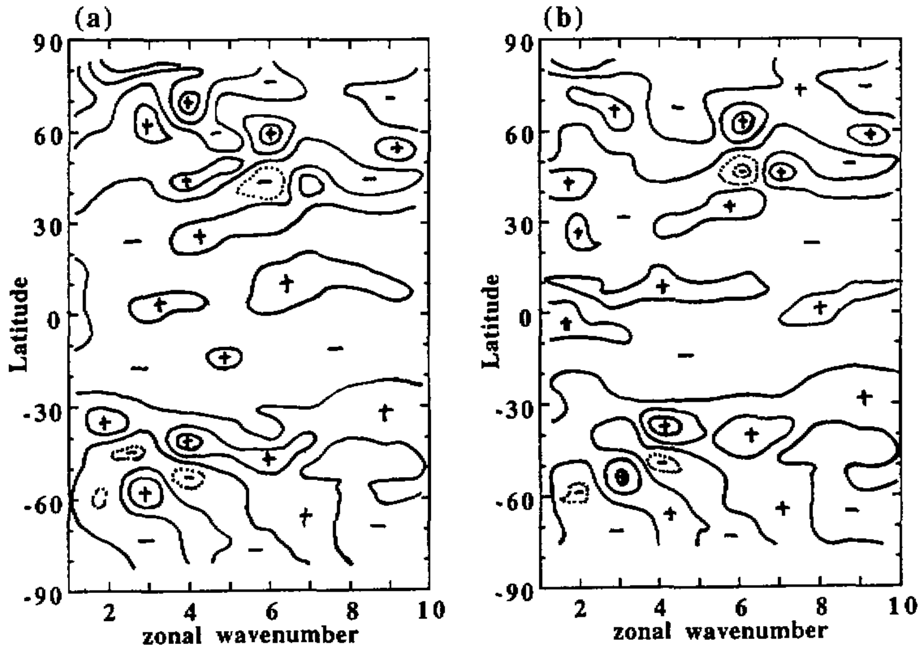

Fig. 7. Same as Fig. 3, except for July 1982.

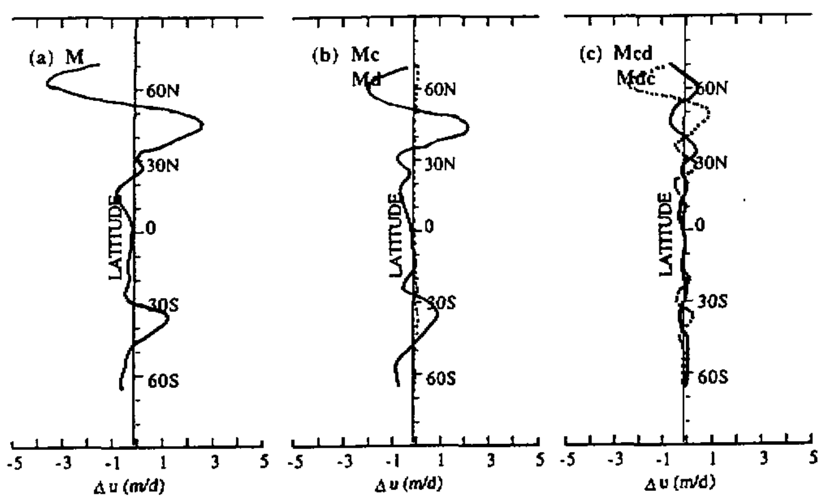

Fig. 8. Same as Fig. 2, except that M, M_c, M_d, M_{cd}, and M_{dc} are calculated from the stationary disturbances of January 1982.

separately calculated as sums of continuous– and discrete–spectrum disturbances. For stationary waves in January 1982, the total zonal flow acceleration is shown in Fig. 8a. Stationary waves also accelerate zonal flow in middle latitudes of the two hemispheres, but the maximum is about 3 m / d which is smaller than that in Fig 2. Figures 8b and 8c are the results of the continuous– and discrete–spectrum parts of the stationary disturbances. It is also seen that the transport from the continuous–spectrum disturbance is more significant than those from other terms. Transport from the cross products becomes negligible partly due to cancellation between the two out–of–phase terms.

Figures 9a–c are the monthly mean results corresponding to the transient disturbances in January 1982. Because transient disturbances are primarily composed of continuous–spectrum disturbances, we note that the convergence of angular momentum transport contributed by continuous–spectrum disturbances can almost match the total convergence, and all other components are negligible. Maximum accelerations of M and M_c occur in the Southern Hemisphere, they are about 5 m / d.

In Figs. 8 and 9, we also note that in the Northern Hemisphere, stationary and transient disturbances make comparable contributions to the transport of angular momentum; but in the Southern Hemisphere, transient disturbances make more contributions. This is related to weak forcing of stationary waves in the Southern Hemisphere.

Figures 10 and 11 show the latitude–wavenumber distribution of zonal flow acceleration (M and M_c) for the stationary disturbances and transient disturbances in January 1982. Major contributions are found due to the planetary scale stationary waves and due to the synoptic–scale transient waves; and both are mostly contributed by continuous–spectrum disturbances. Calculations for other seasons give similar results, and they are not shown here.

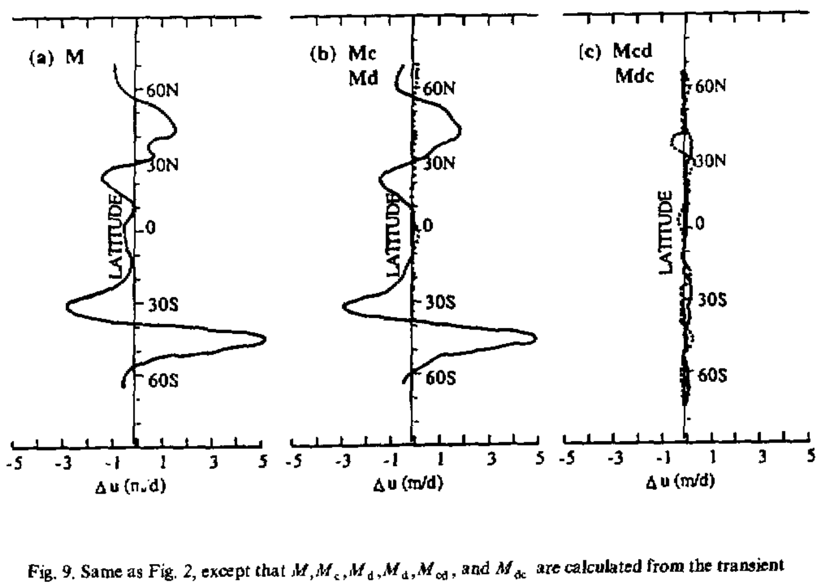

Fig. 9. Same as Fig. 2, except that M, M_c, M_d, M_d, M_{cd}, and M_{dc} are calculated from the transient disturbances of January 1982.

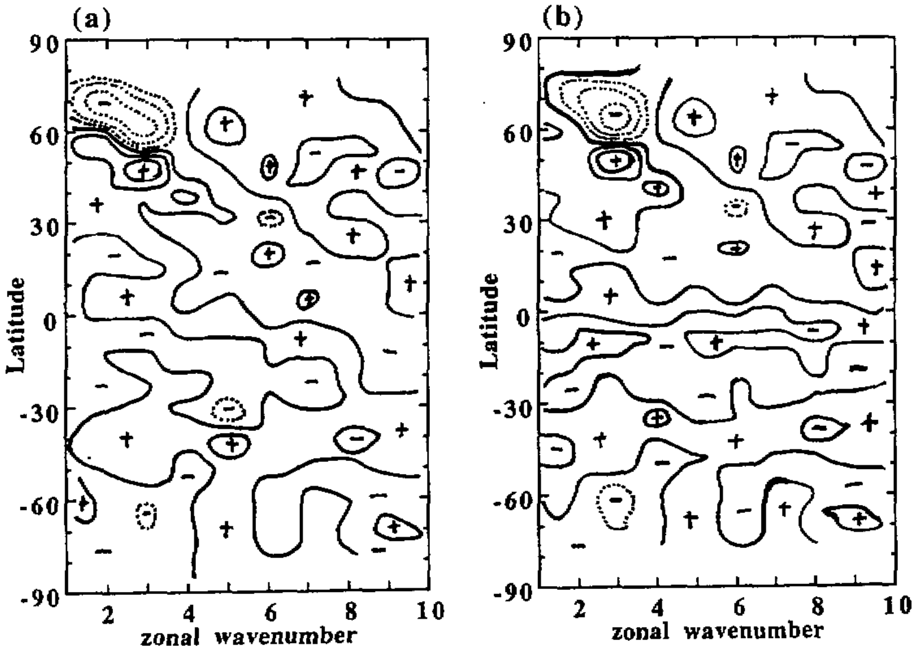

Fig. 10. Same as Fig. 3, except for the stationary disturbances of January 1982.

7. Summary and discussion

Our analysis has shown that the transport of angular momentum by atmospheric disturbances is carried out by both normal modes and non–modal disturbances. The normal modes include a set of neutral traveling waves and exponentially unstable waves; and only the unstable waves can contribute to the maintenance and acceleration of the zonal current. The non–modal continuous–spectrum disturbances can always effectively transport zonal angular momentum in their evolutionary process regardless of the stability of the basic flow.

Our calculations also show that in the real atmosphere, the contribution of continuous–spectrum disturbances to the angular momentum transport dominates the total transport, and thus these disturbances are the primary player in maintaining the westerly flow. Even when the basic flow is unstable, angular momentum transport by discrete–spectrum disturbances is small in comparison with that by continuous–spectrum disturbances. In particular, transport due to transient waves is almost entirely from continuous–spectrum disturbances.

Variability of the atmospheric zonal flow is governed by forcing and dissipation. The forcing in the real atmosphere consists of convergence of meridional and vertical transports of angular momentum by disturbances and by the mean meridional circulation. Since

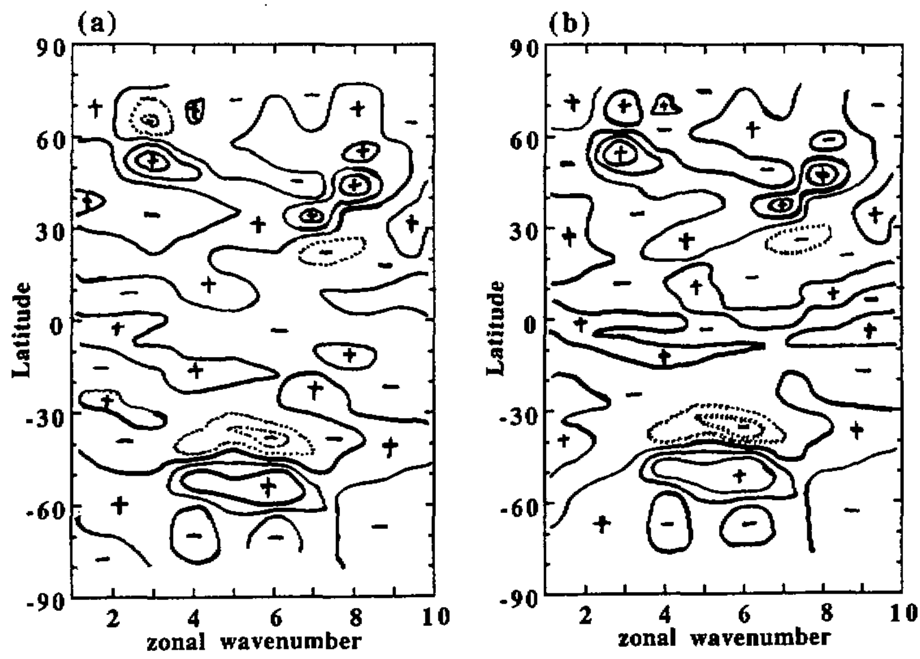

Fig. 11. Same as Fig. 3, except for the transient disturbances of January 1982.

the vertical transport of angular momentum is typically one order of magnitude smaller than the meridional one, and the mean meridional circulation is actually determined by the disturbance angular momentum transport, the results of this study demonstrate the dominant role of the continuous–spectrum disturbances over that of the unstable normal modes in the wave–mean flow interaction problem. The actural transport of angular momentum in the real atmosphere is carried out mainly through the tilted trough–ridge lines of the continuous–spectrum disturbances rather than those of the exponentially decaying normal modes.

We therefore presume a general picture of the general circulation of the atmosphere as follows: The zonal atmospheric flow is baroclinically unstable, leading to transformation of the zonal available potential energy to the disturbance energy; and it interacts with various localized atmospheric forcings. Continuous–spectrum disturbances are favorably excited because of their localized structures; they take the form of wave packets. These packets eventually decay, and they transport angular momentum toward the jet cores, serving to feed energy to the basic zonal flow and to maintain it against dissipation.

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