A Numerical Study on Effects of Land–Surface Heterogeneity from "Combined Approach" on Atmospheric Process Part I: Principle and Method①

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ABSTRACT

A method based on Giorgi (1997a, 1997b) and referred to as 'combined approach', which is a combination of mosaic approach and analytical–statistical–dynamical approach, is proposed. Compared with those of other approaches, the main advantage of the combined approach is that it not only can represent both interpatch and intrapatch variability, but also cost less computational time when the land surface heterogeneity is considered. Because the independent variable of probability density function (PDF) is extended to the single valued function of basic meteorological characteristic quantities, which is much more universal, the analytical expressions of the characteristic quantities (e.g., drag coefficient, snow coverage, leaf surface aerodynamical resistance) affected by roughness length are derived, when the roughness length(and / or the zero plane displacement) heterogeneity has been mainly taken into account with the approach.

On the basis of the rule which the PDF parameters should follow, we choose a function y of the roughness length z_0 as the PDF independent variable, and set different values of the two parameters width ratio α_n and height ratio γ of PDF (here a linear, symmetric PDF is applied) for sensitivity experiments, from which some conclusions can be drawn, e.g., relevant characteristic terms show different sensitivities to the heterogeneous characteristic (i.e., roughness length), which suggests that we should consider the heterogeneities of the more sensitive terms in our model instead of the heterogeneities of the rest, and which also implies that when the land surface scheme is coupled into the global or regional atmospheric model, sensitivity tests against the distribution of the heterogeneous characteristic are very necessary; when the parameter α_n is close to zero, little heterogeneity is represented, and α_n differs with cases, which have an upper limit of about 0.6; in the reasonable range of α_n, a peak–like distribution of roughness length can be depicted by a small value of γ, etc..

Key words: Representation of land surface heterogeneity, "Combined approach", Numerical experiment

1. Introduction

An ideal land surface model must both realistically describe the land surface characteristics (especially the universal heterogeneity) and be computationally inexpensive. Currently popular land surface models are all based on "big leaf" theory, in which a model grid cell is divided into many patches (e.g., a bare soil patch or a vegetation patch; many regular patches divided into according to geographical locations), and in which only interpatch variability is

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taken into account while seldom intrapatch variability is, when the heterogeneity is considered. Even if intrapatch variability is dealt with, the computation costs too much. Among different approaches in the land surface model, the mosaic approach has been mostly used to describe interpatch variability (Deardorff, 1978; Avissar and Piekle, 1989; Koster and Suarez, 1992; Dickinson et al., 1993; Leung and Ghan, 1995), while the analytical type of statistical—dynamical approach (ATSDA) (Moore and Clark, 1981; Entekhabi and Eagleson, 1989) is the best to realistically represent intrapatch variability and computationally costs lest. Because of the limitations of the different approaches, previous researches cannot get both the advantages in computational cost and the inclusion of intrapatch variability, which result in different disadvantages: newly-appiled ATSDA only considers few heterogeneous characteristics, with its PDF less varied in the form; other approaches computationally cost such a lot when intrapatch variability is considered that very seldom work is internationally reported concerning the coupling of a land surface model with intrapatch variability with an atmospheric model up till now. Nevertheless, the coupling is very necessary when considering land-atmosphere interactions.

It should be noted that among the state-of-the-art approaches, say, so called "mixed approach" and fine-mesh approach (Seth, et al., 1994), in spite of their characteristics, many are the variants of the mosaic approach except ATSDA. In the patches divided by the mosaic approach, at most only few characteristics can be assumed to be homogeneous, while some other very important ones, such as roughness lengths and stomatal resistances, are still pronouncedly heterogeneous. Hence, it is very difficult for the mosaic approach to consider intrapatch variability, which suggests other approaches should be found out for better representing land surface heterogeneity.

Giorgi (1997a, 1997b) used ATSDA (hereafter referred to as combined approach) and proposed a land surface model, in which the heterogeneities of the temperature and moisture near the surface and in the soil were considered, and in which the intrapatch variability was represented by the distribution-function approach similar to that of Entekhabi and Eagleson (1989). They chose a linear symmetric function as the probability density function (PDF) to represent first-order effects due to the heterogeneity, and computational efficiency was achieved through the use of analytical rather than numerical solutions. That is to say, they did not numerically integrate the whole set of equations over the distribution, but analytically integrate some nonlinear terms affected by the heterogeneity.

In contrast to that in Giorgi (1997a, 1997b), in this paper we extend the independent variable of the probability density function (PDF) to the single valued function of basic meteorological characteristic quantities, which can make the application of the combined approach more universal. We mainly consider the roughness length heterogeneity, while the paper about other heterogeneities will be presented in the future. After the treatment by the combined approach, we obtain the averages of the characteristic quantities or nonlinear terms affected by every kind of heterogeneous variables, and the averages are used as representative values of the grid cell to enter the calculation of land-atmosphere interaction. As the first part, this paper describes the theoretical treatment of the land surface heterogeneity, while the companion paper will discuss the effects on the short-term weather process due to the land surface heterogeneity.

2. Treatment of heterogeneous characteristics by combined approach

In the following, a commonly-used land surface scheme BATS (Dickinson et al., 1993) is used as an example, in which the roughness length heterogeneity is considered, to investigate the treatment by the combined approach.

2.1 *Treatment of drag coefficient* C_{dn}

In BATS, the representative value of drag coefficient is the weighted average over various kinds of land surface in the grid cell. With respect to a certain type of land surface, the drag coefficient under neutral condition C_{dn} is the function of roughness length z_0 and the height of the lowest level in the atmospheric model z_1:

$$C_{dn} = \left[\frac{k}{\ln[(z_1 - d_0)/z_0]}\right]^2 . \quad (1)$$

Where k is the Karman constant, z_1 varies little in a certain atmospheric model, zero plane displacement height d_0 is introduced when obstacles such as buildings, trees and even crops exist. As a general treatment, z_0 can be regarded as a fraction of the height of an obstacle (e.g., 1 / 10), d_0 also a fraction (e.g., 7 / 10), then d_0 is a multiple of z_0 (i.e., $d_0 = d_{00} \times z_0$, d_{00} is a constant). So, d_0 heterogeneity is taken into account as long as z_0 heterogeneity is, and C_{dn} can be assumed to change only with z_0. In the cases without obstacles (e.g., bare soil, sea surface), $d_0 = 0$. Generally, we assume

$$y = \ln[(z_1 - d_0)/z_0] , \quad (2)$$

then

$$C_{dn} = \left[\frac{k}{\ln(z_1/z_0 - d_{00})}\right]^2 . \quad (3)$$

Here PDF——$f_{pdf}(x)$ is employed as Giorgi (1997a, 1997b), and y is assumed to the independent variable of $f_{pdf}(y)$. Such a transformation is suitable because the variation of z_0 in a certain range corresponds to that of y in the other range, that is, y will exhibit a certain PDF distribution. For simplicity, here $f_{pdf}(y)$ is transformed. Assuming the average, half width and height ratio of y are denoted by y_0, α, γ, respectively, then

$$f_{pdf}(y) = c_1 y + d_1 , \quad y_0 - \alpha \leqslant y \leqslant y_0 \quad (4a)$$

$$f_{pdf}(y) = c_2 y + d_2 , \quad y_0 \leqslant y \leqslant y_0 + \alpha \quad (4b)$$

where

$$c_1 = \alpha_2(1-\gamma)/\alpha , \quad d_1 = \alpha_2[\gamma + (1-\gamma)(-y_0 + \alpha)/\alpha] \quad (5a)$$

if $y_0 - \alpha \leqslant y \leqslant y_0$,

and

$$c_2 = -\alpha_2(1-\gamma)/\alpha , \quad d_2 = \alpha_2[\gamma + (1-\gamma)y_0/\alpha] \quad (5b)$$

if $y_0 \leqslant y \leqslant y_0 + \alpha$.

In the above two Formulas (5a) and (5b), α_2 is the maximum PDF value.

If the heterogeneity operator (Giorgi, 1997a; 1997b)

$$F^x_{pdf}(A) = \int A f_{pdf}(x) dx \quad (6)$$

is applied to the nonlinear term A to represent the average of A after the heterogeneity is considered, then the expression of C_{dn} is obtained after the heterogeneity treatment:

$$
\begin{aligned}
F_{pdf}^{y}(C_{dn}) &= \int C_{dn} f_{pdf}(y)dy = \int (\frac{k}{y})^2 f_{pdf}(y)dy \\
&= k^2 \sum_{i=1}^{2} \int \frac{c_i y + d_i}{y^2} dy \\
&= k^2 [(c_1 \ln y_0 - c_1 \ln y_1 - \frac{d_1}{y_0} + \frac{d_1}{y_1}) + (c_2 \ln y_2 - c_2 \ln y_0 - \frac{d_2}{y_2} + \frac{d_2}{y_0})] \\
&= k^2 \sum_{i=1}^{2} (c_i \ln y - \frac{d_i}{y}) |_{y_{i1}}^{y_{i2}} ,
\end{aligned}
$$

where y_0 is calculated from the value of C_{dn0} which is an output from BATS without the heterogeneity treatment:

$$y_0 = k / \sqrt{C_{DN0}} \ , \quad y_1 = y_0 - \alpha \ , \quad y_2 = y_0 + \alpha \ . \tag{8}$$

Formula (7) consists of 8 terms, where

$$y_{11} = y_1 , \quad y_{12} = y_0 \qquad \text{if} \quad i = 1 \ , \tag{9a}$$

$$\text{and} \quad y_{21} = y_0 , \quad y_{22} = y_2 \quad \text{if} \quad i = 2. \tag{9b}$$

Thus, by the use of Formula (9), the expression similar to Formula (7) after the heterogeneity treatment is very simple. For simplicity, hereafter denotations similar to Formula (7) are applied after the heterogeneity treatment.

Similarly, the expression of z_0 after the heterogeneity treatment is obtained (also see Appendix B):

$$F_{pdf}^{y}(z_0) = \int z_0 f_{pdf}(y)dy = \int z_0 f_{pdf}[y(z_0)](dy / dz_0)dz_0 = \text{E1} + \text{E2} + \text{E3} \ , \tag{10}$$

where each term is derived after complex calculations, with

$$
\begin{aligned}
\text{E}_1 &= \sum_{i=1}^{2} \frac{z_1 d_i}{d_{00}} \ln(\frac{z_1}{d_{00}} - z_0) |_{z_{i1}}^{z_{i2}} \ , \\
\text{E}_2 &= \sum_{i=1}^{2} \frac{z_1 c_i}{2d_{00}} [\ln(z_1 - d_{00} z_0)]^2 |_{z_{i1}}^{z_{i2}} \ , \\
\text{E}_3 &\approx \sum_{i=1}^{2} \sum_{k=1}^{4} c_i (- d_{00} / z_1)^k \frac{z_0^{k+1}}{k+1} (\ln z_0 - \frac{1}{k+1}) |_{z_{i1}}^{z_{i2}} \ .
\end{aligned}
$$

In the above terms, when $i = 1$,

$$z_{11} = z_1 / (\exp(y_0 - \alpha) + d_{00}) \ , \quad z_{12} = z_1 / (\exp(y_0) + d_{00}) \ ; \tag{11a}$$

and when $i = 2$,

$$z_{21} = z_1 / (\exp(y_0) + d_{00}) \ , \quad z_{22} = z_1 / (\exp(y_0 + \alpha_v) + d_{00}) \ . \tag{11b}$$

Because C_{dn} is the single valued function of z_0, the drag coefficient in the neutral condition C_{dn} is completely determined by roughness length z_0. In terms of mathematics, it is necessary to introduce various treatments relevant to z_0 after the heterogeneity treatment of C_{dn}. Furthermore, it may lead to great differences for the nonlinear terms to be treated by the z_0 heterogeneity operator because of the significant heterogeneity of z_0.

2.2 *Treatment of snow coverage* f_{snow}

Snow coverage f_{snow} in BATS is written as

$$f_{snow} = d_{snow} / (d_{snow} + 10z_0) \ , \tag{12}$$

where f_{snow} is the average snow depth. Assume $S_r = 0.1 \times d_{snow}$, then

$$f_{snow} = S_r / (S_r + z_0) \ . \tag{13}$$

In fact, because f_{snow} is affected by z_0 which is pronouncedly heterogeneous, the treatment of f_{snow} may lead to great differences for the nonlinear terms to be treated by the z_0 heterogeneity operator. Hence, f_{snow} in Formula (13) is treated by the operator.

Because of the functional relationship between C_{dn} and z_0, PDF independent variable is chosen according to Formula (2). Then, the distribution of z_0 is determined with that of C_{dn} determined. So, it is necessary to make complex transformations from one variable into another and make some approximations when f_{snow} is treated by the heterogeneity operator to compute $F^y_{pdf}(f_{snow})$, whose final expression is formula A.1 in Appendix A.

2.3 *Treatment of wind speed within the foliage layer* U_{af}

The wind speed within the foliage layer U_{af} in BATS is given by

$$U_{af} = V_a C_D^{1/2} \ , \tag{14}$$

where V_a is the wind speed at the anemometer level, C_D is the drag coefficient,

$$C_D = C_{dn} \times g(R_{iB}) \ . \tag{15}$$

In the above formula, $g(R_{iB})$ is a correction factor changing little with the z_0 heterogeneity. For simplicity, it is taken as the function of the bulk Richardson number R_{iB}.

Then, by the use of expressions (1), (2), (14) and (15),

$$F^y_{pdf}(U_{af}) = \sum_{i=1}^{2} \int \frac{kV_a[g(R_{iB})]^{1/2}(c_i y + d_i)}{y} dy = kV_a[g(R_{iB})]^{1/2} \sum_{i=1}^{2} (c_i y + d_i \ln y)|_{y_{i1}}^{y_{i2}} \ . \tag{16}$$

2.4 *Treatment of aerodynamical resistance at the foliage layer* r_{la}

In BATS, the sensible heat and latent heat transfer coefficients are represented by the inverse of aerodynamical resistance at the foliage layer r_{la}:

$$r_{la}^{-1} = C_f \times (U_{af} / D_f)^{1/2} \ , \tag{17}$$

where C_f is a constant, D_f a leaf characteristic quantity. Then, with the application of expressions (1), (2), (14), (15) and (17), we have

$$\begin{aligned} F^y_{pdf}(r_{la}^{-1}) &= C_f\{kV_a[g(R_{iB})]^{1/2} D_f^{-1}\}^{1/2} \times F^y_{pdf}(y^{-1/2}) \\ &= C_E \times \sum_{i=1}^{2} (\frac{2c_i}{3} y^{3/2} + 2d_i y^{1/2})|_{y_{i1}}^{y_{i2}} \ , \end{aligned} \tag{18}$$

where

$$C_E = C_f\{kV_a[g(R_{iB})]^{1/2} D_f^{-1}\}^{1/2} \ . \tag{19}$$

We can therefore compute the evaporation at the leaf surface by the use of Formula (18).

2.5 Treatment of stomatal resistance r_s

Transpiration rate E_{tr} is calculated by

$$E_{tr} = \rho_a \delta(E_f^{WET}) L_d (\frac{1}{r_{la} + r_s})(q_f^{SAT} - q_{af}) , \quad (20)$$

where ρ_a is the density of surface air, q_f^{SAT} the saturation specific humidity corresponding to the leaf temperature, q_{af} the specific humidity of the air within the foliage, $\delta(E_f^{WET})$ the step function, L_d a weighted factor, r_s the stomatal resistance. r_s is also treated because its heterogeneity plays a very important role in the transpiration process.

In BATS, many factors affecting r_s are highly heterogeneous. For example, r_s has a very complicated relationship with the sky situation, the solar zenith, leaf area index, etc.. Hence, we assume what BATS outputs are averages, on the basis of which the heterogeneity is considered. We assume

$$q = r_s / C_E . \quad (21)$$

Similar to above $f_{pdf}(y)$, we choose PDF as $g_{pdf}(q)$, in which c_{qj}, d_{qj}, γ_q and α_q correspond to c_i, d_i, γ and α in expressions (4) and (5), and in which q_j, q_0, q_{j1} and q_{j2} correspond to y_i, y_0, y_{i1} and y_{i2} in expressions (8) and (9), where q_0 is the q value computed by r_s and C_E, two output values from BATS without heterogeneity treatment, via Equation (21).

Analogously, the corresponding heterogeneity operator is written as

$$G_{pdf}^q(A) = \int A g_{pdf}(q) dq . \quad (22)$$

Because r_{la} and r_s are independent of each other, the treatment of the nonlinear term $\frac{1}{r_{la} + r_s}$ in E_{tr} involves two-fold heterogeneity operator, which can be seen as Formula (A.2) in Appendix A.

3. Numerical experiments with respect to PDF parameters

3.1 General principle of PDF parameter choosing

After a certain independent variable is chosen, its relevant nonlinear terms must also be treated. For example, we choose $y = \ln(z_1 / z_0 - d_{00})$ as a PDF independent variable, and must treat the drag coefficient C_{do}, the snow coverage f_{snow}, the within-foliage wind U_{af} and the transfer coefficient for foliage r_{la}^{-1}, which are nonlinear and relevant to y. Hence, when the term without heterogeneity treatment is replaced by that with heterogeneity treatment, it is necessary to choose suitable half width α and height ratio γ so that PDF can represent the first-order heterogeneity.

Apparently, the greater α, the wider the range of the PDF independent variable change; the closer to 1 γ is, the more uniform the PDF independent variable distributed over the same interval in the distribution range is. This is a general concept for these two independent variables. As for performing simulations, we suggest, it must be followed that the chosen PDF parameters should be relatively consistent with the used model and the properties of the simulated domain.

In fact, different properties of the domain (grid cell) or different physical variables correspond to different α and γ, which even change with time. For instance, the range of roughness–length change differs with the surface types. In a grid cell, according to big–leaf theory, only one type of vegetation is set, and this assumed condition will restrict the corresponding range of roughness–length change; while for the same grid cell, no matter which approach is applied, a mosaic approach in which a grid cell is finely and geographically divided, or a mosaic approach in which a grid cell is topographically regrouped, the range or PDF of roughness–length change differs from that applied common big–leaf theory. In addition, as for the same grid cell, or the same model, or the same characteristic quantity, α and γ may even change with time, e.g., the temperature stratification after noon differs from that at night in a steeply mountainous district, i.e., the temperature lapse rates are different, thereby α's and γ's in surface temperature PDF function $f_{\rm pdf}(T)$ are different.

The numerical models used are not only relevant to the approaches of heterogeneity representation, but also to the options of PDFs and their parameters. Giorgi (1997a) suggested that the linear and symmetric PDF could represent the first–order approximation of the distribution of a variable, and considered the heterogeneity of the "basic variables", i.e., temperature and moisture, which have obviously physical meanings. When we extend the PDF parameters to the single valued function (e.g., $=\ln z_0$), which has no obviously physical meaning, of the "basic variables" with obviously physical meaning (e.g., roughness length z_0), we are faced with how to choose α and γ of the PDF of the single valued function. We propose to do this as follows: Firstly transform the PDF of the single valued function corresponding to α and γ into the other PDF of the "basic variable" with obviously physical meaning, then investigate the reasonableness of the "basic variable" distribution, on this basis, α's and γ's of the PDF of the single valued function are determined.

3.2 Design of experiments

When we consider the roughness–length heterogeneity, we actually start with assuming $y=\ln(z_1/z_0-d_{00})$ as the PDF independent function so as to obtain the analytical expressions of heterogeneity treatment. This is just a mathematical tool. In fact, its physical meaning must be comprehended like this: the roughness–length heterogeneity leads to the heterogeneity of y, i.e., z_0's PDF leads to y's PDF. Therefore, $g_{\rm pdf}(z_0)$, z_0's PDF, can be derived from $f_{\rm pdf}(y)$, y's PDF:

$$g_{\rm pdf}(z_0)=f_{\rm pdf}[y(z_0)]\left|\frac{dy}{dz_0}\right|=\frac{\{c_i[\ln(z_1-d_{00}z_0)-\ln z_0]+d_i\}z_1}{(z_1-d_{00}z_0)z_0} \quad . \tag{23}$$

So, when α and γ of $f_{\rm pdf}(y)$ result in the relatively realistic representation (by $g_{\rm pdf}(z_0)$) of the properties of the simulated domain, we may regard α and γ appropriate. Hence, according to that $C_{\rm dn}$ in Formula (1) is of the order of 10^{-3}, three experiments are designed so as to deduce and obtain the PDF parameters of y and the ranges of characteristic quantities, such as the drag coefficient $C_{\rm dn}$, snow cover $f_{\rm snow}$, within–foliage wind $U_{\rm af}$, transfer coefficient of leaf surface $r_{\rm la}^{-1}$, after treating by heterogeneity operator.

We append subscript "0" to the denotations of the characteristic quantities without heterogeneity treatment, i.e., $C_{\rm dn0}$, $f_{\rm snow0}$, $U_{\rm af0}$, $r_{\rm la0}^{-1}$ and z_{00} represent un–treated drag coefficient, snow cover, within–foliage wind and transfer coefficient of leaf surface, respectively; while appended with subscript " 1", C_{dn1}, f_{snow1}, U_{af0}, r_{la1}^{-1} and z_{01} denote those with heterogeneity treatment. r_{cdn}, r_{fsn}, r_{uaf}, r_{rai} and r_{z0} are, respectively, the ratios of C_{dn1}, f_{snow1}, U_{af1}, r_{la1}^{-1} and z_{01} to C_{dn0}, f_{snow0}, U_{af0}, r_{la0}^{-1} and z_{00}. For the convenience of experiments, here an input parameter, α is replaced by width ratio α_n, a ratio of α to the PDF independent variable without heterogeneity treatment.

Following is a primary statement of the experiments.

Exp.1: corresponds to Table 1, where $z_1 = 40$ m, $z_{00} = 2.69 \times 10^{-2}$ m, $s_r = 0.01$ m, $\gamma = 0.8$.

Exp.2: corresponds to Table 2, where $z_1 = 80$ m, $z_{00} = 2.69 \times 10^{-2}$ m, with different α_n and γ.

Exp.3: corresponds to Table 3, where $z_1 = 80$ m, $z_{00} = 5.39 \times 10^{-2}$ m, $s_r = 0.01$ m, with different α_n and γ.

Table 1. Results of heterogeneity treatment in Exp.1 ($z_1 = 40$ m, $z_{00} = 2.69 \times 10^{-2}$ m, $s_r = 0.01$ m, $\gamma = 0.8$)

	$\alpha_n = 0.001$	$\alpha_n = 0.006$	$\alpha_n = 0.02$	$\alpha_n = 0.04$	$\alpha_n = 0.1$	$\alpha_n = 0.2$	$\alpha_n = 0.3$	$\alpha_n = 0.45$	$\alpha_n = 0.6$
z_{0min} (m)	2.67×10^{-2}	2.58×10^{-2}	2.33×10^{-2}	2.01×10^{-2}	1.30×10^{-2}	6.25×10^{-3}	3.01×10^{-3}	1.01×10^{-3}	3.37×10^{-4}
z_{0max} (m)	2.71×10^{-2}	2.81×10^{-2}	3.12×10^{-2}	3.61×10^{-2}	5.59×10^{-2}	1.16×10^{-1}	2.41×10^{-1}	7.21×10^{-1}	2.15
z_{01} (m)	2.70×10^{-2}	2.69×10^{-2}	2.70×10^{-2}	2.73×10^{-2}	2.92×10^{-2}	3.70×10^{-2}	5.27×10^{-2}	1.04×10^{-1}	2.30×10^{-1}
C_{dn1} ($\times 10^{-3}$)	2.9999	3.00012	3.00114	3.0045	3.029	3.118	3.28	3.72	4.58
f_{snow1} (%)	27.058	27.062	27.089	27.179	27.788	29.690	32.136	35.682	38.466
r_{cdn}	0.99998	1.00002	1.00038	1.0015	1.0095	1.3930	1.0932	1.2388	1.5266
r_{z0}	0.99999	1.00031	1.00336	1.0135	1.0862	1.3727	1.9548	3.8669	8.5625
r_{fsn}	0.99960	0.9997	1.00071	1.0040	1.0265	1.0968	1.1872	1.3182	1.4210
r_{uaf}	0.99998	1.00002	1.00073	1.0005	1.0032	1.0129	1.0299	1.0726	1.1459
r_{rai}	0.99998	1.00001	1.00005	1.0002	1.0012	1.0048	1.0111	1.0263	1.0509

Mathematically α_n and γ are independent of each other, but physically they are not, e.g., PDF, α_n and γ cannot be arbitrarily set for a given simulated domain. By the sensitivity experiments mentioned above, we try to comprehend the sensitivity of roughness– length PDF, which has more obviously physical meaning and is not a linear, symmetric one, to different PDF parameters of y. From this, a theoretical foundation is provided for the use of combined approach to consider relatively realistic PDFs of heterogeneous distributions of various physical quantities.

3.3 *Analysis of experiments*

3.3.1 *Analysis of Exp.1*

Table 1 gives the comparison of Exp.1 between results without and with heterogeneity treatment, where the cases when $\alpha_n = 0.001$, 0.006, 0.02 respectively correspond to lines A, B, C in Fig. 1a and Fig. 1b (hereafter lines A, B, C in Fig. na, the y PDF curves, respectively correspond to lines A, B, C in Fig. nb, the z_0 PDF curves). It can be seen that when width ratio α is very small, not only $f_{pdf}(y)$ shows linearity and symmetry, i.e., a peak–like distribution with the small front and back parts while the large center part, but also $g_{pdf}(z_0)$, z_0's PDF, does; $\alpha_n = 0.04$, 0.1, 0.2 correspond to lines A, B, C in Fig. 2a and Fig. 2b, respectively, which implies that the linearity and symmetry of $g_{pdf}(z_0)$ vanish with α_n being greater, instead, a feature appears in which the distribution probability of small z_0 values is greater than that of large z_0 values. This means that the area of relatively smooth surface is greater, while the area with tall obstacles is smaller. Considering the variation range of z_0 is realistic,

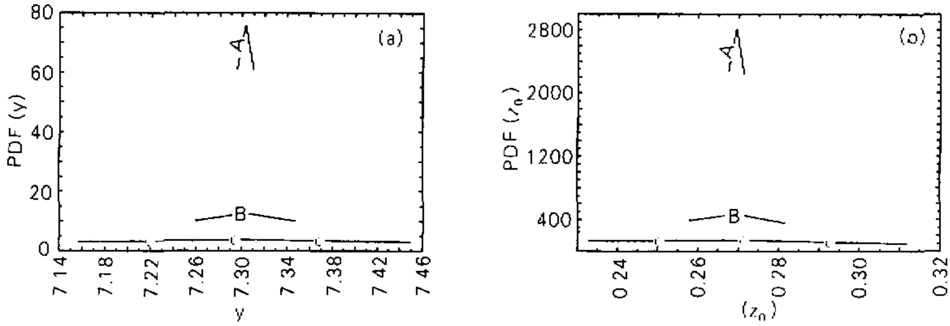

Fig. 1. Probability density function (PDF) in Exp. 1 for a certain curve type (e. g., two curves labeled with ' A'), a PDF and its independent variable $y(y=\ln[(z_1-d_0)/z_0])$ in Fig. 1a correspond to the other PDF (m^{-1}) and independent variable z_0 (m), the roughness length, in Fig. 1b.

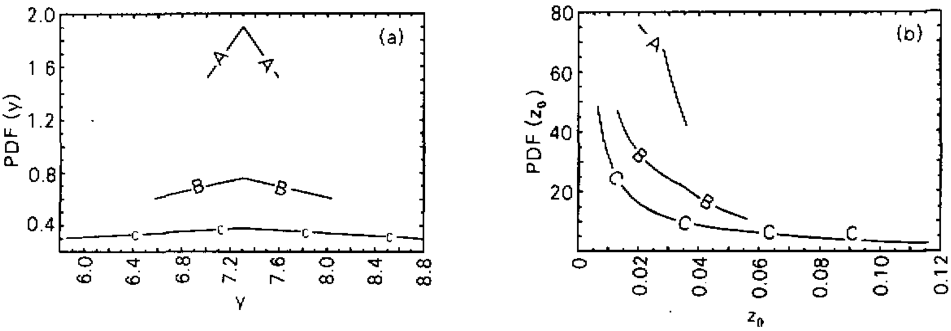

Fig. 2. Same as Fig. 1 except for different PDF parameters.

these PDFs are representative to some extent. When α_n is very small, say $\alpha_n=0.001$, $z_{01}\rightarrow z_{00}, C_{dn1}\rightarrow C_{dn0}$, it shows that the case is nearly equivalent to that without heterogeneity treatment. When α_n increases up to 0.6, $z_{0min}=3.37\times 10^{-4}$ m (close to 2.4×10^{-4} m, roughness length at water surface. Obviously, if α_n increases furthermore, the range of z_0 distribution is not realistic any more, which limits α_n of y distribution to the upper bound of about 0.6), $z_{0max}=2.15$ m (close to the roughness length of tall trees or buildings about 20 m high). In this case, z_{01} $(=2.3\times 10^{-1}$ m) is 8 times of $z_{00}(=2.69\times 10^{-2}$ m), and this leads to the drag coefficient to increase by 52.7% after heterogeneity treatment, while snow cover is 1.42 times of the original value. All of these imply the great influence of heterogeneity treatment. Physically, it is very necessary to consider the roughness–length heterogeneity, the above differences also show the importance of this consideration mathematically. Note that the maximum r_{uaf} is 1.146, and generally the increase extent of U_{af} is smaller than 10%, which shows the influence of heterogeneity on within–foliage windspeed is not great; while maximum r_{rai} is 1.051, i.e., the maximum extent of increase is 5.1%, showing the transfer coefficient of leaf surface influenced least by heterogeneity treatment. All the above conclusions can also be drawn from the following experiments.

3.3.2 *Analysis of Exp.2*

The aim of Exp.2 is to investigate the nonlinear changes caused by different height ratio γ, when width ratio α_n is fixed at certain values. From Table 2, when $\alpha_n = 0.04$, with γ changing from 0.001 to 0.999, C_{dn} changes from 3.0025×10^{-3} to 3.0048×10^{-3}, whose range is smaller than 1%; when $\alpha_n = 0.3$ with γ changing from 0.001 to 0.999, C_{dn} increases from 3.1715×10^{-3} to 3.2966×10^{-3}, whose range is 4%; when $\alpha_n = 0.5$ with γ changing from 0.01 to 0.999, C_{dn} increases from 3.4630×10^{-3} to 3.9997×10^{-3}, and the range is 15%. So, the greater width ratio α_n (i.e., the greater range of y or z_0), the greater average and range of drag coefficient C_{dn}; the greater height ratio γ (i.e., the more convergent to the average the y distribution), the greater average of drag coefficient C_{dn}. Meanwhile it can be seen that when γ has a value within (0, 1), the greater change range of C_{dn} corresponds to greater α_n. The greater α_n and γ, the greater C_{dn1}, therefore the more heterogeneous z_0, the greater average of drag coefficient C_{dn}. Besides, once α_n is determined, the range of z_0 distribution is then determined, together with the minimum C_{dn1} which has a certain deviation from C_{dn0} when α_n is not small enough. This illustrates the heterogeneity of the variable. While once γ is determined, no matter what value it has within (0,1), when α_n is small, the nonlinear characteristic quantity C_{dn1} can be very close to C_{dn0}. Therefore this shows that in the PDF parameters, α_n may be more important than γ. This may hint for a general case that the effective or representative range of a PDF independent variable is more important than the form of the probability distribution. Nevertheless this does not mean that γ is unimportant in the PDF representation. In fact, the γ value directly determines the range of the concentrated distribution of a PDF independent variable, as can be seen from the above cases where z_{01} is different corresponding to different γ's with the same α_n.

Table 2. Results of heterogeneity treatment in Exp.2 ($z_1 = 80$ m, $z_{00} = 5.39 \times 10^{-1}$ m)

	γ	z_{0min} (m)	z_{0max} (m)	z_{01} (m)	C_{dn1} ($\times 10^{-3}$)
$\alpha_n = 0.04$	0.01	4.02×10^{-2}	7.22×10^{-2}	5.43×10^{-2}	3.0025
	0.1	4.02×10^{-2}	7.22×10^{-2}	5.45×10^{-2}	3.0038
	0.4	4.02×10^{-2}	7.22×10^{-2}	5.46×10^{-2}	3.0043
	0.7	4.02×10^{-2}	7.22×10^{-2}	5.46×10^{-2}	3.0046
	0.9	4.02×10^{-2}	7.22×10^{-2}	5.47×10^{-2}	3.0047
	0.999	4.02×10^{-2}	7.22×10^{-2}	5.47×10^{-2}	3.0048
$\alpha_n = 0.3$	0.0001	6.03×10^{-3}	4.82×10^{-1}	7.92×10^{-2}	3.144
	0.01	6.03×10^{-3}	4.82×10^{-1}	7.98×10^{-2}	3.147
	0.02	6.03×10^{-3}	4.82×10^{-1}	8.03×10^{-2}	3.150
	0.03	6.03×10^{-3}	4.82×10^{-1}	8.09×10^{-2}	3.153
	0.1	6.03×10^{-3}	4.82×10^{-1}	8.45×10^{-2}	3.172
	0.2	6.03×10^{-3}	4.82×10^{-1}	8.90×10^{-2}	3.195
	0.6	6.03×10^{-3}	4.82×10^{-1}	1.012×10^{-1}	3.258
	0.9	6.03×10^{-3}	4.82×10^{-1}	1.071×10^{-1}	3.289
	0.99	6.03×10^{-3}	4.82×10^{-1}	1.086×10^{-1}	3.297
$\alpha_n = 0.5$	0.01	1.39×10^{-3}	2.70	1.50×10^{-1}	3.46
	0.1	1.39×10^{-3}	2.70	1.73×10^{-1}	3.55
	0.7	1.39×10^{-3}	2.70	2.60×10^{-1}	3.90
	0.9	1.39×10^{-3}	2.70	2.77×10^{-1}	3.97
	0.999	1.39×10^{-3}	2.70	2.80×10^{-1}	4.00

3.3.3 *Analysis of Exp.3*

In Exp.3, we choose several α_n and calculate the magnitudes of characteristic quantities corresponding to $\gamma=0.1$, 0.5, 0.8. In Table 3 from the changes of $z_{0\min}$ and $z_{0\max}$ with α_n, an upper bound 0.6 is found out for α_n. Meanwhile it can be seen that with γ increasing, especially with α_n increasing, r_{cdn}, r_{fsn}, r_{z0}, r_{uaf} and r_{rai} increase gradually, i.e., the more significant the heterogeneity, the farther away the nonlinear magnitudes treated from those un–treated, which may be seen from all the experiments in the paper. For example, when $\alpha_n=0.45$ and $\gamma=0.5$, $\gamma_{cdn}=1.208$, $r_{fsn}=1.615$, $r_{z0}=3.474$, $r_{uaf}=1.064$; when $\alpha_n=0.6$ and $\gamma=0.1$, $\gamma_{cdn}=1.298$, $r_{fsn}=1.661$, $r_{z0}=4.983$, $r_{uaf}=1.086$; and when $\alpha_n=0.6$ and $\gamma=0.8$, $r_{cdn}=1.527$, $r_{fsn}=1.965$, $r_{z0}=8.56$, $r_{uaf}=1.146$; but r_{rai} is smaller than 1.051 all the time. So, when the heterogeneity of a certain variable is taken into account (say roughness length z_0 in the paper), some of the relevant nonlinear terms are very sensitive to the heterogeneity (e.g., C_{dn} and f_{snow} in the paper. Why C_{dn} is so sensitive is that C_{dn} is proportional to y^{-2} where the index $|-2|>1$), some are not sensitive (say, in the paper U_{af} is generally smaller than 1.1, only in the extreme case $1.1<r_{uaf}<1.2$. This is because U_{af} is proportional to y^{-1} where the index $|-1|=1$), some are extremely insensitive (say, in the paper r_{rai} is smaller than 1.051 all the time, which is because r_{lai}^{-1} is proportional to $y^{-1/2}$ where $|-1/2|<1$). The sensitivities are associated with the exponential indexes of the heterogeneous variable in the nonlinear terms, and the greater the absolute value of the index compared with 1, the more sensitive to heterogeneity the nonlinear terms; while the smaller the absolute value of the index compared with 1, the less sensitive the nonlinear terms. This actually gives us an enlightenment that we do not need to consider the heterogeneity of the terms which are insensitive or even not very sensitive. Obviously, the sensitivity experiments are the preconditions of the above conclusion. This further shows that it is very necessary to perform sensitivity experiments on the stand–alone terms similar to those in the paper, before the heterogeneously–treated analytical expressions are used for a land surface model.

Comparing the case when $\alpha_n=0.45$ and $\gamma=0.5$ with the case when $\alpha_n=0.6$ and $\gamma=0.1$, we find that the nonlinear terms with heterogeneity treatments in the latter case are farther away from those without treatments. This also demonstrates the conclusion drawn from Exp.2, i.e., α_n is more important than γ in the parameters representing heterogeneity.

Fig. 3 to Fig. 6 correspond to Exp.3, where Fig. 3 and Fig. 4 depict the cases of $\alpha_n=0.006$ and 0.1 respectively. It can be seen that the peak–like distribution of z_0 can be obtained in the case of small α_n, while when $\alpha_n=0.1$, the peak–like distribution of z_0 is obtained only in the case of $\gamma=0.1$. Turning to Exp.2, we can see that when $\alpha_n=0.3$, the peak–like distributions of z_0 are also shown in the cases of $\gamma=0.001$, 0.01, 0.02 (figure not shown). This provides us a way that, in the land surface model, no matter what value width ratio α_n (with respect to y) has in the reasonable range, we can get a peak–like distribution of z_0 which reflects high heterogeneity by choosing very small height ratio γ.

Fig. 5 and Fig. 6 correspond to the cases of $\alpha_n=0.2$ and $\alpha_n=0.6$, from which we can see that the distribution probability mainly concentrates over the small–value range of z_0 in spite of the great extent of z_0 change with the increase of α_n.

From the above analysis of experiments, we can see that in a certain condition, the roughness–length heterogeneity can cause relatively large deviations from those without heterogeneity treatment of factors such as drag coefficient and snow coverage, etc..

Table 3. Results of heterogeneity treatment in Exp.3 (z_l = 80 m, z_{00} = 5.39×10^{-2} m, s_r = 0.01 m, and the three values of the same quantity with the same α_n respond to γ = 0.1, 0.5, 0.8, respectively)

	α_n = 0.006	α_n = 0.02	α_n = 0.04	α_n = 0.1	α_n = 0.2	α_n = 0.3	α_n = 0.45	α_n = 0.6
z_{0min} (m)	5.16×10^{-2}	4.66×10^{-2}	4.02×10^{-2}	2.60×10^{-2}	1.25×10^{-2}	6.03×10^{-3}	2.01×10^{-3}	6.74×10^{-4}
z_{0max} (m)	5.63×10^{-2}	6.24×10^{-2}	7.22×10^{-2}	1.12×10^{-1}	2.32×10^{-1}	4.82×10^{-1}	1.44	4.31
	5.39×10^{-2}	5.40×10^{-2}	5.43×10^{-2}	5.68×10^{-2}	6.62×10^{-2}	8.45×10^{-2}	1.40×10^{-1}	2.68×10^{-1}
z_{01} (m)	5.39×10^{-2}	5.40×10^{-2}	5.45×10^{-2}	5.80×10^{-2}	7.15×10^{-2}	9.88×10^{-2}	1.87×10^{-1}	4.00×10^{-1}
	5.39×10^{-2}	5.40×10^{-2}	5.47×10^{-2}	5.85×10^{-2}	7.40×10^{-2}	1.05×10^{-1}	2.08×10^{-1}	4.61×10^{-1}
	3.000	3.0007	3.0028	3.018	3.073	3.17	3.43	3.90
$C_{dnl}\times10^{-3}$	3.000	3.0009	3.0040	3.025	3.104	3.25	3.63	4.36
	3.000	3.0011	3.0045	3.029	3.118	3.28	3.72	4.58
	15.65	15.67	15.72	16.1	17.4	19.3	22.7	26.0
f_{snowl} (%)	15.65	15.68	15.76	16.3	18.2	20.8	25.3	29.3
	15.66	15.69	15.77	16.4	18.5	21.5	26.5	30.8
	1.00002	1.0002	1.0009	1.006	1.024	1.057	1.142	1.298
r_{cdn}	1.00003	1.0003	1.0013	1.008	1.034	1.082	1.208	1.455
	1.00004	1.0004	1.0015	1.010	1.039	1.093	1.239	1.527
	1.00019	1.0021	1.0084	1.0536	1.223	1.569	2.616	4.983
r_{z0}	1.00027	1.0030	1.0119	1.0759	1.327	1.833	3.474	7.437
	1.00031	1.0034	1.0135	1.0862	1.373	1.949	3.867	8.562
	0.9997	1.0008	1.0044	1.029	1.113	1.236	1.449	1.661
r_{fsn}	1.0000	1.0015	1.0067	1.042	1.160	1.331	1.615	1.869
	1.0001	1.0019	1.0077	1.048	1.182	1.374	1.691	1.965
	1.00001	1.00008	1.00032	1.0020	1.0080	1.019	1.044	1.086
r_{uaf}	1.00001	1.00011	1.00045	1.0028	1.0114	1.026	1.064	1.127
	1.00002	1.00013	1.00050	1.0032	1.0129	1.030	1.073	1.146
	1.00000	1.00003	1.00012	1.0007	1.0030	1.0069	1.0161	1.0306
r_{rai}	1.00000	1.00004	1.00017	1.0010	1.0042	1.0097	1.0231	1.0445
	1.00001	1.00005	1.00019	1.0012	1.0048	1.0111	1.0263	1.0509

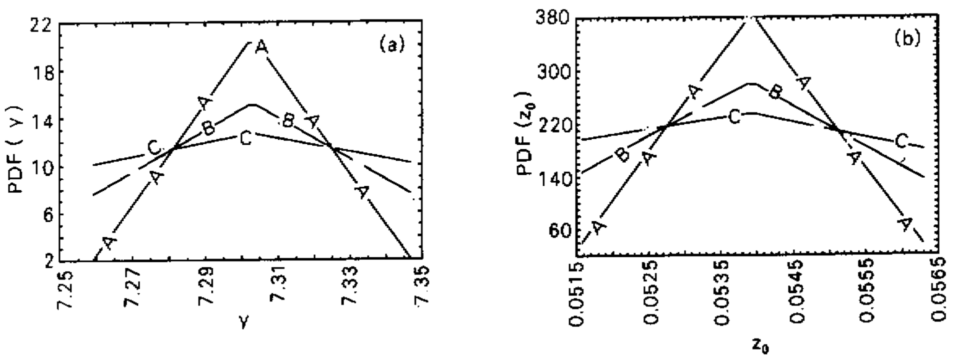

Fig. 3. Same as Fig. 1 except for different PDF parameters in Exp.3.

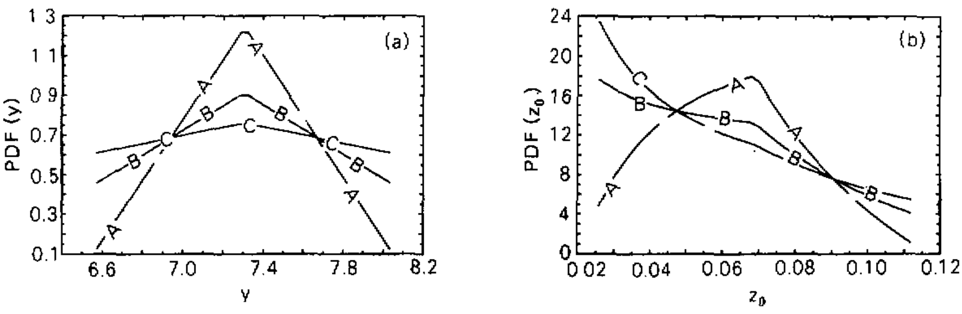

Fig. 4. Same as Fig. 3 except for different PDF parameters.

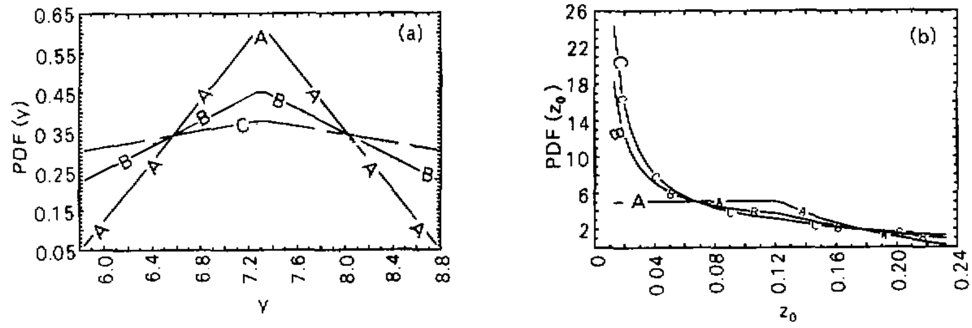

Fig. 5. Same as Fig. 3 except for different PDF parameters.

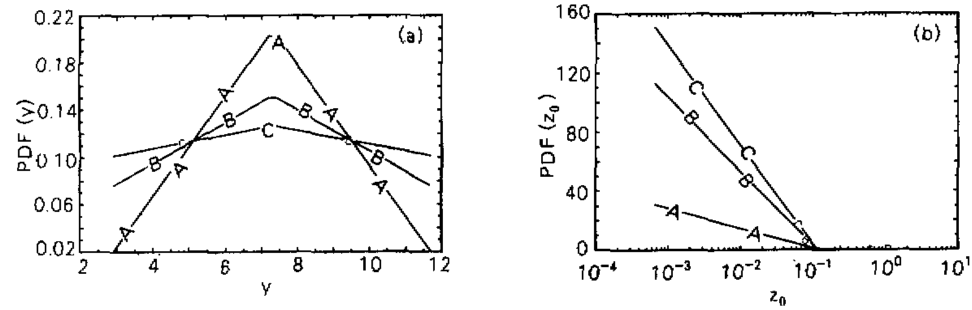

Fig. 6. Same as Fig. 3 except for different PDF parameters.

These deviations can change the land surface upward fluxes, i.e., the fluxes of momentum, sensible heat and latent heat, which are the main aim for a land surface model to calculate. Hence, heterogeneity treatment is very important in land surface models.

4. Summary and conclusions

Based on Giorgi (1997a, 1997b), in this paper, we have proposed a "combined approach", which is a combination of mosaic approach and analytical–statistical–dynamical approach, thereby it not only can represent both interpatch and intrapatch variability, but also cost less computational time when the land surface heterogeneity is taken into account. Because the independent variable of probability density function (PDF) is extended to the single valued function of basic meteorological characteristic quantities, which is much more universal as compared to Giorgi (1997a, 1997b). By the use of the combined approach, analytical expressions of characteristic quantities (e.g., drag coefficient, snow coverage, within–foliage windspeed and leaf surface aerodynamical resistance) affected by roughness length are derived, when the roughness length (and / or the zero plane displacement) heterogeneity is considered, and then sensitivity experiments against different values of width ratio α_n and height ratio γ of PDF (here a linear, symmetric PDF) are carried out.

We firstly suggest the rule which should be followed when the parameters width ratio α_n and height ratio γ of the distribution of the heterogeneous characteristic quantities are chosen, i.e., the chosen PDF parameters should be relatively consistent with the used model and the properties of the simulated domain. On the basis of the rule, we choose a function y of the roughness length z_0 as the PDF independent variable, and set different values of the two parameters width ratio α_n and height ratio γ, which are mathematically independent of each other, to construct a variety of PDFs. We designed three experiments, in which we judged and obtained the range of α_n and γ as well as the heterogeneity–caused changes of drag coefficient, snow coverage, within–foliage windspeed and leaf surface aerodynamical resistance, according to the reasonable range of z_0 and its distributions. By the above sensitivity experiments, we try to choose different PDF width ratio α_n's and height ratio γ's corresponding to y, and to comprehend the sensitivity of roughness– length PDF, which has more obviously physical meaning, to different PDF parameters . Therefore, a theoretical foundation is provided for the use of combined approach to consider relatively realistic PDFs of heterogeneous distributions of various physical quantities.

Meanwhile, some main conclusions can be drawn as follows:

(1) When the heterogeneity of a certain variable is considered (say roughness length z_0 in the paper), some of the relevant nonlinear terms are very sensitive to the heterogeneity (e.g., C_{dn} and f_{snow} in the paper), some are not very sensitive (say U_{af} in the paper), some are extremely insensitive (say, in the paper r_{rai}). So, we need to consider the heterogeneity of the terms which are sensitive, and do not need to consider the heterogeneity of the terms which are especially insensitive or even not very sensitive. This further demonstrates that it is very necessary to perform sensitivity experiments on the stand–alone terms similar to those in the paper, before the heterogeneously–treated analytical expressions are used for an off–line land surface model or a land surface model coupled to an atmospheric model.

(2) When the parameter α_n is very small, little heterogeneity is represented, which is almost equivalent to the case without heterogeneity treatment.

(3) Different cases have different α_n upper limits, which are generally smaller than about 0.6.

(4) In the reasonable range of α_n, no matter what a value it has, a peak–like distribution of roughness length z_0 can be depicted by a small value of γ.

(5) In the PDF parameters which represent the heterogeneity, α_n is more important than

y. This may hint a general case that the effective or representative range of a PDF independent variable is more important than the form of the distribution.

(6) The more heterogeneous roughness length z_0, the greater differences between the characteristics with and without heterogeneity treatment.

In this paper, some of the results or conclusions, e.g., conclusions (1) and (2), have universal meanings, while some are relevant to the used heterogeneity treatment, e.g., drag coefficient, within–foliage windspeed and leaf surface aerodynamical resistance become greater after heterogeneity treatment. The companion paper will discuss the experiments of applying results in this paper to the land surface model coupled to the atmospheric model.

APPENDIX A

Heterogeneity treatment results of two characteristic quantities

(1) Treatment of snow coverage f_{snow}

After using snow coverage f_{snow} formula in BATS, we make transformations of the integral variables and make approximations for the series, then obtain

$$F^{y}_{pdf}(f_{snow}) = \int f_{snow} f_{pdf}(y)dy = \int \frac{S_r}{z_0 + S_r} f_{pdf}(y)dy = \sum_{i=1}^{2} \int_{y} \frac{S_r(c_i y + d_i)}{z_0 + S_r} dy$$

$$\overset{}{\underset{y=\ln(z_1/z_0 - d_{00})}{=\!=\!=\!=}} \sum_{i=1}^{2} (\mathrm{M}_1 + \mathrm{M}_2 + \mathrm{M}_3 + \mathrm{M}_4) \ , \qquad \text{(A.1)}$$

where

$$\mathrm{M}_1 = c_i d_{00} S_r / (z_1 + d_{00} S_r)\{\frac{1}{2}[\ln(z_1 - d_{1,00} z_0)]^2 - \ln(z_1 + d_{00} S_r)\ln(d_{00} z_1 + d_{00} S_r) + \sum_{k=1}^{4} \{(d_{00} z_1 + d_{00} S_r)/[k(z_1 + d_{00} S_r)]\}^k \}\Big|_{z_{i1}}^{z_{i2}} \ ,$$

$$\mathrm{M}_2 \approx d_{00} S_r d_i / (z_1 + d_{00} S_r)\{\ln[z_1 - d_{1,00} z_0)/(d_{00} z_0 + d_{00} S_r)] - \frac{c_i}{d_i}[\ln(z_i / d_{00})\ln(z_1 / d_{00} - z_0) + \frac{1}{2}\ln^2(z_0 + S_r)] + \frac{c_i}{d_i} \sum_{k=1}^{4} \{S_r / [k(z_0 + S_r)]\}^k - \frac{c_i}{d_i} \sum_{k=1}^{4} [(z_1 - d_{1,00} z_0)/(kz_1)]^k \}\Big|_{z_{i1}}^{z_{i2}} \ ,$$

$$\mathrm{M}_3 = S_r c_i \{\ln z_1 \ln z_0 - \ln(z_1 + d_{00} S_r)\ln[(z_0 + S_r)d_{00}] - \frac{1}{2}\ln^2 z_0 + \frac{1}{2}\ln^2(z_0 + S_r)\}\Big|_{z_{i1}}^{z_{i2}} \ ,$$

$$\mathrm{M}_4 \approx S_r d_i \ln[z_0 / (z_0 + S_r)]\Big|_{z_{i1}}^{z_{i2}} + S_r c_i \left\{ \sum_{k=1}^{4} \{S_r / [k(z_0 + S_r)]\}^k - [(d_{1,00} z_0 / (kz_1)]^k + \left[(d_{1,00}(z_0 + S_r)/[kz_1(z_1 + d_{00} S_r)] \right]^k \right\}\Bigg|_{z_{i1}}^{z_{i2}} \ .$$

(2) Treatment of stomatal resistance r_s

Utilizing the formulas in the text of the paper, we can treat stomatal resistance and aerodynamical resistance in Formula (20) as follows:

$$G^q_{\text{pdf}}\{F^y_{\text{pdf}}[(r_{\text{la}} + r_s)^{-1}]\} = 2C_E \times \sum_{j=1}^{2}\sum_{i=1}^{2}(\text{N}_1 + \text{N}_2 - \text{N}_3 - \text{N}_4 + \text{N}_5) \,, \tag{A.2}$$

where

$$\text{N}_1 = \int_q \int_x x(c_i x^2 + d_i)c_{qj}dxdq = (c_{qj}c_i\frac{x^4}{4} + d_i c_{qj}\frac{x^2}{2})|_{x_{i1}}^{x_{i2}} \quad q|_{q_{j1}}^{q_{j2}} \,,$$

$$\text{N}_2 = \ln(x + q_{j2})(Ax^5 + Bx^4 + Cx^3 + Dx^2)|_{x_{i1}}^{x_{i2}} \,,$$

$$\text{N}_3 = \ln(x + q_{j1})(Ax^5 + Bx^4 + Cx^3 + Dx^2)|_{x_{i1}}^{x_{i2}} \,,$$

in above three terms, $x = y^{1/2}$;

$$\text{N}_4 = [\frac{As^5}{5} + \frac{(-5Aq_{j2} + B)s^4}{4} + \frac{(10Aq_{j2}^2 - 4Bq_{j2} + C)s^3}{3} - \frac{(10Aq_{j2}^3 - 6Bq_{j2}^2 + 3Cq_{j2} - D)s^2}{2} + (5Aq_{j2}^4 - 4Bq_{j2}^3 + 3Cq_{j2}^2 - 2Dq_{j2})s + (-Aq_{j2}^5 + Bq_{j2}^4 - Cq_{j2}^3 + Dq_{j2}^2)|\ln s\,]|_{x_{i1}+q_{j2}}^{x_{i2}+q_{j2}} \,,$$

$s = x + q_{j2}$ in above terms;

$$\text{N}_5 = [\frac{As^5}{5} + \frac{(-5Aq_{j1} + B)s^4}{4} + \frac{(10Aq_{j1}^2 - 4Bq_{j1} + C)s^3}{3} - \frac{(10Aq_{j1}^3 - 6Bq_{j1}^2 + 3Cq_{j1} - D)s^2}{2} + (5Aq_{j1}^4 - 4Bq_{j1}^3 + 3Cq_{j1}^2 - 2Dq_{j1})s + (-Aq_{j1}^5 + Bq_{j1}^4 - Cq_{j1}^3 + Dq_{j1}^2)\ln s\,]|_{x_{i1}+q_{j1}}^{x_{i2}+q_{j1}} \,,$$

$s = x + q_{j1}$ in above terms.

APPENDIX B

Transformation between PDFs corresponding to two relevant heterogeneous variables

After PDF function $f_{\text{pdf}}(y)$ corresponding to independent variable y is chosen, and if $y = y(x)$, PDF function $g_{\text{pdf}}(x)$ corresponds to independent variable x, then

$$1 = \int_{xx_1}^{xx_2} g_{\text{pdf}}(x)dx \,, \tag{B.1}$$

$$1 = \int_{y_1}^{y_2} f_{\text{pdf}}(y)dy = \int_{x_1}^{x_2} f_{\text{pdf}}[y(x)]\frac{dy(x)}{dx}dx \,, \tag{B.2}$$

where $xx_2 > xx_1, y_2 > y_1, f_{\text{pdf}}(y) \geq 0, y_1 = y(x_1), y_2 = y(x_2)$.

When we follow the rule that the independent variable of PDF is extended to the single valued function of the basic meteorological characteristic quantity, and if $\frac{dy(x)}{dx} > 0$, $x_2 > x_1$ in Formula (B.2), we can obtain

$$g_{pdf}(x) = f_{pdf}[y(x)]\frac{dy(x)}{dx} \quad . \tag{B.3}$$

While if $\frac{dy(x)}{dx} < 0$, then $x_2 < x_1$, and

$$1 = \int_{x_1}^{x_2} f_{pdf}[y(x)]\frac{dy(x)}{dx}dx = \int_{x_2}^{x_1} f_{pdf}[y(x)][-\frac{dy(x)}{dx}]dx = \int_{x_2}^{x_1} f_{pdf}[y(x)]\left|\frac{dy(x)}{dx}\right|dx \ .$$

From formula (B.1), we have the form

$$g_{pdf}(x) = f_{pdf}[y(x)]\left|\frac{dy(x)}{dx}\right| \ . \tag{B.4}$$

Hence, PDF function $g_{pdf}(x)$ with the form as Formula (B.4) is validated in varied cases, and satisfies the meaning $g_{pdf}(x) > 0$. Therefore, when the PDF corresponding to independent variable y and the quantitative relationship between y and x are given, the PDF corresponding to x can be derived. It is notable that the integral upper limit should be greater than the lower limit when Formula (B.4) is used for the integration with respect to x, e.g., in Formula (B.1), $xx_2 > xx_1$ must be satisfied.

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