Response of Atmospheric Low–frequency Wave to
Oceanic Forcing in the Tropics[①]

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ABSTRACT

The effects of oceanic forcing on the atmospheric low–frequency wave (LFW for short) in the tropics are analyzed, where ocean and atmosphere are taken as an independent system, respectively. Here oceanic effects are parameterized as evaporation–wind feedback (EWF for short) and forcing of SST. Under the modulation of EWF, forcing of SST plays a different role from that without EWF. So LFWs are diabatic waves, forced by the interactions of multiple factors, in the tropics.

Key words: Atmospheric LFW, Oceanic forcing, EWF, Radiative cooling, SST

1. Introduction

Investigations about atmospheric LFW have been a focus of research since Madden and Julian's outstanding analysis works (1971, 1972). Many dynamical and thermal mechanisms (Chao et al., 1996; Fu et al., 1998; Hendon et al., 1998; Krishnamurti et al., 1988; Lau and Chan, 1988) have been advised to explain LFW. Among them are oceanic effects, such as SST effect, thermal forcing and others. Usually atmosphere and ocean are taken as a coupled system, which is used to explain ENSO or the relation between El Nino and intraseasonal oscillations (Hirst and Lau, 1990; Lau and Chan, 1988; Lau et al., 1989; Li and Li, 1996; Li and Liao, 1998; Tziperman et al., 1997; Zhang, 1995). Oceanic effects are also taken as forcing to atmosphere, which is considered as a single system (Kessler et al., 1995; Kleeman, 1991; Liu et al., 1993; Webster, 1981). Investigations have shown that there is a correlation between SST anomaly and atmospheric potential height (Li and Li, 1996; Li and Liao, 1998 and Wu et al., 1996). In this paper, therefore, we parameterize oceanic effects as two forcing factors, SST effect and EWF mechanism, to analyze the characteristics of forced atmospheric LFW.

2. Basic equations and analysis

In the equatorial β–plane and long–wave approximation framework, the pure baroclinic model containing diabatic heating can be written as

$$\frac{\partial \hat{u}}{\partial t} - \beta y \hat{v} = - \frac{\partial \hat{\varphi}}{\partial x} \ , \tag{1a}$$

$$\beta y \hat{u} = - \frac{\partial \hat{\varphi}}{\partial y} \ , \tag{1b}$$

$$\frac{\partial \hat{\varphi}}{\partial t} + c_{11}^2 \left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} \right) = Q' \ . \tag{1c}$$

Equations (1) can be taken as the formulas about pure baroclinic components in a two–layer model, details of derivation can be obtained in Fu et al. (1998). c_{11} represents baroclinic wave speed, the term Q' at the right side of (1c) represents diabatic heating rate, in the context of this paper, it is expressed as

$$Q' = \lambda_a [1 - e(T_s)]c_{11}^2 \hat{u} - \lambda_c \hat{\varphi} + e(T_s)c_{11}^2 \left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} \right) \ , \tag{2}$$

where $- \lambda_c \hat{\varphi}$ and $e(T_s)c_{11}^2 \left(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} \right)$ are parameterizations for SST effects caused by radiative cooling and the precipitation related to SST effects, respectively; $\lambda_a [1 - e(T_s)c_{11}^2 \hat{u}$ is the parameterization of EWF, coupled with SST effects.

Thus, the solutions to Equations (1) can be assumed in the following form:

$$(\hat{u}, \hat{v}, \hat{\varphi}) = [u(y), v(y), \varphi(y)]e^{i(kx - \omega t)} \ . \tag{3}$$

where ω is angular frequency and k is wave number in the x–direction.

Substituting (3) into (1) yields

$$\frac{d^2 v}{dy^2} - \frac{i\lambda_a \beta y}{\omega} \frac{dv}{dy} + \left[- \frac{\beta k + i\lambda_a \beta}{\omega} - \left(1 + \frac{i\lambda_c}{\omega} \right) \frac{\beta^2}{c_1^2} y^2 \right] v = 0 \ , \tag{4}$$

where $c_1^2 = [1 - \exp(T_s)]c_{11}^2$, it is obvious that SST effects make the forced waves propagate slowly for $e(T_s) < 1$; the higher SST, the slower their phase speeds (The value of $e(T_s)$ corresponding to different SST is given by Liu and Zhuang (1993)).

Equation (4) can be transformed as Weber–type equation

$$\frac{d^2 p}{dy^2} + \left[- \frac{2\beta k + i\lambda_a \beta}{2\omega} - \left(1 + \frac{i\lambda_c}{\omega} - \frac{\lambda_a^2 c_1^2}{4\omega^2} \right) \frac{\beta^2}{c_1^2} y^2 \right] p = 0 \ , \tag{5}$$

when transformation $v(y) = p(y)e^{\frac{i\lambda_a \beta}{4\omega} y^2}$ is chosen.

Equation (5) satisfying the following condition $y \to \pm \infty$, $p \to 0$ gives eigenvalue

$$- \left(1 + \frac{i\lambda_c}{\omega} - \frac{\lambda_a^2 c_1^2}{4\omega^2} \right)^{-\frac{1}{2}} c_1 \frac{i\lambda_a \beta + 2k\beta}{\beta \quad 2\omega} = 2n + 1 \ , \quad (n = 0,1,2\cdots) \tag{6}$$

and corresponding eigenfunction

$$p_n(y) = A_n H_n(y / L_2) \exp^{-\frac{y^2}{2L_2^2}} \quad , \quad (n = 0,1,2\cdots) \tag{7}$$

with $L_2^4 = \left(1 + \frac{i\lambda_c}{\omega} - \frac{\lambda_a^2 c_1^2}{4\omega^2}\right)\frac{\beta^2}{c_1^2}$, where A_n is arbitrary constant. $H_n(y / L_2)$ is Hermitian polynomial. Thus

$$v_n(y) = A_n H_n(y / L_2) \exp^{-\frac{y^2}{2L_2^2} + \frac{i\lambda_a \beta}{4\omega}y^2} \quad , \quad (n = 0,1,2\cdots) \tag{8}$$

which satisfying the condition $y \to \pm\infty$, $v \to 0$ yields the constraint on ω

$$Re\left(\frac{i(n+1)\lambda_a + k}{\omega}\right) < 0 \ . \tag{9}$$

From Equation (6), we get

$$\omega^2 + i\lambda_c \omega = \frac{[n(n+1)\lambda_a^2 + k^2 + i\lambda_a k]c_1^2}{(2n+1)^2} \ . \tag{10}$$

Setting

$$\omega = \omega_r + i\omega_i \ . \tag{11}$$

where ω_r is disturbance frequency and ω_i is disturbance growth rate.

When $e(T_x) < 1$, substituting (11) into (10), we obtain

$$\omega_r^2 - \omega_i^2 - \lambda_c \omega_i = \frac{[n(n+1)\lambda_a^2 + k^2]c_1^2}{(2n+1)^2} \ . \tag{12a}$$

$$2\omega_r \omega_i + \lambda_c \omega_r = \frac{\lambda_a k c_1^2}{(2n+1)^2} \ . \tag{12b}$$

from which the following solutions can be obtained:

$$\omega_r = -\left[\frac{G}{2} - \frac{\lambda_c^2}{8} + \frac{1}{2}\sqrt{[G - \frac{\lambda_c^2}{4}]^2 + [\frac{\lambda_a k c_1^2}{(2n+1)^2}]^2}\right]^{\frac{1}{2}} \ . \tag{13a}$$

$$\omega_i = -\frac{\lambda_c}{2} + \frac{\lambda_a k c_1^2}{2(2n+1)^2 \omega_r} \ . \tag{13b}$$

where $G = \frac{[n(n+1)\lambda_a^2 + k^2]c_1^2}{(2n+1)^2}$, on account of that when $\lambda_c = \lambda_a = e(T_x) = 0$, Equation (13) must take the following forms:

$$\omega_r = \omega_1 \equiv -\frac{kc_{11}}{2n+1} \ , \quad (n = 0,1,2\cdots) \tag{14a}$$

$$\omega_i = 0 \ . \tag{14b}$$

Substituting (11) into (9) yields the constraint on ω_r and ω_i

$$k\omega_r + (n+1)\lambda_a \omega_i < 0 \ . \tag{15}$$

In order to get low frequency in (13), the following condition

$$\lambda_c^2 \leqslant \frac{4[n(n+1)\lambda_a^2 + k^2]c_1^2}{(2n+1)^2} + S_0 \tag{16}$$

must be valid for λ_c, where $S_0 > 0$. From Eq.(16), we know that the LFW would be the results forced by stronger SST forcing (λ_c is larger) under the modulation of EWF, when the atmosphere is keeping an equilibrium of stronger radiation and convection (λ_a is small but not zero). Then we can obtain smaller ω_r and $\omega_i > 0$.

We know that solutions (13) can be also extended to

$$\omega_r = \left[\frac{k^2 c_1^2}{2} - \frac{\lambda_c^2}{8} + \frac{1}{2}\sqrt{\left(k^2 c_1^2 - \frac{\lambda_c^2}{4}\right)^2 + \lambda_a^2 k^2 c_1^4} \right]^{\frac{1}{2}} , \tag{17a}$$

$$\omega_i = -\frac{\lambda_c}{2} + \mathrm{sgn}(\lambda_a)\left[-\left(\frac{k^2 c_1^2}{2} - \frac{\lambda_c^2}{8}\right) + \frac{1}{2}\sqrt{\left(k^2 c_1^2 - \frac{\lambda_c^2}{4}\right)^2 + \lambda_a^2 k^2 c_1^4} \right]^{\frac{1}{2}} , \tag{17b}$$

for Kelvin wave. Similarly, in order to get low frequency in (17), the following condition

$$\lambda_c^2 \leqslant 4k^2 c_1^2 + S_0 \tag{18}$$

must be valid for λ_c. Under the forcing of SST effect and EWF, forced Kelvin wave is LFW and unstable.

Similarly, when $\lambda_a = 0$, from (10), we can obtain

$$\omega_r = -\sqrt{-\frac{\lambda_c^2}{4} + \frac{k^2 c_1^2}{(2n+1)^2}} \ , \tag{19a}$$

$$\omega_i = -\frac{\lambda_c}{2} \ , \tag{19b}$$

for Rossby wave and

$$\omega_r = \sqrt{-\frac{\lambda_c^2}{4} + k^2 c_1^2} \ , \tag{20a}$$

$$\omega_i = -\frac{\lambda_c}{2} \ , \tag{20b}$$

for Kelvin wave.

Eq.(19) and (20) are the results when only SST forcing is considered. From (19), we know that $\omega_i \neq 0$ and $|\omega_r| < |\omega_1|$ are valid for $\lambda_c \neq 0$ or $e(T_s) \neq 0$, so disturbance is unstable and its speed can be slowed down. When $\lambda_c > 0$, $\omega_i < 0$, which means that the positive anomaly of sea temperature damps the waves. Furthermore, ω_r is real, so the following condition

$$\lambda_c^2 \leqslant \frac{4k^2 c_1^2}{(2n+1)^2} \qquad (n = 0,1,2\cdots) \tag{21}$$

must be valid for λ_c. From (21), we know that the lower mode is easier to reach its valid λ_c and it can also have broader value limit, which shows that the forced modes in the tropics are lower modes.

Similarly, the above analysis is also valid for (20), and the constraint

$$\lambda_c^2 \leqslant 4k^2 c_1^2 \tag{22}$$

is the limitation for λ_c. Comparing (21) with (22) shows that the Kelvin wave is most probably forced among all modes. This explains why the Kelvin wave plays an important role in the LFW in the tropics.

When $\lambda_c = e(T_c) = 0$, from (10), we have

$$\omega_r = -\frac{c_{11}}{\sqrt{2}(2n+1)}\left[n(n+1)\lambda_a^2 + k^2 + \sqrt{[n(n+1)\lambda_a^2 + k^2]^2 + \lambda_a^2 k^2}\right]^{\frac{1}{2}}, \tag{23a}$$

$$\omega_i = -\frac{c_{11}\,\mathrm{sgn}(\lambda_a)}{\sqrt{2}(2n+1)}\left[\sqrt{[n(n+1)\lambda_a^2 + k^2]^2 + \lambda_a^2 k^2} - [n(n+1)\lambda_a^2 + k^2]\right]^{\frac{1}{2}}, \tag{23b}$$

which is respectively the expression of angular frequency and growth rate of the Rossby wave when only EWF mechanism is considered. From (23), we know that $\omega_i \neq 0$ and $|\omega_r| > |\omega_1|$ are valid for $\lambda_a \neq 0$, so disturbance may be unstable but its speed cannot be slowed down.

If we define the non–dimensional coefficient of EWF mechanism as $n_k \equiv \frac{\lambda_a}{k_1}$, the damping scale as $\tau \equiv \frac{1}{\lambda_c}$ and the non–dimensional zonal wavenumber as $k_m \equiv \frac{k}{k_1}$, where k_1 is zonal wavenumber one. So the wave period can be expressed as

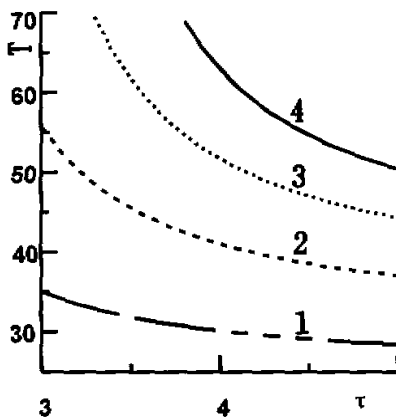$$T = \frac{2\pi}{|\omega_r|}. \tag{24}$$

Fig. 1. Variation of T with τ.   Fig. 2. Variation of T with k_m.
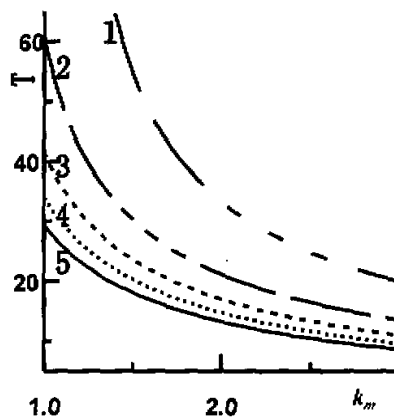
From the results based on (13), we can obtain some numerical analysis about the forced LFW. It shows that the forced wave is not always damped, when the basic current is westerly, the LFW can be amplified (figure not shown). Fig. 1 shows the variation of wave period with damping scale τ, the labels 1, 2, 3 and 4 are corresponding to the choices $n_k = 0.8$, $n_k = 0.5$, $n_k = 0.2$, and $n_k = 0.1$, for $k_m = 1$ and $n = 1$, respectively. They indicate that the wave period is slowed down by the increasing EWF effect when the radiative cooling effect is taken into account, in which EWF mechanism plays a modulating role. The variation of T is slowed down with the increasing damping scale. Fig. 2 shows the variation of T with k_m for $n = 0$ and $\tau = 4$ days, in which labels 1, 2, 3, 4 and 5 denote SST corresponding to 24°C, 23°C, 22°C, 21°C and 20°C, respectively. It is clear that the increasing SST makes the wave period longer and the increasing zonal wavenumber slows down the period. The results of Fig. 1, and Fig. 2, are the same in that the heating from the SST effect is beneficial to the generation of LFW in the tropics.

3. Conclusion

In this paper, we analyze the characteristics of LFW forced by SST and EWF forcing when $e(T_s) < 1$. Comparisons between (16) and (21), (18) and (22) show that under the modulation of EWF, forced LFW is different from that forced only by SST forcing. Constraints (16) and (18) are looser than constraints (21) and (22), respectively. EWF cannot slow down the speed of waves in the tropics solely, but it can make the waves unstable and make other factors play as different roles. So there exist different mechanisms in the oceanic forcing, the excitement mechanism ($e(T_s)$ effect) and modulation mechanism (long wave radiation cooling and EWF effect) play different roles in generation and development of LFW. The numerical results show that the diabatic heating from SST effect is really necessary to generate LFW in the tropics.

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