Multi–scale Fractal Characteristics of Atmospheric Boundary–Layer Turbulence①

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ABSTRACT

The turbulence data are decomposed to multi–scales and its respective fractal dimensions are computed. The conclusions are drawn from investigating the variation of fractal dimensions. With the level of decomposition increasing, the low–frequency part extracted from the turbulence signals tends to be simple and smooth, the dimensions decrease; the high–frequency part shows complex, the dimensions are fixed, about 1.70 on the average, which indicates clear self–similarity characteristics.

Key words: Discrete wavelet, Fractal dimension, Multi–scale, Turbulence data

1. Introduction

The observed signals of atmospheric boundary–layer turbulence are very complex. Many researches demonstrated that the spatial–temporal development pattern of turbulence data has fractal characteristics. Analysis with the Fourier and wavelet transform, it can be revealed that the turbulence data contain rich scales, scaling–law and self–similarity exist in some scales.

How can these scales be distinguished so as to investigate the regularity? It is known that a signal can be decomposed and reconstructed successively using the discrete wavelet transform, then these scales can be distinguished and the information can be compressed, and so on (Li, 1998; Hu, 1998). Using the discrete wavelet as a filter, the original signal can be decomposed to the ' approximation' (low–frequency) and ' detailed' (high–frequency) part. The multi–scale decomposition tree is given in Fig. 1, it is shown that the successive approximation parts are decomposed in turn.

In this paper, the discrete wavelet transform is used to decompose the turbulence signals to multiple scales, then the fractal dimensions of the turbulence signals for various scales are computed using the " variation method" (Dubuc et al., 1989). It is proved that the atmospheric boundary–layer turbulence has self–similarity characteristics for some scales.

2. Discrete wavelet transform

For dealing with such discrete signals as turbulence, we usually construct a discrete orthogonal wavelet function series (Chui, 1992; Daubechies, 1992). When the sampling τ of

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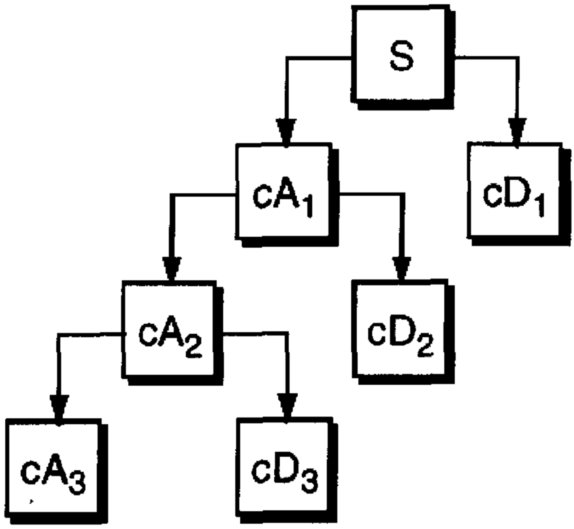


Fig. 1. The multiple–level decomposition trees (S–original signal, A–approximation part, D–detailed part).

the observation data is constant and the time series is $f(k)$, $k=1,2,\cdots N$, the discrete wavelet transform is as below

Normal transform: $$W_f(i,j)=\tau\sum_{k=1}^{N}\psi_{ij}(k)f(k). \quad (1)$$

Inverse transform: $$f(x)=\sum_{j=-\infty}^{+\infty}\sum_{i=-\infty}^{+\infty}W_f(i,j)\psi_{ij}(x)2^{j}. \quad (2)$$

There is fast algorithm for the discrete wavelet transform, such as the Mallat algorithm (Mallat, 1989). Generally the discrete orthogonal wavelet function could not be expressed by specific mathematical formula, but its picture can be drawn, such as the noted Daubechies wavelet.

Some physical criteria, such as isotropic, are respectively proposed to determine the time scale that is related to large scale and small scale parts of turbulence (Hu, 1998). In addition, based on abundant numerical experiment, it is found that the eighth level (namely, 2^9 time scale) decomposed is pretty for our analysis to atmospheric boundary–layer turbulence signal.

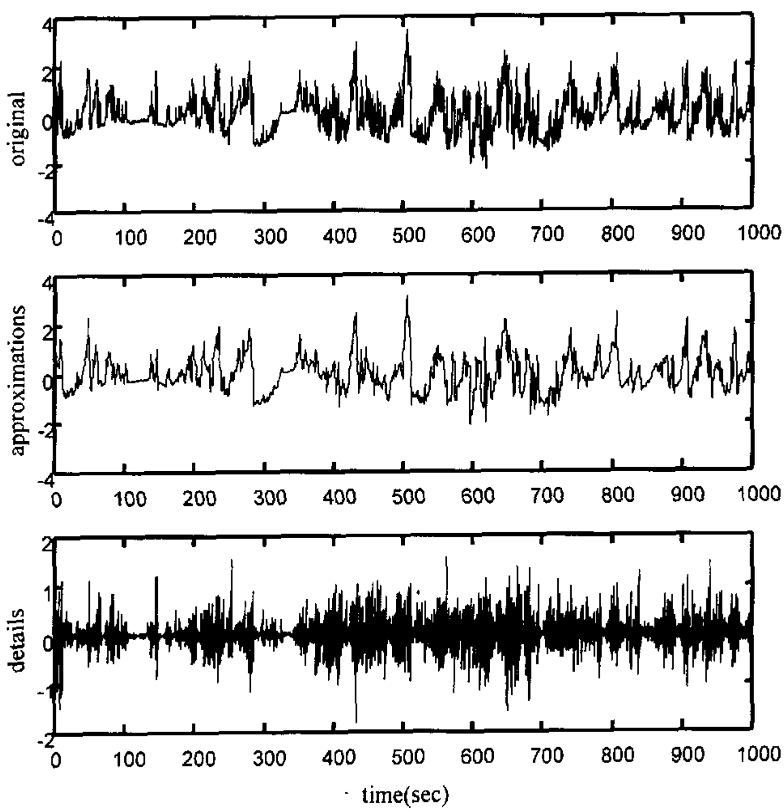
Here we decompose the wind velocity and temperature data of the atmospheric turbulence using the third–order Daubechies wavelet. The time series came from the ultrasonic observation data of the HEIFE experiment in 1992. The observed data at noon on August 13 are given as an example. It must be noted in the process of decomposition, the detailed (high–frequency) part of each level is the sum of all level detailed parts above. For the third layer decomposition as an example, the approximation part A_3 is obtained from the wavelet transform, and the detailed parts $D_i (i= 1,2,3)$ of all the levels above are added up so as to constitute the signal S to this level,

$$S= A_3 + D_3 + D_2 + D_1. \tag{3}$$

The signal of all variables has been decomposed to the eighth level. The decomposition of temperature data to the forth and eighth levels is given in Fig. 3. It can be seen that the approximation (low-frequency) part shows simple and smooth, the detailed (high-frequency) part shows complex. This can be indicated in the section below.

3. Fractal dimension

The fractal dimension characterizes the degree of complexity of a geometrical object. The fractal dimension is larger as the object is more complex. It is of significance in revealing stringent or statistical regularity of complex geometrical object having no characteristic scale such as the observed curve of atmospheric turbulence. Here we introduce a simple method for computing the fractal dimension of curve (Dubuc et al., 1989).



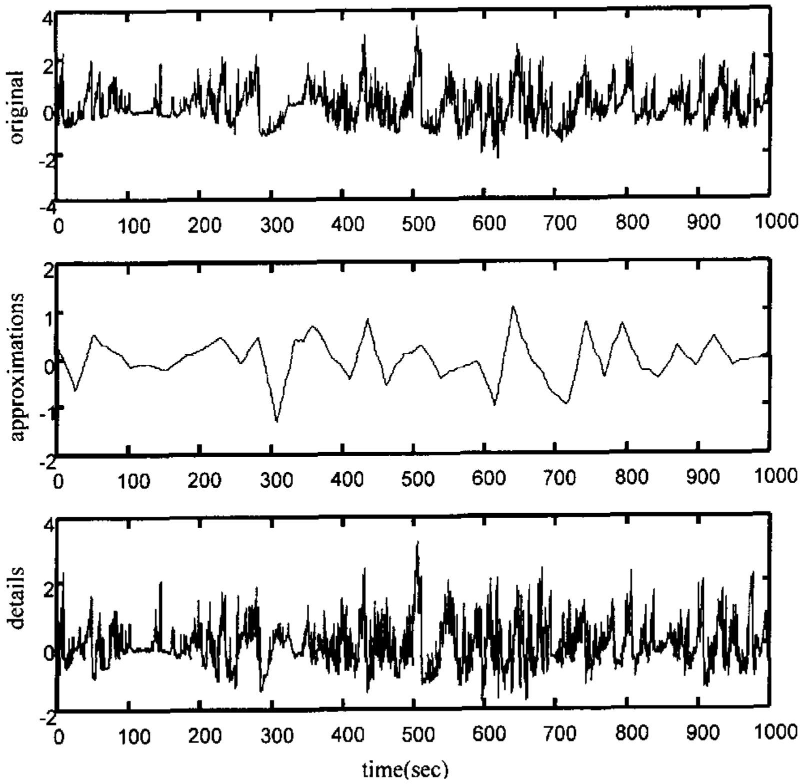


Fig. 2. (a) The fourth level decomposition of real atmospheric turbulence data (T, HEIFE, 1992). (b) The eighth level decomposition of real atmospheric turbulence data (T, HEIFE, 1992).

The variation method determines the fractal dimension of a curve by measuring the total area of boxes needed to cover the entire curve as a function of the length of the box's base. Let $f(x)$ be fractal and cover it with boxes with bottom edge equal to 2ε. The vertical edge of the box, called the ε oscillation $v(x,\varepsilon)$ of $f(x)$, is

$$v(x,\varepsilon)=\sup f(x')-\inf f(x'),\qquad x'\in[x-\varepsilon,x+\varepsilon], \tag{4}$$

where sup (superior) is the local maximum and inf (inferior) is the local minimum.

The total area of the boxes with bottom edge 2ε needed to cover the curve is called the ε variation $V(\varepsilon,f)$ of $f(x)$ and is

$$V(\varepsilon,f)=\int v(x,\varepsilon)dx. \tag{5}$$

In this equation, $v(x,\varepsilon)dx$ is the area of one box with dx equal to 2ε. Integration of equation

(5) gives the area covered by boxes with bottom edge 2ε. A different area would be obtained with a different ε and the fractal dimension D that describes this dependence is given by

$$D = \lim_{\varepsilon \to 0}[2 - \frac{\log V(\varepsilon, f)}{\log \varepsilon}]. \tag{6}$$

In practice, D is obtained from the slope of the least square line passing through the points $(\log\frac{1}{\varepsilon}, \log(\frac{1}{\varepsilon^2} V(\varepsilon, f)))$.

We used the above method to compute the fractal dimensions of the curves of all variables decomposed to various scales. The results are shown in Tables 1 and 2.

Table 1. The fractal dimension of turbulence for various scale (The approximation part)

Levels \ Variables	U	V	W	T
1	1.61	1.65	1.74	1.69
2	1.59	1.63	1.72	1.68
3	1.55	1.60	1.68	1.65
4	1.50	1.55	1.63	1.59
5	1.44	1.47	1.53	1.51
6	1.36	1.36	1.45	1.41
7	1.28	1.29	1.35	1.36
8	1.20	1.23	1.24	1.26

Table 2. The fractal dimension of turbulence for various scale (the detailed part)

Levels \ Variables	U	V	W	T
1	1.74	1.68	1.73	1.68
2	1.75	1.73	1.75	1.69
3	1.75	1.74	1.76	1.70
4	1.74	1.75	1.76	1.70
5	1.73	1.76	1.76	1.70
6	1.71	1.72	1.77	1.70
7	1.67	1.69	1.75	1.70
8	1.67	1.67	1.75	1.70

It can be seen from the tables, with the level of decomposition increasing, the fractal dimension of the approximation (low–frequency) part decreases successively, the minimal value is 1.20, indicating that it tends to be simple and smooth. The fractal dimension of the detailed (high–frequency) part does not always increase, but it tends to be almost fixed, reaching a value of 1.70 or so, specially the vertical fluctuating component of wind velocity and temperature do. $D = 1.70$ is also in agreement with the widely accepted relation between the power spectral density (PSD) of atmospheric turbulence and frequency f

$$PSD \sim 1/f^{\beta},$$

where $\beta = 5/3$ (Panofsky and Dutton, 1984). Recently, it is shown in general $\beta = 5 - 2D$ (West and Shlesinger, 1990), which predicts that D for wind speed should be 1.67, less than the value found for the present data, but the difference is not considered significantly.

It can be indicated that the turbulence signal of various scales have obvious self–similarity characteristics.

4. Conclusions

(1) We can decompose the atmospheric turbulence signals to various scales using the discrete wavelet transform, the ' mathematical microscope' , to investigate the variation of outward appearance effectively.

(2) One of chaotic characteristics of turbulence, the multi–scale self–similarity structure, is proved through computing the fractal dimensions of turbulence signals decomposed to various scales with the orthogonal wavelet transform.

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大气边界层湍流多尺度分形特征的研究

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摘 要

运用离散正交小波变换将湍流信号分解为不同尺度，计算其分数维．考察其分数维的变化得出：随着分解层次得增加，提取湍流信号得低频部分趋于简单光滑，分数维不断减小，高频部分呈现复杂，分数维趋于定值，平均为 1.70 左右．说明大气边界层湍流信号在某些尺度上，存在明显的自相似性特征．

关键词：离散小波，分维数，多尺度，湍流信号