Monotonic Digit Filter for Limited-Area Model[1]

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ABSTRACT

Numerical diffusion or filter are used in most numerical models in order to eliminate small-scale (near two-grid intervals in wavelength) waves. However, conventional diffusion or filter schemes introduce the noise, and indeed few people realized, by filters themselves. For instance, most filters are troubled when they are put to use on meteorological fields with sharp gradient or with steep slope and consequently, the recurrence of undesirable numerical high-frequent oscillations (overshooting and undershooting) seems to be inevitable. Particularly when diffusion or filter is implemented in limited-area models, serious side effects on the limited-area boundaries often contaminate the modeling results.

The merits and demerits are surveyed for commonly used diffusion or filter operations. A new type of monotonic digit filter is suggested to prevent overshooting and undershooting (due to the computational shock and Gibbs oscillation) nearby the discontinuous or nearly discontinuous locations when the filtering process was carried out, meanwhile the high selective property of damping is retained. Moreover, the new filter is designed on the implicit framework so that it can easily handle the problem of boundary diminishing in limited-area modeling.

Key words: Monotonic digit filter, Limited-area model, Boundary diminishing, Overshooting Undershooting

1. Introduction

In numerical modeling, two-grid-size waves appearing as numerical noise is an obstacle to maintain computational stability and accuracy of computation, and thus must be eliminated. Many classical smoothing techniques such as diffusion term added on the prognostic equation of atmospheric model were widely employed (Hamming 1977; Vichnevetsky and Bowles 1982). Discussions on the use of diffusion to filtering out the numerical noise can be found among others from Shapiro (1970, 1971, 1975), Wu (1977), Raymond and Garder (1976, 1988), Xue (2000). Numerical representation with various orders of diffusion has been usually selected in different circumstances, for instance, second-order (2d) and fourth-order (4d) representations (also called three- and five-point smoothers in Chinese) are mostly used in gridpoint models (Klemp and Wilhelmson 1978) and the higher order is commonly used in spectral models (Hoskins 1980). In addition, zero-order representation of diffusion (often used for damping or relaxation) is frequently implemented in the lateral and upper boundary

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treatment (Chen 1973; Davies and Turner 1977).

However, some typical lower–order diffusion operators, like Shapiro 2d and 4d schemes (Haltiner and Williams 1979) have heavy damping effect on longer waves. To avoid this, a series of de–smoothing must be done (e.g., Wang 1997), which cost expensive arithmetic operations. The higher order diffusions on the other hand damp waves being more selectively, but unfortunately also cause high–frequent oscillation known as Gibbs phenomenon. Similarly with such oscillation, computational shock occurs when higher order diffusions deal with sharp–changed fields (Smolarkiewicz and Grabowski 1990). Both Gibbs oscillation and numerical shock create overshooting / undershooting. Most of numerical representation of diffusion is in explicit framework, in which it employed certain grid points locally and operated recursively. It implies that for limited–area modeling, whatever the order chosen for, they have to face the difficult problem of boundary diminishing. Such boundary diminishing will possibly creat erroneous perturbations in the boundary zone, which in turn propagate very quickly into the interior of the domain (Baumhefner and Perkey 1982), and finally contaminate the modeling results.

Quite different with the diffusion term, Pepper et al. (1979) proposed one –dimensional spatial digit filter. Pepper's spatial filter can be introduced by applying it periodically to the predicted field and its effect is often equivalent to adding diffusion in prognostic equation and applying it in a period of time (Xue 2000). In the view of arithmetic point, Pepper's spatial filter is an implicit operator that can be formulated in tri–diagonal matrix so that the computation cost is less expensive. The boundary set in the tri–diagonal matrix is of simplicity and accuracy and that is in favor of limited–area modeling (Raymond and Garder 1988). Compared with diffusion term, the advantages and possible disadvantages of Pepper's digit filter can be illustrated in Fig. 1. In the figure, we select the diffusion type of common three–point operator,

$$\bar{f}_i = (1-s)f_i + s(f_{i-1} + f_{i+1})/2, \tag{1}$$

where $s=0.5$ is taken into account. Pepper's digit filter is written as

$$(1-\varepsilon)\bar{f}_{i-1} + 2(1+\varepsilon)\bar{f}_i + (1-\varepsilon)\bar{f}_{i+1} = f_{i-1} + 2f_i + f_{i+1}, \tag{2}$$

where $\varepsilon = 0.002$ is chosen. Both are now taken to smooth a special–designed one–dimensional step function, i.e., a cylinder–shape function in discrete form:

$$f_i = \begin{cases} 100 & 8 \leqslant i \leqslant 14 \\ 0 & \text{elsewhere} \end{cases} \qquad i = 1,2,3,\cdots,21 \tag{3}$$

Just as shown in Fig. 1 by 200 iterations (the reason taking many times of the operations is simply due to its usual multi–implements in real modeling circumstance), the three–point operator has significantly imposed huge damping on the original peak in which it is decreased from magnitude of 100 to 20. On the contrary, Pepper's filter kept the shape of underlying function that is much better than the diffusion. Since the underlying function in Fig. 1 has a sharp–changing slope and not surprisingly, the filtered result by Pepper's operator has severe overshooting and undershooting near the steeps of the initial profile. Such computational shock is absolutely fictitious, it occurs frequently wherever underlying function varies steeply, and indeed being hardly eliminated.

In the following sections, a new digit filter is designed and tested in order to alleviate, and to delete completely the undesirable effects of heavy damping and fictitiously

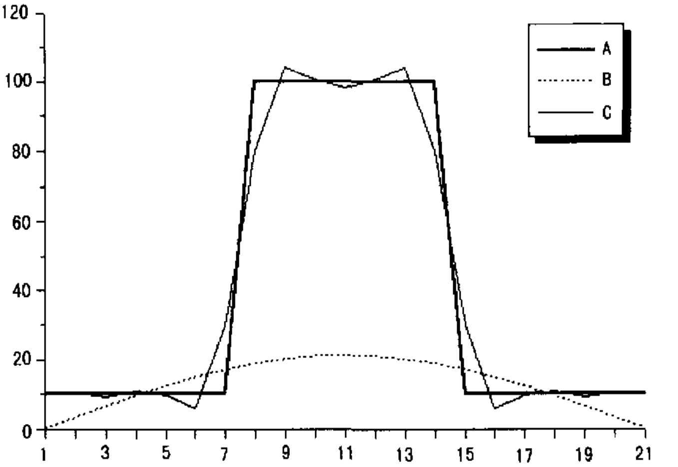


Fig. 1 Comparison of the smoothing effects between diffusion operator and digit filter. Curve A Before filtering. Curve B: using 3-point diffusion operator, and Curve C using Pepper's digit filter. The abscissa is the grid position and the ordinate represents the magnitude of initial profile. The detail explanations are referred in the context.

overshooting / undershooting, all being created by diffusion or filter themselves. The proposed filter has especially been considered to use in limited-area modeling and furthermore the lateral boundary condition should be elaborately designed to suit for the factual circumstance. For this reason, ideal periodic boundary condition is excluded in our consideration. An important improvement on digit filter we mention here is the introduction of monotonic constraint into the digit operator and thus formulates monotonic digit filter. Being monotonic, it means that the digit filter will not create new extrema that are not previously presented in the field. Therefore the filtered or smoothed field will be free of numerical shocks and Gibbs phenomena. Meanwhile we can ensure that the monotonic constraint do preserve very well the shape of longer than two-grid length waves.

2. Digit filter design

Without the loss of generality, the digit filter is expressed in compact form for arbitrary k-dimension space:

$$L_i^{\varepsilon} L_j^{\varepsilon} \cdots L_k^{\varepsilon} \tilde{f}_{i,j,\cdots,k} = L_i^0 L_j^0 \cdots L_k^0 f_{i,j,\cdots,k} . \tag{4}$$

where L_i^{ε} for instance represents filtering operator with wave-selectivity parameter ε for ith dimension; similarly the superscript zero for L_i^0 corresponds to $\varepsilon = 0$ which means that the operator does not execute the filtering process. Consequently, $\tilde{f}_i$ is the filtered field and f_i is unfiltered field. For simplicity, the design of the digit filter is presented in two-dimensional case for illustration purpose

$$L_i^{\varepsilon} L_j^{\varepsilon} \tilde{f}_{i,j} = L_i^0 L_j^0 f_{i,j}. \tag{5}$$

If we write it explicitly in each dimension within the mesh denoted by $(1 \leq i \leq M, 1 \leq j \leq N)$, then

$$\begin{cases} L_i^{\varepsilon} \tilde{f}_{i,j} = (1-\varepsilon)\tilde{f}_{i-1,j} + 2(1+\varepsilon)\tilde{f}_{i,j} + (1-\varepsilon)\tilde{f}_{i+1,j}, \\ L_j^{\varepsilon} \tilde{f}_{i,j} = (1-\varepsilon)\tilde{f}_{i,j-1} + 2(1+\varepsilon)\tilde{f}_{i,j} + (1-\varepsilon)\tilde{f}_{i,j+1}, \\ L_i^0 f_{i,j} = f_{i-1,j} + 2f_{i,j} + f_{i+1,j}, \\ L_j^0 f_{i,j} = f_{i,j-1} + 2f_{i,j} + f_{i,j+1} \end{cases} \tag{6}$$

In each one–dimensional case, it is actually reduced into Pepper's scheme. We should note that repeated applications of Pepper's one–dimensional filter in each dimension do not make true two–dimensional filter expressed as Eq. (5). The correct and efficient way for two–dimensional operator is schematically demonstrated in the following splitting fashion (the splitting algorithm can be found, e.g., Staniforth and Mitchell (1977))

$$L_i^{\varepsilon}[L_j^{\varepsilon}\tilde{f}_{i,j}] = [L_i^0 L_j^0 f_{i,j}] \rightarrow L_i^{\varepsilon}\tilde{f}_{i,j}^* = D_{i,j} \text{(first step)} \rightarrow L_j^{\varepsilon}\tilde{f}_{i,j} = \tilde{f}_{i,j}^* \text{(second step)},$$

where $D_{i,j} = L_j^0 L_i^0 f_{i,j}$ is expressed as

$$D_{i,j} = f_{i-1,j+1} + 2f_{i,j+1} + f_{i+1,j+1} + 2f_{i-1,j} + 4f_{i,j} + 2f_{i+1,j} + f_{i-1,j-1} + 2f_{i,j-1} + f_{i+1,j-1}. \tag{7}$$

And $\tilde{f}_{i,j}^* = [L_j^{\varepsilon}\tilde{f}_{i,j}]$ is intermediate variable and it satisfies

$$\underset{2 \leq i \leq M-1}{(1-\varepsilon)\tilde{f}_{i-1,j}^* + 2(1+\varepsilon)\tilde{f}_{i,j}^* + (1-\varepsilon)\tilde{f}_{i+1,j}^* = D_{i,j}}. \tag{8}$$

The final filtered $\tilde{f}_{i,j}$ being in search can be obtained from the following operator

$$\underset{2 \leq j \leq N-1}{(1-\varepsilon)\tilde{f}_{i,j-1} + 2(1+\varepsilon)\tilde{f}_{i,j} + (1-\varepsilon)\tilde{f}_{i,j+1} = \tilde{f}_{i,j}^*}. \tag{9}$$

Implicit scheme requires both Eq.(8) and Eq.(9) to formulate a system of tri–diagonal matrices, which can be readily solved if the set of boundary conditions are rightly given. The boundary values for Eq.(8) should satisfy the following condition that is from Eq.(9)

$$\begin{cases} (1-\varepsilon)\tilde{f}_{1,j-1} + 2(1+\varepsilon)\tilde{f}_{1,j} + (1-\varepsilon)\tilde{f}_{1,j+1} = \tilde{f}_{1,j}^*, \\ (1-\varepsilon)\tilde{f}_{M,j-1} + 2(1+\varepsilon)\tilde{f}_{M,j} + (1-\varepsilon)\tilde{f}_{M,j+1} = \tilde{f}_{M,j}^* \end{cases} \tag{10}$$

Moreover, to meet the required condition of Eq.(10), the boundary values of $D_{i,j}$ in Eq.(8) are adjusted to

$$\begin{cases} D_{1,j} = (1-\varepsilon)\tilde{f}_{1,j-1} + 2(1+\varepsilon)_{1,j} + (1-\varepsilon)\tilde{f}_{1,j+1}, \\ D_{M,j} = (1-\varepsilon)\tilde{f}_{M,j-1} + 2(1+\varepsilon)_{M,j} + (1-\varepsilon)\tilde{f}_{M,j+1} \end{cases}, \tag{11}$$

where various choices for Eq. (11) exist. But in the case that the boundary values have not necessarily to be infected by filtering process, then the boundary condition Eq.(11) through setting $\varepsilon = 0$ yields

$$\begin{cases} D_{1,j} = f_{1,j-1} + 2f_{1,j} + f_{1,j+1}, \\ D_{M,j} = f_{M,j-1} + 2f_{M,j} + f_{M,j+1} \end{cases}. \tag{12}$$

in which the following inherent relation is employed

$$\begin{cases} \tilde{f}_{1,j-1} + 2\tilde{f}_{1,j} + \tilde{f}_{1,j+1} = f_{1,j-1} + 2f_{1,j} + f_{1,j+1}, \\ \tilde{f}_{M,j-1} + 2\tilde{f}_{M,j} + \tilde{f}_{M,j+1} = f_{M,j-1} + 2f_{M,j} + f_{M,j+1} \end{cases} \quad (13)$$

according to Eq.(5) and Eq.(6). Thus, Eq.(8) with the specified boundary condition Eq.(12) formulates a well-posed solution (such well-poseness can be checked to see if the filtered and unfiltered variables are identical everywhere by setting $\varepsilon=0$ in following Eq.(14) and Eq.(18)) of tri-diagonal matrix

$$(\tilde{f}^*_{1,j}, \tilde{f}^*_{2,j} \cdots \tilde{f}^*_{M-1,j}, \tilde{f}^*_{M,j})^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & & & \\ 1-\varepsilon & 2(1+\varepsilon) & 1-\varepsilon & & \\ & & \cdots & & \\ & & \cdots & & \\ & & 1-\varepsilon & 2(1+\varepsilon) & 1-\varepsilon \\ & & & 0 & 1 \end{bmatrix}^{-1} (D_{1,j}, D_{2,j} \cdots D_{M-1,j}, D_{M,j})^{\mathrm{T}}. \quad (14)$$

$2 \leqslant j \leqslant N-1$

Following the similar way, on the other hand, the boundary values for Eq.(9) should satisfy the condition that meets Eq. (8)

$$\begin{cases} (1-\varepsilon)\tilde{f}^*_{i-1,1} + 2(1+\varepsilon)\tilde{f}^*_{i,1} + (1-\varepsilon)\tilde{f}^*_{i+1,1} = D_{i,1}, \\ (1-\varepsilon)\tilde{f}^*_{i-1,N} + 2(1+\varepsilon)\tilde{f}^*_{i,N} + (1-\varepsilon)\tilde{f}^*_{i+1,N} = D_{i,N} \end{cases} \quad (15)$$

The compatibility requirement of $D_{i,j}$ and the condition by setting $\varepsilon=0$ yield

$$\begin{cases} \tilde{f}^*_{i-1,1} + 2\tilde{f}^*_{i,1} + \tilde{f}^*_{i+1,1} = D_{i,1} = f_{i-1,1} + 2f_{i,1} + f_{i+1,1}, \\ \tilde{f}^*_{i-1,N} + 2\tilde{f}^*_{i,N} + \tilde{f}^*_{i+1,N} = D_{i,N} = f_{i-1,1} + 2f_{i,1} + f_{i-1,1} \end{cases} \quad (16)$$

Therefore

$$\tilde{f}^*_{i,1} = f_{i,1}, \qquad \tilde{f}^*_{i,N} = f_{i,N}. \quad (17)$$

Thus,

$$(\tilde{f}_{i,1}, \tilde{f}_{i,2} \cdots \tilde{f}_{i,N-1}, \tilde{f}_{i,N})^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & & & \\ 1-\varepsilon & 2(1+\varepsilon) & 1-\varepsilon & & \\ & & \cdots & & \\ & & \cdots & & \\ & & 1-\varepsilon & 2(1+\varepsilon) & 1-\varepsilon \\ & & & 0 & 1 \end{bmatrix}^{-1} (f_{i,1}, \tilde{f}^*_{i,2} \cdots \tilde{f}^*_{i,N-1}, f_{i,N})^{\mathrm{T}}. \quad (18)$$

$2 \leqslant i \leqslant M-1$

It is not difficult to find, by the way, that the above two-dimensional scheme can be readily expanded into multi-dimensional space. Furthermore, for the purpose of comparison, the simplest diffusion operator is adopted here and expressed as $\tilde{f}_{i,j} = f_{i,j} + \alpha(f_{i-1,j} + f_{i+1,j} + f_{i,j-1} + f_{i,j+1} - 4f_{i,j})$ (two dimensional five-points smoother); and its corresponding response amplitude, as well as the amplitude profile of the proposed digit filter, for certain input of harmonic wave is shown in Fig. 2.

Evidently it is illustrated in Fig. 2 that both operators are well-defined low-pass filters since two-grid-scale waves are completely eliminated. For longer waves, however, the

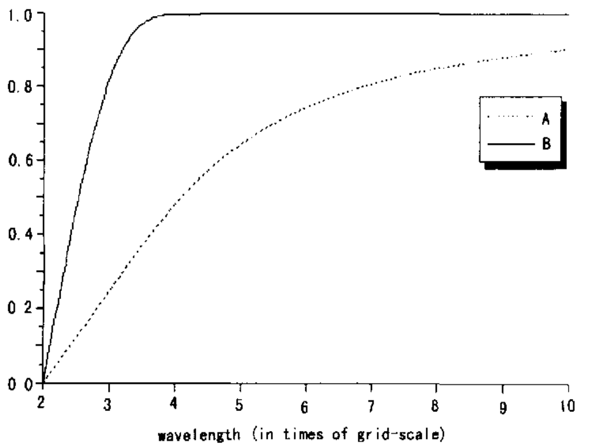


Fig. 2. Amplitude profiles of responses for two-dimensional digit filter and diffusion operator (Curve A: two-dimensional diffusion operator with $\alpha = 0.125$; and Curve B: the digit filter proposed, $\varepsilon = 0.002$, after 200 times operations).

amplitude of response for new digit filter approaches unity more quickly than that of the diffusion smoother. Therefore, the new filter has much less damping effect on longer waves. This merit of the digit filter and the demerit of diffusion is much clear if one applying both operators as many times as possible. For instance, the amplitude of wave with ten times grid length will access to zero nearly after 200 times of diffusion operator. On the contrary, the amplitude of new digit filter will only vary from 0.9996 to 0.9231 during the same period. It is worth noting while, higher-order diffusion operators can also improve the heavy damping effect in some ways, but meanwhile it also faces many difficulties such as boundary diminishing and Gibbs oscillation as we discussed in Introduction. Therefore higher order diffusion is not a wise choice although possible solution to those problems could be found in Xue (2000).

3. Monotonic digit filter

In order to delete the aforementioned overshooting / overshooting problem, the design of the digit filter in Section 2 should be further framed to the so-called monotonic digit filter, i.e., the digit filter constrained by monotonic constraint. Being monotonic digit filter, we mean that digit filter will not create new extrema in the filtered field, and that are not presented previously in the unfiltered field. The consideration thus leads to the following monotonic constraint:

$$\tilde{f}_k = \begin{cases} f_{max} & \tilde{f}_k > f_{max} \\ f_{min} & \tilde{f}_k < f_{min} \\ \tilde{f}_k & \text{otherwise} \end{cases}, \tag{19}$$

where

$$\begin{cases} f_k^{max} = \max[f_{k-1}, f_k, f_{k+1}], \\ f_k^{min} = \min[f_{k-1}, f_k, f_{k+1}] \end{cases} \tag{20}$$

Clearly, it is one-dimensional constraint. But for multidimensional case it is just straightforward since the splitting technique has been employed in the last section. The arithmetic expression in Eq.(19) and Eq.(20) can be analytically formulated into, namely, monotonic operator $C_k\{\ \}$ at kth dimension, that is

$$C_k\{\ \} = f_k^{max} H[\{\ \} - f_k^{max}] + f_k^{min} H[f_k^{min} - \{\ \}] + \{\ \} H[(f_k^{max} - \{\ \})(\{\ \} - f_k^{min})], \tag{21}$$

here $H[t]$ represents Heaviside function which is defined as zero for $t < 0$, 1 for $t > 0$ and is not defined at $t = 0$. For two-dimensional monotonic filter, it is expressed in compact form as

$$L_i^1 L_j^1 C_i C_j \tilde{f}_{i,j} = L_i^0 L_j^0 f_{i,j}. \tag{22}$$

We also require that the setting of boundary condition for Eq.(22) should be neither influenced by filtering nor by monotonic operating. The numerical scheme as well as the boundary conditions described in the last section can be therefore simply used for Eq.(22)

4. Numerical test

In the previous sections, the digit filter with and without monotonic constraint has been designed. The following experiments show the performance of monotonic digit filter and the superiority of monotonic constraint. Two analytical functions are designed for testing cases in which Case 1 presents a rectangle field with sharp–changing corners; and Case 2 represents a comprehensive field of weather pattern, i.e., the saddle pattern with inherent continuity.

Case 1:

$$H_{i,j} = \begin{cases} 100 & 8 \le i, j \le 14 \\ 0 & \text{elsewhere.} \end{cases} \tag{23}$$

Case 2:

$$S_{i,j} = 100\sin[2\pi(i-1)\Delta]\sin[2\pi(j-1)\Delta] + 25\sin[5\pi(i-1)\Delta]\sin[4\pi(j-1)\Delta].$$

where Δ is the grid resolution at i as well as j direction.

It is mainly through the two cases that we could hopefully distinguish the merits and demerits of the diffusion operator and the new digit filter, especially the latter one with and without monotonic limitation. As shown in Fig. 3, three experiments for Case 1 in which the 2d diffusion operator, the digit filter with and without monotonic limitation are implemented 200 times. The smoothing results have clearly demonstrated that the diffusion operator was gravely damping the underlying field (Fig. 3b) and distorting the underlying pattern; on the other hand, the digit filter without monotonic limitation preserved the shape of rectangle better, but the creating of numerical overshooting / undershooting is inevitable (Fig. 3c), such numerical shock however can be successfully prevented by the monotonic limitation (Fig. 3d).

Although the numerical test of Case 1 has shown some benefits of monotonic constraint very clearly, the remaining question still exists does the performance of monotonic constraint generally do well for the continuously varying field, or does it work perfectly for the special designation of the field varying steeply in Case1 that is just in favor of the monotonic limitation? The following experiments of Case 2 will make clear–cut for such doubts. As shown in Fig. 4 after 200 operations, the diffusion operator unfortunately and terribly exerts

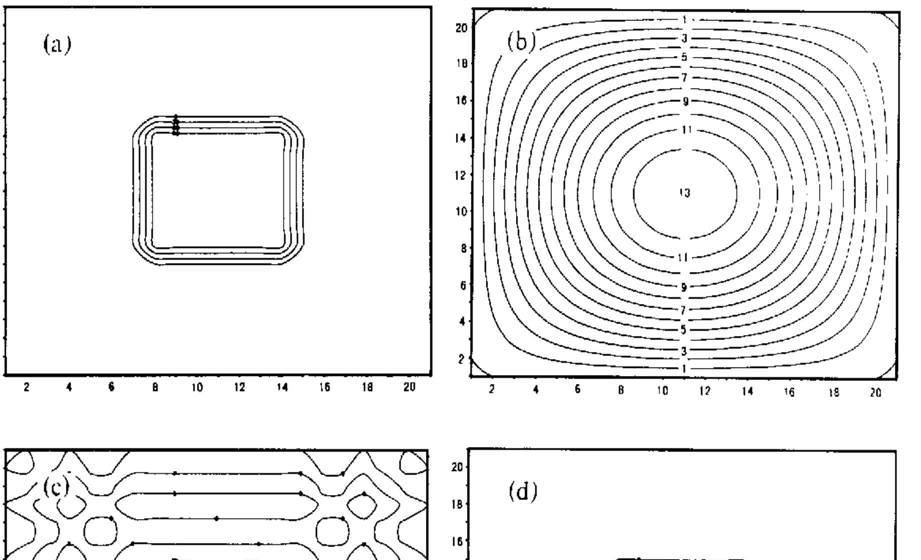


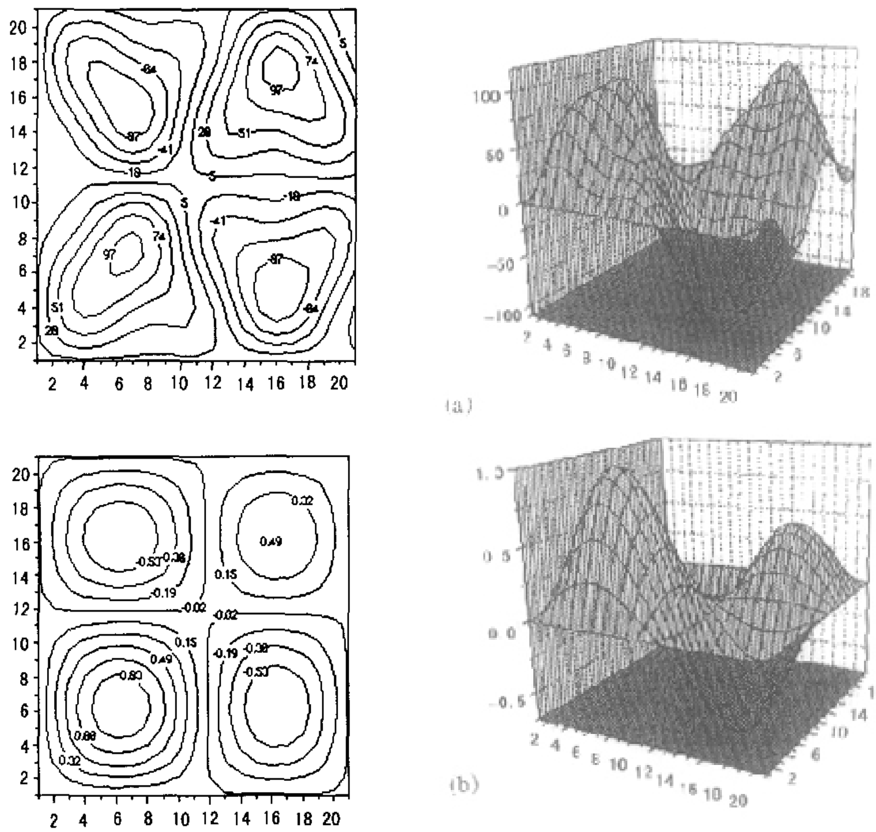
Fig. 3. The comparison of filtering results among digit filter with and without monotonous limitation, as well as five-point diffusion operator in Case 1 (after 200 operations). (a) before filtering, (b) diffusion ($\alpha = 0.125$); (c) digit filter without monotonic limitation ($\varepsilon = 0.002$); (d) digit filter with monotonic limitation ($\varepsilon = 0.002$).

huge damping on the filtered field (Fig. 4b), the distortion even creates the erroneous negative values. On the other hand, the digit operators as we expected, preserve the underlying physical extrema and prevent the numerical extrema very well; meanwhile do not create heavy damping on the smoothing field. Most interesting, the digit operators do not make a significant difference between ones with or without the monotonic limitation (Figs. 4c, 4d) for the field varying continuously. The numerical test for Case 2 is of profound importance since in practice, we can not a prior make judgment that the field to be filtered is varying steeply or continuously and therefore it is difficult to determine whether or not the monotonic limitation should be used in advance. But now we can be sure of the monotonic digit filter, i.e., the filter with monotonic constraint can be used wherever the filtering process is required.

This generality of monotonic digit filter is further proved in Fig. 5 and Fig. 6. The profiles of RMSE vary with respect to a series of operations (Fig. 5) and the changing parameter (Fig. 6) made no difference when using or not using the monotonic limitation in the continually varying field of Case 2, which indicate the monotonic limitation, basically designed for steep varying field, did not distort smoothing field in such gently varying field. On the other hand, the assessment of the error levels in Fig. 5 demonstrated that the digit filter with or without monotonic limitation is superior to the diffusion operator.

We should note that the elimination of numerical shock in the other way could be done through enlarging the value of parameter ε. But the increase of ε will create more damping and consequently increasing the inaccuracy of filtered field as shown in Fig. 6. In fact, as we tested (not shown), the RMSE (Root Mean Square Error) is quickly increased up to one-order magnitude while the parameter ε is ranged in nearly one-order magnitude, which implies the fast and grave attenuation applying on longer waves. It is the point that the digit filter with monotonic limitation proposed can not only prevent the numerical shock, but also deal with the heavy attenuation successfully, and both of them indeed are pertinacious problems in filtering techniques.

Conclusively, the monotonic limitation does preserve the solution near the gently gradient and eliminate the overshooting / undershooting nearby the sudden gradient, while the highly selective property of smoothing is however successfully retained.



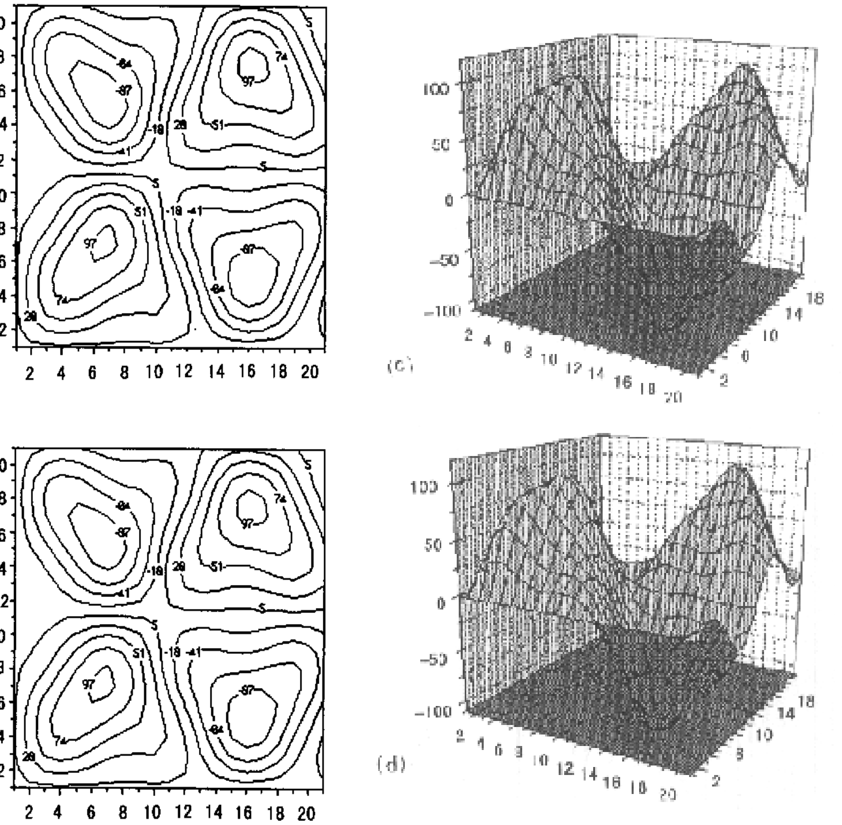


Fig. 4. The filtering results among digit filter with and without monotonous limitation, as well as five-points diffusion operator in the experiments of Case 2 (after 200 operations) (a) before filtering, (b) diffusion ($\alpha = 0.125$), (c) digit filter without monotonic limitation ($\varepsilon = 0.002$), (d) digit filter with monotonic limitation ($\varepsilon = 0.002$)

5. Conclusion

In this paper, a monotonic digit filter is proposed and examined. The idea of such filter is inspired from Pepper's original one–dimensional algorithm and then it is further constrained by our proposal of the so–called monotonic limitation. Unlike common used filters or diffusions as we already reviewed in introduction section, the monotonic digit filter is perfectly free of overshooting and undershooting created either from numerical shock or from Gibbs oscillation, meanwhile the highly selective property of smoothing is however very successfully retained.

It is in evidence that the proposed filter can be used in any circumstances where and when the smoothing process is whatever required. It means that the design of monotonic constraint can automatically detect the fictitious extrema and efficiently eliminate it during smoothing process, and consequently, the overshooting / undershooting created nearby the sharp gradient of function will be filtered out from the solution, whereas the solution near the

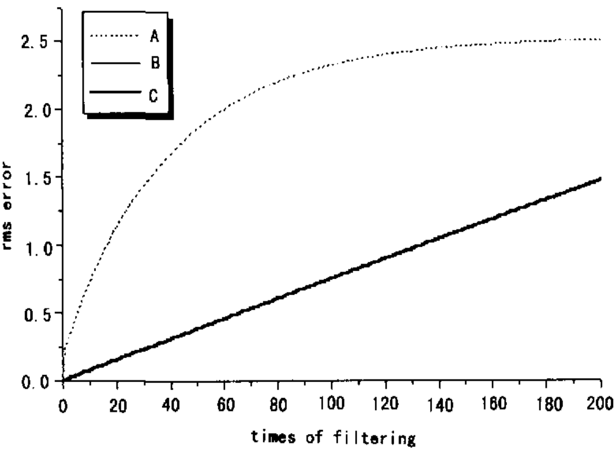


Fig. 5. The RMSE assessment for filtering results varying with iterations of operation for Case 2. A: the five–point diffusion operator (the values have been divided by 20, $\alpha = 0.125$); B: the digit filter without monotonous limitation; and C: the digit filter with monotonous limitation (Curves B and C are actually superposed, and $\varepsilon = 0.002$).

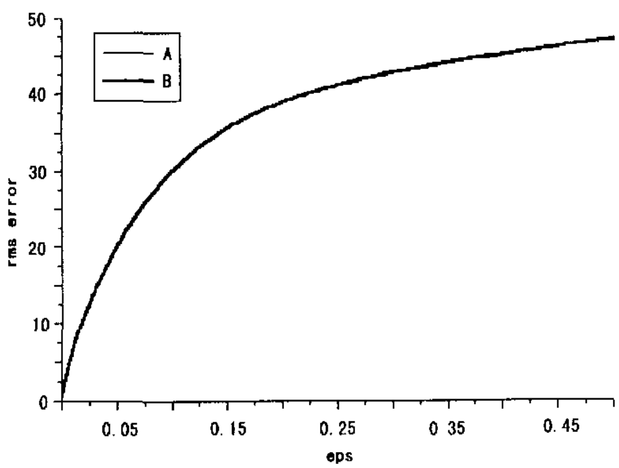


Fig 6. The RMSE assessment of filtering results varying with the parameter ε for Case 2 (after 200 operations). A: the digit filter without monotonous limitation; B: the digit filter with monotonous limitation (Curves B is reasonably superposed).

gently gradient will not be changed, so that the underlying profile or pattern of curve or field to be smoothed will be very well preserved.

Since the proposed algorithm is based on tri–diagonal matrix calculation, its computation is thus quite efficient. For the same reason it can be readily expanded into multidimensional case, though we only made the two-dimensional experiments for the purpose of illustration. It is unlike 3-point, 5-point or 9-point explicit diffusion operators which employed certain grid points locally and operated recursively; instead, the implicit monotonic digit filter is assigned globally—it means the filter put all of valid grids in use simultaneously. Therefore it is of high accuracy that has been proved during the numerical examination.

In fact, the design of the monotonic digit filter is mainly focused on the filtering process employed in regional or mesoscale limited-area model. Under the implicit framework, the setting of well-posed boundary conditions in the matrix arithmetic of digit filter guarantees that the filtering process is totally free of the problem of boundary diminishing, and such well-known problem does exist in common used diffusion, particularly when high-order diffusion operators are implemented. Moreover, the proposed boundary condition on limited area can be easily converted into that of global domain if one can put down periodic boundary condition in the proposed algorithm (the way is actually straightforward); therefore, the proposed filter can be used in global model, too.

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有限区域模式中的单调性数位滤波器

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摘 要

在大多数数值模式中，为了消除小尺度(波长接近两倍网格距)的波动，必须使用数值耗散或滤波技术。然而，很少有人意识到，常规的耗散或滤波方案自身会引入噪声。例如，大部分滤波器在对梯度变化剧烈或存在陡峭坡度的气象场进行滤波时会遇到困难，即在其结果中不可避免地出现无意义的高频数值振荡(上冲和下冲)。特别是当耗散或滤波应用于有限区域模式时，错误的边值效应往往会严重破坏模式解。

本文分析了常用的耗散或滤波方法的优缺点，提出了一种新型的单调性数位滤波器。它可以防止在物理场出现不连续或者接近不连续时由于计算激波和吉布斯振荡引起的上冲和下冲现象，与此同时仍能保持滤波的高选择特性。此外，新滤波器还采用了隐式计算方案，因而能够轻而易举地解决有限区域模式中的边界退化问题。

关键词：单调性数位滤波器，有限区域模式，边界退化，上冲／下冲