On Problem of Nonlinear Symmetric Instability in Zonal Shear Flow[1]

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ABSTRACT

This paper is focused on the problem of nonlinear symmetric instability in a baroclinic basic flow. The limited amplitude characteristics of unsteady wave were investigated with the aid of equations of adiabatic, inviscid, nonlinear symmetric disturbance and a multi-scale singular perturbation technique. Evidence suggests that the limited amplitude of unsteady wave exhibits an oscillatory trend of its intensity: the amplitude of the symmetric disturbance displays periodical variation both in super- and sub-critical shear case, and the duration of the periods is related not only to the stability parameters of the basic field and wave properties but to the amplitude of initial disturbance and its time-varying change rate as well.

Key words: Symmetric instability, Nonlinear, Limited amplitude

1. Introduction

In the past decades many scientists have made ceaseless efforts on the problem of linear symmetric instability. Stone (1966) discovered that the growth rates of different types of disturbance were the functions of Richardson number (Ri) and symmetric instability is prevailing at $0.25 < Ri < 0.95$. Emanuel (1979), Bennetts and Hoskins (1979), and Xu (1986) made studies of both linear symmetric instability in a viscous fluid and conditional symmetric instability, obtaining the criteria of conditional symmetric instability in a boundless atmosphere and indicating that such instability is likely to be the genesis mechanism for band-like precipitation structures in midlatitude extratropical cyclones, and prefrontal squall lines. Zhang (1988) showed that the condition of symmetric instability in a baroclinic basic flow was derived with the aid of rigid boundary constraints and illustrated that morphologically symmetric instability is actually inertially convective instability in a baroclinic atmosphere and falls rightly on a meso-β spectral band between convective and inertial motions.

The above studies of linear theory on symmetric instability show that as a consequence of the linearization and the formal constancy of the mean state, the growth rate for the wave is also constant, leading inevitably to exponential growth for the perturbation. No matter how small the initial amplitude of the disturbance is, eventually this exponential growth will

yield a perturbation amplitude so great that nonlinear effects can no longer be ignored. Walton (1975) first made efforts at weak nonlinear evolution of limited–amplitude symmetric perturbation inside a viscous fluid under rigid unmovable boundary conditions in the context of a static equilibrium model by means of the normally–used singular perturbation technique, i.e., a high–order solution is derived step by step from a lower–order linear analytical solution, followed by eliminating the resonance terms to get the expression of limited–amplitude disturbance evolution.

Yet the wave dynamics in most instability models of oceanographic and meteorological relevance are inviscid, so that wave dissipation plays an unimportant role (Pedlosky 1979). In such cases, where the lack of dissipation makes the physical processes essentially reversible with time, how do the wave amplitude and structure evolve after reaching their limiting value, assuming, of course, that one exists? This is a problem that deserved our efforts. In view of the fact that inviscid linear symmetric instability has to yield a rigorous analytical solution, a condition that helps us greatly deal with the evolution of nonlinear symmetric instability, that is our starting point.

2. Analysis of linear stability

Comparison of the dynamic properties of atmospheric models shows that a model applicable to meso–scale motion is one of f–plane nonstatic equilibrium, sound wave filtering that takes the assumption of homogeneous uncompressibility in the equation of continuity. In that case, the complete motion equations in an adiabatic, frictionless atmosphere are

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv \ . \tag{1}$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu \ . \tag{2}$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \ . \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \ . \tag{4}$$

$$\frac{d\theta}{dt} = 0 \ . \tag{5}$$

We assume $u = \bar{u} + u'$, $v = v'$, $w = w'$, $p = \bar{p} + p'$, $\rho = \bar{\rho} + \rho'$ and $\theta = \bar{\theta} + \theta'$, which are put into (1)–(5). Also, we define the atmospheric stratification stability parameter $N^2 = \frac{g}{\theta}\frac{\partial\bar{\theta}}{\partial z}$, the intertial stability parameter $F^2 = f(f - \frac{\partial\bar{u}}{\partial y})$, and the baroclinic stability parameter $S^2 = f\frac{\partial\bar{u}}{\partial z} = -\frac{g}{\theta}\frac{\partial\bar{\theta}}{\partial y}$, all being constants. If boussinesq approximation is used and the physical quantities are symmetric about the x–axis ($\frac{\partial}{\partial x} = 0$), then (1)–(5) have their nonlinear symmetric perturbation form

$$\left(\frac{\partial}{\partial t} + v'\frac{\partial}{\partial y} + w'\frac{\partial}{\partial z}\right)(fu') = F^2 v' - S^2 w' \ . \tag{6}$$

$$\left(\frac{\partial}{\partial t} + v'\frac{\partial}{\partial y} + w'\frac{\partial}{\partial z}\right)v' = -\frac{\partial}{\partial y}\left(\frac{p'}{\rho_0}\right) - fu' \ . \tag{7}$$

$$\left(\frac{\partial}{\partial t} + v'\frac{\partial}{\partial y} + w'\frac{\partial}{\partial z}\right)w' = -\frac{\partial}{\partial z}\left(\frac{p'}{\rho_0}\right) + \frac{\theta'}{\theta_0}g \ . \tag{8}$$

$$\frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \ . \tag{9}$$

$$\left(\frac{\partial}{\partial t} + v'\frac{\partial}{\partial y} + w'\frac{\partial}{\partial z}\right)\left(\frac{\theta'}{\theta_0}g\right) = S^2 v' - N^2 w' \ . \tag{10}$$

in which $\bar{\rho}$ and $\bar{\theta}$ are set to be ρ_0 and θ_0, the reference density and potential temperature, respectively, for static atmospheric state. for convenience. the term $-\frac{1}{\rho_0}\frac{\partial p'}{\partial z}$ of (8) is approximately set to be $-\frac{\partial}{\partial z}(\frac{p'}{\rho_0})$. Linearization of (6)–(10) yields

$$\frac{\partial}{\partial t}(fu') = F^2 v' - S^2 w' \ . \tag{11}$$

$$\frac{\partial v'}{\partial t} = -\frac{\partial}{\partial y}\left(\frac{p'}{\rho_0}\right) - fu' \ . \tag{12}$$

$$\frac{\partial w'}{\partial t} = -\frac{\partial}{\partial z}\left(\frac{p'}{\rho_0}\right) + \frac{\theta'}{\theta_0}g \ . \tag{13}$$

$$\frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \ . \tag{14}$$

$$\frac{\partial}{\partial t}\left(\frac{\theta'}{\theta_0}g\right) = S^2 v' - N^2 w' \ . \tag{15}$$

Considering (14), we introduce disturbance streamfunction ψ, yielding $(v',w') = \left(-\frac{\partial\psi}{\partial z}, \frac{\partial\psi}{\partial y}\right)$, and by eliminating u', p' and θ', we find a partial differential equation of ψ

$$\frac{\partial^2}{\partial t^2}\nabla^2\psi = -N^2\frac{\partial^2\psi}{\partial y^2} - 2S^2\frac{\partial^2\psi}{\partial y\partial z} - F^2\frac{\partial^2\psi}{\partial z^2} \ . \tag{16}$$

where $\nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator.

By letting $\psi = \bar{\psi}(y,z)e^{\sigma t}$, we change (16) into a form containing cross–derivative term $\frac{\partial^2\bar{\psi}}{\partial y\partial z}$, viz.,

$$(\sigma^2 + N^2)\frac{\partial^2 \tilde{\psi}}{\partial y^2} + 2S^2\frac{\partial^2 \tilde{\psi}}{\partial y \partial z} + (\sigma^2 + F^2)\frac{\partial^2 \tilde{\psi}}{\partial z^2} = 0 \ . \tag{17}$$

In order to eliminate such term, conversion of independent variables $\xi = y + az$ and $\zeta = z$ (where a is the coefficient to be determined) is made, resulting in (18) of the form

$$[\sigma^2 + N^2 + (\sigma^2 + F^2)a^2 + 2S^2 a]\frac{\partial^2 \tilde{\psi}}{\partial \xi^2} + [2a(\sigma^2 + F^2) + 2S^2]\frac{\partial^2 \tilde{\psi}}{\partial \xi \partial \zeta} + (\sigma^2 + F^2)\frac{\partial^2 \tilde{\psi}}{\partial \zeta^2} = 0 \tag{18}$$

Set the coefficient of $\frac{\partial^2 \tilde{\psi}}{\partial \xi \partial \zeta}$ to be zero and we get $a = -S^2 / (\sigma^2 + F^2) < 0$. Hence, (18) is simplified into a standard differential equation, viz.,

$$(\sigma^2 + N^2 - \frac{S^4}{\sigma^2 + F^2})\frac{\partial^2 \tilde{\psi}}{\partial \xi^2} + (\sigma^2 + F^2)\frac{\partial^2 \tilde{\psi}}{\partial \zeta^2} = 0 \ . \tag{19}$$

which is reduced, under the assumption of wave solution $\tilde{\psi} = \hat{\psi}(\zeta)e^{im\xi}$, to

$$(\sigma^2 + F^2)\frac{d^2 \hat{\psi}}{d\zeta^2} - m^2(\sigma^2 + N^2 - \frac{S^4}{\sigma^2 + F^2})\hat{\psi} = 0 \ . \tag{20}$$

Let $\hat{\psi} = A\sin\frac{n\pi}{H}\zeta$ (which satisfies the rigid boundary condition $\hat{\psi}|_{z=0,H} = 0$) and we find the dispersion relation of linear symmetric disturbance in the following form

$$(1 + \alpha)\sigma^4 + [\alpha N^2 + (2 + \alpha)F^2]\sigma^2 + \alpha(N^2 F^2 - S^4) + F^4 = 0 \ . \tag{21}$$

$$\sigma^2 = -\frac{p^*}{2} + \sqrt{\left(\frac{p^*}{2}\right)^2 - q^*} \ . \tag{22}$$

where $\alpha = \left(\frac{mH}{n\pi}\right)^2$, $p^* = [\alpha N^2 + (2 + \alpha)F^2] / (1 + \alpha)$, $q^* = [\alpha(N^2 F^2 - S^4) + F^4] . (1 + \alpha)$.
Note that the roots from a negative radical sign have been dropped because they are caused by the coordinate transformation. Analysis of (22) shows that at $q^* > 0$, $\sigma^2 < 0$, and at $q^* < 0$, $\sigma^2 > 0$. For neutral disturbance, $q^* = 0$ and S^2 is set to be $S_c^2 = (N^2 F^2 + F^4 / \alpha)^{\frac{1}{2}}$, in which case the critical value of vertical wind shear is $(\overline{U}_z)_c = \frac{1}{f}(N^2 F^2 + F^4 / \alpha)^{\frac{1}{2}}$. For the stability parameters of a given basic field N^2 and F^2, if the baroclinic stability parameter S^2 is changed by a tiny amout on the basis of S_c^2, that is, $S^2 = S_c^2 (1 + |\Delta|)$ with $0 < |\Delta| \ll 1$, meaning $\overline{u}_z = (\overline{u}_z)_c(1 + |\Delta|)$, we have $q^* \sim -\frac{2\alpha}{1 + \alpha}S_c^4 |\Delta|$, which is then substituted into (22), yielding $\sigma^2 \sim \frac{2\alpha S_c^4}{(1 + \alpha)p^*}|\Delta|$, with $\sigma \sim O[|\Delta|^{\frac{1}{2}}]$. It is the basis where-upon the analysis of nonlinear stability will be undertaken in the following.

3. Transformation of nonlinear disturbance equations

To facilitate the analysis of nonlinear stability it is necessary first to decrease the number of dependent quantities of (6)–(10), we define the opeartor $\mathcal{L} = \frac{\partial}{\partial t} + v'\frac{\partial}{\partial y} + w'\frac{\partial}{\partial z}$, which

used to find derivatives of both sides of (7)–(8), leading to

$$\mathcal{L}(\mathcal{L}v') = -\mathcal{L}\left[\frac{\partial}{\partial y}\left(\frac{p'}{\rho_0}\right)\right] - \mathcal{L}(fu') \ . \tag{23}$$

$$\mathcal{L}(\mathcal{L}w') = -\mathcal{L}\left[\frac{\partial}{\partial z}\left(\frac{p'}{\rho_0}\right)\right] + \mathcal{L}\left(\frac{\theta'}{\theta_0}g\right) \ . \tag{24}$$

(6) and (10) are put into (23) and (24) such that

$$\mathcal{L}(\mathcal{L}v') = -\mathcal{L}\left[\frac{\partial}{\partial y}\left(\frac{p'}{\rho_0}\right)\right] - F^2 v' + S^2 w' \ , \tag{25}$$

$$\mathcal{L}(\mathcal{L}w') = -\mathcal{L}\left[\frac{\partial}{\partial z}\left(\frac{p'}{\rho_0}\right)\right] + S^2 v' - N^2 w' \ . \tag{26}$$

It is of particular note that for any unknown function φ, $\mathcal{L}(\mathcal{L}\varphi) \neq \mathcal{L}^2\varphi$. We know from analysis of linear stability that as the basic–field stability parameter S^2 increases by a tiny quantity $\sim O(|\Delta|)$, the growth rate of linear disturbance is of the order of magnitude of $O(|\Delta|^{\frac{1}{2}})$, which suggests that it is necessary to introduce the slowly–varying time scale $T = |\Delta|^{\frac{1}{2}}t$ in dealing with the analysis of nonlinear stability by virtue of the multi–scale method. Thus we have

$$\frac{\partial}{\partial t} = |\Delta|^{\frac{1}{2}}\frac{\partial}{\partial T} \qquad \frac{\partial^2}{\partial t^2} = |\Delta|\frac{\partial^2}{\partial T^2} \ . \tag{27}$$

Based on (9), disturbances streamfunction ψ is introduced, making

$$v' = -\frac{\partial \psi}{\partial z} \qquad w' = \frac{\partial \psi}{\partial y} \ . \tag{28}$$

Now (27)–(28) are inserted into (25)–(26), leading to a closed system of equations containing only ψ and p', namely,

$$\mathcal{L}[\mathcal{L}(-\frac{\partial \psi}{\partial z})] = -(|\Delta|^{\frac{1}{2}}\frac{\partial}{\partial T} - \frac{\partial \psi}{\partial z}\frac{\partial}{\partial y} + \frac{\partial \psi}{\partial y}\frac{\partial}{\partial z})[\frac{\partial}{\partial y}(\frac{p'}{\rho_0})] + F^2\frac{\partial \psi}{\partial z} + S^2\frac{\partial \psi}{\partial y} \ . \tag{29}$$

$$\mathcal{L}[\mathcal{L}(\frac{\partial \psi}{\partial y})] = -(|\Delta|^{\frac{1}{2}}\frac{\partial}{\partial T} - \frac{\partial \psi}{\partial z}\frac{\partial}{\partial y} + \frac{\partial \psi}{\partial y}\frac{\partial}{\partial z})[\frac{\partial}{\partial z}(\frac{p'}{\rho_0})] - S^2\frac{\partial \psi}{\partial z} - N^2\frac{\partial \psi}{\partial y} \ . \tag{30}$$

By letting $S^2 = S_c^2(1+\Delta) = S_c^2[1 + \mathrm{sgn}(\Delta)|\Delta|]$ and series expanding ψ and p'/ρ_0 in terms of $\varepsilon = |\Delta|^{\frac{1}{2}}$ and find

$$\psi = |\Delta|^{\frac{1}{2}}\psi_1 + |\Delta|\psi_2 + |\Delta|^{\frac{3}{2}}\psi_3 + |\Delta|^2\psi_4 + \cdots \ , \tag{31}$$

$$p' / p_0 = |\Delta|^{\frac{1}{2}} p_1 + |\Delta| p_2 + |\Delta|^{\frac{3}{2}} p_3 + |\Delta|^2 p_4 + \cdots . \tag{32}$$

which are substituted into (29)–(30). Compare the coefficients of terms of the same order $|\Delta|^{\frac{1}{2}}$, $|\Delta|$, $|\Delta|^{\frac{3}{2}}$ and $|\Delta|^2$ on both sides of the equations and we get the approximation equations of these orders.

For $O(|\Delta|^{\frac{1}{2}})$ case

$$F^2 \frac{\partial \psi_1}{\partial z} + S_t^2 \frac{\partial \psi_1}{\partial y} = 0 . \tag{33}$$

$$S_t^2 \frac{\partial \psi_1}{\partial z} + N^2 \frac{\partial \psi_1}{\partial y} = 0 . \tag{34}$$

For $O(|\Delta|)$ case

$$F^2 \frac{\partial \psi_2}{\partial z} + S_t^2 \frac{\partial \psi_2}{\partial y} = \frac{\partial^2 p_1}{\partial T \partial y} . \tag{35}$$

$$S_t^2 \frac{\partial \psi_2}{\partial z} + N^2 \frac{\partial \psi_2}{\partial y} = -\frac{\partial^2 p_1}{\partial T \partial z} . \tag{36}$$

For $O(|\Delta|^{\frac{3}{2}})$ case

$$F^2 \frac{\partial \psi_3}{\partial z} + S_t^2 \frac{\partial \psi_3}{\partial y} = \frac{\partial^2 p_2}{\partial T \partial y} + R_1 = \frac{\partial^2 p_2}{\partial T \partial y} + (-\frac{\partial \psi_2}{\partial z}\frac{\partial}{\partial y} + \frac{\partial \psi_2}{\partial y}\frac{\partial}{\partial z})(\frac{\partial p_1}{\partial y}) . \tag{37}$$

$$S_t^2 \frac{\partial \psi_3}{\partial z} + N^2 \frac{\partial \psi_3}{\partial y} = -\frac{\partial^2 p_2}{\partial T \partial z} + R_2 = -\frac{\partial^2 p_2}{\partial T \partial z} - (-\frac{\partial \psi_2}{\partial z}\frac{\partial}{\partial y} + \frac{\partial \psi_2}{\partial y}\frac{\partial}{\partial z})(\frac{\partial p_1}{\partial z}) . \tag{38}$$

For $O(|\Delta|^2)$ case

$$F^2 \frac{\partial \psi_4}{\partial z} + S_t^2 \frac{\partial \psi_4}{\partial y} = \frac{\partial^2 p_3}{\partial T \partial y} + R_3 = \frac{\partial^2 p_3}{\partial T \partial y} + (-\frac{\partial \psi_2}{\partial z}\frac{\partial}{\partial y} + \frac{\partial \psi_2}{\partial y}\frac{\partial}{\partial z})(\frac{\partial p_2}{\partial y})$$
$$+ (-\frac{\partial \psi_3}{\partial z}\frac{\partial}{\partial y} + \frac{\partial \psi_3}{\partial y}\frac{\partial}{\partial z})(\frac{\partial p_1}{\partial y}) - \frac{\partial^3 \psi_2}{\partial T^2 \partial z} - S_t^2 \,\mathrm{sgn}(\Delta)\frac{\partial \psi_2}{\partial y} . \tag{39}$$

$$S_t^2 \frac{\partial \psi_4}{\partial z} + N^2 \frac{\partial \psi_4}{\partial y} = -\frac{\partial^2 p_3}{\partial T \partial z} + R_4 = -\frac{\partial^2 p_3}{\partial T \partial z} - (-\frac{\partial \psi_2}{\partial z}\frac{\partial}{\partial y} + \frac{\partial \psi_2}{\partial y}\frac{\partial}{\partial z})(\frac{\partial p_2}{\partial z})$$
$$- (-\frac{\partial \psi_3}{\partial z}\frac{\partial}{\partial y} + \frac{\partial \psi_3}{\partial y}\frac{\partial}{\partial z})(\frac{\partial p_1}{\partial z}) - \frac{\partial^3 \psi_2}{\partial T^2 \partial y} - S_t^2 \,\mathrm{sgn}(\Delta)\frac{\partial \psi_2}{\partial z} . \tag{40}$$

Note that the condition of $\frac{\partial \psi_1}{\partial y} = 0$, $\frac{\partial \psi_1}{\partial z} = 0$ has been applied before obtaining

(35)–(40). In fact, the condition is none other than the solution to the case of $O(|\Delta|^{\frac{1}{2}})$.

4. Solution to the approximations for these orders

For the case $O(|\Delta|^{\frac{1}{2}})$ the determinant of coefficients of (33)–(34) $\begin{vmatrix} F^2 & S_i^2 \\ S_i^2 & N^2 \end{vmatrix} = N^2 F^2$

$- S_i^4 = -\frac{1}{\alpha} F^4 \neq 0$ leads to $\frac{\partial \psi_1}{\partial y} = 0$, $\frac{\partial \psi_1}{\partial z} = 0$.

For the case $O(|\Delta|)$, eliminating variable p_1 of (35)–(36) yields the only differential equation of ψ_2:

$$N^2 \frac{\partial^2 \psi_2}{\partial y^2} + 2S_i^2 \frac{\partial^2 \psi_2}{\partial y \partial z} + F^2 \frac{\partial^2 \psi_2}{\partial z^2} = 0 , \tag{41}$$

whose solution is obtained with the aid of linear theory, i.e.,

$$\psi_2 = A(T)\varphi(z)e^{imy} + \text{c.c.} , \tag{42}$$

where $\varphi = \sin\frac{n\pi}{H}ze^{ima_i z}$, $a_i = -S_i^2 / F^2$ and c.c. means the conjugate complex number for the preceding part.

Now, putting (42) into (35)–(36) yields

$$\frac{\partial p_1}{\partial y} = (F^2 \frac{d\varphi}{dz} + imS_i^2\varphi)\int_{T_0}^{T} AdTe^{imy} + \text{c.c.} , \tag{43}$$

$$\frac{\partial p_1}{\partial z} = -(S_i^2 \frac{d\varphi}{dz} + imN^2\varphi)\int_{T_0}^{T} AdTe^{imy} + \text{c.c.} , \tag{44}$$

where T_0 is a constant.

We now proceed to give the way to solve the problem $O(|\Delta|^{\frac{3}{2}})$. Putting (42)–(44) into the expressions of R_1 and R_2, eliminating p_2 of (37)–(38), we get the differential equation of ψ_3 in the form

$$N^2 \frac{\partial^2 \psi_3}{\partial y^2} + 2S_i^2 \frac{\partial^2 \psi_3}{\partial y \partial z} + F^2 \frac{\partial^2 \psi_3}{\partial z^2} = Q_1 = \frac{\partial R_1}{\partial z} + \frac{\partial R_2}{\partial y} . \tag{45}$$

where

$$Q_1 = Q_{11} + Q_{12} = [imF^2 (\varphi\frac{d^3\varphi}{dz^3} - \frac{d\varphi}{dz}\frac{d^2\varphi}{dz^2}) - 2m^2 S_i^2 (\varphi\frac{d^2\varphi}{dz^2} - \frac{d\varphi}{dz}\frac{d\varphi}{dz})]A\int_{T_0}^{T} AdTe^{2imy}$$
$$+ \text{c.c}$$

$$- [imF^2 \frac{d^2}{dz^2}(\overline{\varphi}\frac{d\varphi}{dz}) - m^2 S_i^2 \frac{d^2}{dz^2}(\varphi\overline{\varphi})]\overline{A}\int_{T_0}^{T} AdT + \text{c.c} . \tag{46}$$

To obtain the solution to ψ_3 of (45), it is necessary to find the forms of Q_{11} and Q_{12}, and we denote $g_1 = ma_c$, $g_2 = \frac{n\pi}{H}$, $g_3 = -m^2 a_c^2 - \frac{n^2\pi^2}{H^2}$, $g_4 = 2ma_c\frac{n\pi}{H}$, $g_5 = -m^3 a_c^3 - 3ma_c\frac{n^2\pi^2}{H^2}$ and $g_6 = -3m^2 a_c^2\frac{n\pi}{H} - \frac{n^3\pi^3}{H^3}$, where g_1 through g_6 are all real numbers, leading to

$$Q_{11} = (g_7\sin^2\frac{n\pi}{H}z + ig_8\sin\frac{n\pi}{H}z\cos\frac{n\pi}{H}z + g_9\cos^2\frac{n\pi}{H}z)A\int_{T_0}^{t} AdTe^{2im(c+a_c z)} + c.c. , \qquad (47)$$

$$Q_{12} = \frac{d^2}{dz^2}g_{10}\sin^2\frac{n\pi}{H}z + ig_{11}\sin\frac{n\pi}{H}z\cos\frac{n\pi}{H}z)A\int_{T_0}^{t} AdT + c.c. , \qquad (48)$$

where $g_7 = mF^2(g_1g_3 - g_5) - 2m^2 S_c^2(g_3 + g_1^2)$, $g_8 = mF^2(g_6 + g_1g_4 - g_2g_3) - 2m^2 S_c^2(g_4 - 2g_1g_2)$, $g_9 = mF^2 g_2g_4 + 2m^2 S_c^2 g_2^2$, $g_{10} = mF^2 g_1 + m^2 S_c^2$ and $g_{11} = -mF^2 g_2$, all being real constants.

The coordinate conversion ($\xi = y + a_3 z$, $\zeta = z$) preformed of inhomogeneous equation (45) makes it in the form

$$(N^2 + F^2 a_3^2 + 2S_c^2 a_3)\frac{\partial^2\psi_3}{\partial\xi^2} - (2a_3 F^2 + 2S_c^2)\frac{\partial^2\psi_3}{\partial\xi\partial\zeta} + F^2\frac{\partial^2\psi_3}{\partial\zeta^2} = Q_{11} + Q_{12} . \qquad (49)$$

Eliminating $\frac{\partial^2\psi_3}{\partial\xi\partial\zeta}$ and setting $2a_3 F^2 + 2S_c^2 = 0$, we have $a_3 = -S_c^2 / F^2 = a_c$ so that (49) is reduced to its standard form

$$(N^2 - \frac{S_c^4}{F^2})\frac{\partial^2\psi_3}{\partial\xi^2} + F^2\frac{\partial^2\psi_3}{\partial\zeta^2} = Q_{11} + Q_{12} . \qquad (50)$$

Under the assumption of $\psi_3 = \hat{\psi}_3(\zeta)e^{im_3\xi}$, (50) reduces to

$$F^2\frac{d^2\hat{\psi}_3}{d\zeta^2} - m_3^2(N^2 - \frac{S_c^4}{F^2})\hat{\psi}_3 = (Q_{11} + Q_{12})e^{-im_3\xi} . \qquad (51)$$

in which setting $m_3 = 2m$ for the former part Q_{11} of the inhomogeneous term Q_{11} yields the cooresponding solution

$$\hat{\psi}_3 = (b_1 + ib_2\sin\frac{2n\pi}{H}\zeta + b_3\cos\frac{2n\pi}{H}\zeta)A\int_{T_0}^{T} AdT + c.c. , \qquad (52)$$

where $b_1 = \dfrac{g_7 + g_9}{-8m^2(N^2 - S_c^2 / F^2)}$, $b_2 = \dfrac{g_8}{-8m^2(N^2 - S_c^4 / F^2) - 2F^2(\frac{2n\pi}{H})^2}$.

$$b_3 = \frac{g_7 - g_9}{-8m^2(N^2 - S_c^4 / F^2) - 2F^2(\frac{2n\pi}{H})^2} \; .$$

and setting $m_3 = 0$, (51) has its solution to Q_{12} in the form

$$\hat{\psi}_3 = (b_4 + ib_5 \sin\frac{2n\pi}{H}\zeta + b_6 \cos\frac{2n\pi}{H}\zeta)\overline{A}\int_{T_0}^{t} A dT + \text{c.c} \; . \tag{53}$$

in which $b_4 = g_{10} / (2F^2)$, $b_5 = g_{11} / (2F^2)$, $b_6 = -g_{10} / (2F^2)$.

Combination of (52) and (53) gives the population solution

$$\begin{aligned}
\psi_3 &= \varphi_{31}(z)A\int_{T_0}^{T} A dT e^{2imy} + \text{c.c} + \varphi_{32}(z)\overline{A}\int_{T_0}^{T} A dT + \text{c.c} \\
&= (b_1 + ib_2\sin\frac{2n\pi}{H}z + b_3\cos\frac{2n\pi}{H}z)e^{2ima_c z}A\int_{t_0}^{T} A dT e^{2imy} + \text{c.c} \\
&\quad + (b_4 + ib_5\sin\frac{2n\pi}{H}z + b_6\cos\frac{2n\pi}{H}z)\overline{A}\int_{T_0}^{T} A dT + \text{c.c} \; .
\end{aligned} \tag{54}$$

We denote the z—related terms of the expressions of R_1 and R_2 as $R_{ij}(z)$, where $i = 1,2$ and $j = 1,2$, so that

$$R_1 = R_{11}(z)A\int_{T_0}^{T} A dT e^{2imy} + \text{c.c} + R_{12}(z)\overline{A}\int_{T_0}^{t} A dT + \text{c.c} \; , \tag{55}$$

$$R_2 = R_{21}(z)A\int_{T_0}^{t} A dT e^{2imy} + \text{c.c} + R_{22}(z)\overline{A}\int_{t_0}^{T} A dT + \text{c.c} \; . \tag{56}$$

Substitution of (54)–(56) into (37)–(38) leads to

$$\begin{aligned}
\frac{\partial p_2}{\partial y} &= (F^2\frac{d\varphi_{31}}{dz} + 2imS_c^2\varphi_{31} - R_{11})\int_{T_0}^{t}(A\int_{T_0}^{t} A dT)dT e^{2imy} + \text{c.c} \\
&\quad + (F^2\frac{d\varphi_{32}}{dz} - R_{12})\int_{T_0}^{T}(\overline{A}\int_{t_0}^{T} A dT)dT + \text{c.c} \; ,
\end{aligned} \tag{57}$$

$$\begin{aligned}
\frac{\partial p_2}{\partial z} &= (-S_c^2\frac{d\varphi_{31}}{dz} - 2imN^2\varphi_{31} + R_{21})\int_{T_0}^{T}(A\int_{T_0}^{T} A dT)dT e^{2imy} + \text{c.c} \\
&\quad + (-S_c^2\frac{d\varphi_{32}}{dz} + R_{22})\int_{T_0}^{T}(\overline{A}\int_{T_0}^{T} A dT)dT + \text{c.c} \; .
\end{aligned} \tag{58}$$

Now we turn to the case $O(|\Delta|^2)$. Eliminating p_3 of (39)–(40) yields the differential equation of ψ_4 as

$$N^2\frac{\partial^2\psi_4}{\partial y^2} + 2S_c^2\frac{\partial^2\psi_4}{\partial y \partial z} + F^2\frac{\partial^2\psi_4}{\partial z^2} = Q_2 = \frac{\partial R_3}{\partial z} + \frac{\partial R_4}{\partial y} \; . \tag{59}$$

We insert (42)–(44). (54) and (57)–(58) into the expressions of R_3 and R_4 and then write R_3^* and R_4^* for terms of R_3 and R_4 in proportion to e^{imt}. respectively. Analysis indicates that Q_2 on the *rhs* of (59) contains induced terms, thus causing secular terms involved in the solution of the problem. Therefore, to make the set problem have a homogeneous, effective, asymptotically–evolved solution, it is necessary to impose condition on Q_2. Here the secular terms are eliminated by means of the orthogonalization method, which requires

$$(Q_2, \bar{\psi}_4) = 0 . \tag{60}$$

where $\bar{\psi}_4$ stands for the solution of the adjoint homogeneous equation of (59).

It can be inferred from (60) that

$$\left(\frac{\partial R_3^*}{\partial z} + \frac{\partial R_4^*}{\partial y} , \bar{\psi}_4 \right) = 0 . \tag{61}$$

which can be rewritten as

$$\int_0^H \int_0^L \left(\frac{\partial R_3^*}{\partial z} + \frac{\partial R_4^*}{\partial y} \right) \sin \frac{n\pi}{H} z e^{-imt_3 + a_c z} dy \, dz = 0 \tag{62}$$

The expressions of φ, φ_{31}, φ_{32}, R_{11}, R_{12}, R_{21} and R_{22} are put into (62). After large volume of operations and with the aid of definitions of $b_i (i = \overline{1,6})$ and $g_i (= \overline{1,11})$, we find the equation of nonlinearly evolving amplitude of symmetric disturbance, viz.,

$$\left(m^2 + m^2 a_c^2 + \frac{n^2 \pi^2}{H^2} \right) \frac{d^2 A}{dT^2} + 2m^2 a_c S_c^2 \mathrm{sgn}(\Delta) A + \gamma_1 \bar{A} \int_{T_0}^{T} (A \int_{T_0}^{T} A dT) dT$$

$$+ \gamma_2 A \int_{T_0}^{T} (\bar{A} \int_{T_0}^{T} A dT) dT + \gamma_3 A \int_{T_0}^{T} (A \int_{T_0}^{T} \bar{A} dT) dT + \gamma_4 \bar{A} (\int_{T_0}^{T} A dT)^2$$

$$+ \gamma_5 A \left| \int_{T_0}^{T} A dT \right|^2 = 0 . \tag{63}$$

which is actually a homogeneous nonlinear integral equation of complex amplitude A but merely with unity as the integral kernel and the detailed solution will be given in the following section.

5. Nonlinear evolution symmetric disturbance

The amplitude equation (63), if its nonlinear terms are ignored, degenerates into the form

$$(1 + a_c^2 + \frac{1}{a}) \frac{d^2 A}{dT^2} + 2a_c S_c^2 \mathrm{sgn}(\Delta) A = 0 . \tag{64}$$

whose solution has the form dependent on the sign of Δ

$$A = A(0)\exp\left(\sqrt{\frac{-2a_c S_i^2}{1+a_c^2+\frac{1}{a}}}T\right) \quad (\Delta > 0 \text{ for super}-\text{critical shear}) \ . \quad (65)$$

$$A = A(0)\cos\left(\sqrt{\frac{-2a_c S_i^2}{1+a_c^2+\frac{1}{a}}}T\right) \quad (\Delta < 0 \text{ for sub}-\text{critical shear}) \ . \quad (66)$$

The equalities $a_c = -S_c^2/F^2$ and $\chi(N^2F^2 - S_i^4)+ F^4 = 0$ are put into (65) and (66), lead-ing analytically to the fact that (63) has degenerated to the case shown by linear theory.

If nonlinear terms are considered in (63), and the conversion $B = \int_{T_0}^{T} AdT$ is performed, then the equation is changed to the differential equation of variable B , viz.,

$$(1+ a_c^2+\frac{1}{a})\frac{d^3B}{dT^3} + 2a_c S_c^2 \text{sgn}(\Delta)\frac{dB}{dT} + \gamma_1^* \frac{d\overline{B}}{dT}\int_{T_0}^{T}(\frac{dB}{dT}B)dT + \gamma_2^* \frac{dB}{dT} \ . \quad (67)$$

$$\int_{T_0}^{T}(\frac{d\overline{B}}{dT}B)dT + \gamma_3^* \frac{dB}{dT}\int_{T_0}^{T}(\frac{dB}{dT}\overline{B})dT + \gamma_4^* \frac{d\overline{B}}{dT}B^2 + \gamma_5^* \frac{dB}{dT}|B|^2 = 0 \ . \quad (68)$$

where $\gamma_i^* \ (i= \overline{1,5})$ and $\gamma_i (i= \overline{1,5})$ are related by $\gamma_i^* = \gamma_i / m^2 (i= \overline{1,5})$.

After a large volume of operations, we arrive at the differential equation of B, viz.,

$$(1+ a_c^2+\frac{1}{\alpha})\frac{d^3B}{dT^3} + 2a_c S_c^2 \text{sgn}(\Delta)\frac{dB}{dT} - 2(\frac{n\pi}{H})^4 F^2 |B|^2\frac{dB}{dT} = 0 \ . \quad (69)$$

Assume $B= p(T)+ iq(T)$, $d_1 = 1+ a_c^2+\frac{1}{\alpha}$, $d_2 = 2a_c S_c^2 \text{sgn}(\Delta)$ and $d_3 = 2(\frac{n\pi}{H})^4 F^2$ that are then put into (69), followed by the separation of the real and imaginary part, and we have

$$d_1\frac{d^3p}{dT^3} + d_2\frac{dp}{dT} = d_3(p^2+ q^2)\frac{dp}{dT} \ . \quad (70)$$

$$d_1\frac{d^3q}{dT^3} + d_2\frac{dq}{dT} = d_3(p^2+ q^2)\frac{dp}{dT} \ . \quad (71)$$

which are, in fact, a system of ordinary differential equations that do not allow to get its pre-cise analytical solution but to perform numerical computation in a particular range of initial values.

We now attempt to get the analytical solution in a special case. Assume the initial conditions to be $q|_{T=0} = \frac{dq}{dT}\Big|_{T=0} = \frac{d^2q}{dT^2}\Big|_{T=0} = \cdots = 0$ and we know from the Taylor ex-pansion of $q(T)$ that $q(T)= 0$ for a smaller domain of T, during which case (70)–(71)

degenerate to

$$d_1 \frac{d^3 p}{dT^3} + d_2 \frac{dp}{dT} = d_3 p^2 \frac{dp}{dT} \quad . \tag{72}$$

For the variable p, we set $T_0 = 0$ and initial conditions $p|_{t=0} = 0$, $\frac{dp}{dT}\big|_{t=0} = A(0) = c_1$ and $\frac{d^2 p}{dT^2}\big|_{T=0} = \frac{dA}{dT}\big|_{T=0} = c_2$, in which case (72) is integrated once with respect to T so that

$$(\frac{dp}{dT})^2 = \frac{d_3}{6d_1} p^4 - \frac{d_2}{d_1} p^2 + 2c_2 p + c_1^2 \quad . \tag{73}$$

We shall discuss the problem in two cases in view of the fact that signs of Δ differ.

(I) At $\Delta < 0$, $\bar{u}_c < (\bar{u}_c)$, suggestive of sub-critical shear. At this time, $d_1 > 0$, $d_2 > 0$ and $d_3 > 0$. Further, set the initial conditions of perturbation to satisfy (66), which $c_1 = A(0) > 0$ and $c_2 = 0$ used, and (73) degenerates into the form

$$(\frac{dp}{dT})^2 = \frac{d_3}{6d_1} p^4 - \frac{d_2}{d_1} p^2 + c_1^2 \quad . \tag{74}$$

As the initial amplitude $A(0)$ is not too large, the right-hand side of (74) is factorized into $(c_1 - a_1 p^2)(c_1 - a_2 p^2)$ where a_1 and a_2 are positive real numbers that meet the needs of (75) and (76). In this case we might put $a_1 > a_2$.

$$a_1 + a_2 = \frac{d_2}{d_1 c_1} \quad . \tag{75}$$

$$a_1 a_2 = \frac{d_3}{6d_1} \quad . \tag{76}$$

Letting $E = \frac{c_1}{a_1}$ and $k^2 = \frac{a_2}{a_1} < 1$, (74) is converted into the following form

$$\left[\frac{dp}{d(\sqrt{a_1 c_1} \, T)} \right]^2 = \frac{1}{E^2} (E^2 - p^2)(E^2 - k^2 p^2) \quad . \tag{77}$$

whose solution is

$$p = E \mathrm{sn}(\sqrt{a_1 c_1} \, T) = \sqrt{c_1 / a_1} \, \mathrm{sn}(\sqrt{a_1 c_1} \, T) \quad . \tag{78}$$

so that we have the disturbance amplitude

$$A = \frac{dp}{dT} = c_1 \, \text{cn}(\sqrt{a_1 c_1}\,T) \text{dn}(\sqrt{a_1 c_1}\,T) = A(0) \text{cn}(\sqrt{a_1 A(0)}\,T) \text{dn}(\sqrt{a_1 A(0)}\,T) \tag{79}$$

Analysis of (79) shows that for the sub−critical shear $(\Delta < 0)$ the symmetric disturbance amplitude under weak nonlinear effect experiences periodic variation whose characteristics are related to Jacobi elliptic cosine function and Jacobi ellipic function of the third kind, and the period length is given as $\dfrac{4K}{\sqrt{a_1 A(0)}} = \dfrac{4}{\sqrt{a_1 A(0)}} \displaystyle\int_0^1 \dfrac{1}{\sqrt{1 - \xi^2)(1 - k^2 \xi^2}}\,d\xi$, where K represents the Legendre complete elliptic integral of the first kind. It is noted that the period duration is dependent not just on the basic−field parameters $(N^2,\ F^2,\ S_c^2)$ but, more importantly, on the initial amplitude $A(0)$ as well, a situation that differs from the amplitude undergoing its periodic variation as trignometric function of (66) under linear effect.

(II) $\bar{u}_z > (\bar{u}_z)_c$ for $\Delta > 0$, indicative of super critical hear, for which $d_1 > 0$, $d_2 < 0$ and $d_3 > 0$. Besides, assuming the initial disturbance conditions satisfy (65) at $c_1 = A(0) > 0$ and $c_2 = \dfrac{dA}{dT}\Big|_{T=0} = A(0)\sqrt{-2a_1 S_c^2 / (1 + a_i^2 + \dfrac{1}{a})} > 0$ and (73) is in the form

$$\left(\frac{dp}{dT}\right)^2 = \frac{d_3}{6d_1}p^4 - \frac{d_2}{d_1}p^2 + 2c_2 p + c_1^2 = F(p) . \tag{80}$$

We can set $F(p) = 0$ to have four real roots with $\alpha_1 > \beta_1 > \alpha_2 > \beta_2$ (other cases will be treated in a similar way). $F(p)$ is written as

$$F(p) = \frac{d_3}{6d_1}(p - \alpha_1)(p - \beta_1)(p - \alpha_2)(p - \beta_2) . \tag{81}$$

For $p \geqslant \alpha_1$ or $p \leqslant \beta_2$ (i.e., $F(p) \geqslant 0$), we set

$$\frac{p - \alpha_1}{p - \beta_1} = \frac{\alpha_1 - \beta_2}{\beta_1 - \beta_2}\xi^2 \qquad k^2 = \frac{(\alpha_1 - \beta_2)(\beta_1 - \alpha_2)}{(\alpha_1 - \alpha_2)(\beta_1 - \beta_2)} < 1 . \tag{82}$$

Now (80) is converted to

$$C \int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(1 - \xi^2)(1 - k^2\xi^2)}} = T . \tag{83}$$

where $C = 2\sqrt{6d_1} / \sqrt{d_3(\alpha_1 - \alpha_2)(\beta_1 - \beta_2)}$.

$$\xi_1 = \sqrt{\frac{\alpha_1}{\beta_1}\frac{\beta_1 - \beta_2}{\alpha_1 - \beta_2}} \qquad \xi_2 = \sqrt{\frac{\beta_1 - \beta_2}{\alpha_1 - \beta_2}\frac{p - \alpha_1}{p - \beta_1}} \quad .$$

Using the designator $r = \displaystyle\int_0^{\xi_1} \frac{d\xi}{\sqrt{(1 - \xi^2)(1 - k^2\xi^2)}}$, we get from (82)

$$\xi_2 = \text{sn}(\frac{T}{C} + r) \ .$$ (84)

$$p = \frac{\alpha_1(\beta_1 - \beta_2) - \beta_1(\alpha_1 - \beta_2)\text{sn}^2(\frac{T}{C} + r)}{(\beta_1 - \beta_2) - (\alpha_1 - \beta_2)\text{sn}^2(\frac{T}{C} + r)} \ .$$ (85)

As a result, the perturbation amplitude is in the form

$$A = \frac{dp}{dT} = \frac{2(\alpha_1 - \beta_1)(\alpha_1 - \beta_2)(\beta_1 - \beta_2)\text{sn}(\frac{T}{C} + r)\text{cn}(\frac{T}{C} + r)\text{dn}(\frac{T}{C} + r)}{C[(\beta_1 - \beta_2) - (\alpha_1 - \beta_2)\text{sn}^2(\frac{T}{C} + r)]^2} \ .$$ (86)

It can be proved that (86) takes the form $\sqrt{\frac{d_3}{6d_1}\alpha_1\alpha_2\beta_1\beta_2} = c_1 = A(0)$ at $T = 0$ which

represents the initial amplitude. It is seen from (86) that $\Delta > 0$ case the weak nonlinear symmetric disturbance amplitude experiences periodic variation whose characteristics bear a close relation to Jacobi elliptic function, quite different from the case in which the amplitude exhibits exponential growth (Eq.(65)) given by linear theory. The amplitude has its period of the form

$$2CK = \frac{4\sqrt{6d_1}}{\sqrt{d_3(\alpha_1 - \alpha_2)(\beta_1 - \beta_2)}} \int_0^1 \frac{d\xi}{\sqrt{(1 - \xi^2)(1 - k^2\xi^2)}} \ .$$ (87)

where K denotes Legendre complete elliptic intergral of the first kind. The period length is related not only to the basic−field stability parameters (N^2, F^2, S_i^2) and circulation features (m,n,H) but to the initial amplitude $A|_{T=0}$ and its change rate $\frac{dA}{dT}\Big|_{T=0}$ as well. The periodic variation of the nonlinear meso−scale symmetric disturbance amplitude seems different from that of large−scale nonlinear baroclinic instability dominantly in the oscillatory form by (86) more intricate than the large−scale counterpart and is quite complicated as compared to that of large−scale nonlinear Rossby wave. As for the periodic variation of large−scale unsteady baroclinic wave amplitude, it is formally associated only with Jacobi elliptic function of the third kind.

Of course, we are allowed to address, if we want to, the periodic variation of perturbation amplitude at $\alpha_2 \leqslant p \leqslant \beta_1$ (i.e., $F(p) \geqslant 0$) in a similar way.

6. Concluding remarks

We have investigated the evolution of limited amplitude by virtue of a system of nonlinear symmetric perturbation dynamical equations in an adiabatic, frictionless fluid. Results suggest that due to the interaction between disturbances, on one side, and perturbation and basic flow, on the other, the nonlinear effect will eventually cause the growth of unsteady symmetric disturbance to cease, and the limited amplitude of unstable wave to display an

oscillatory trend, a situation that utterly differs from the unlimited exponential growth, as shown in linear hypotheses. Both in super- and sub- critical shear, the symmetric disturbance amplitude exbilits periodic variation, whose characteristics are relative to Jacobi elliptic function, and the period length depends not merely on the basic-field stability parameters and wave properties but on the initial amplitude and its change rate. The periodic variation of all meso-scale limited amplitudes is much more intricate than the oscillatory behaviors of large-scale nonlinear baroclinic instability.

The present study excluded the heating through vapor condensation in an attempt to simplify the problem. The introduction of vapor as an important factor proposed in the late 1970s aimed dominantly at reducing the Richardson number of the real atmosphere for the sake of the theory on symmetric instability. For meso-scale motions, diabatic heating effect is perhaps of particular importance so that external source forcings should include vapor effect as far as possible in nonlinear symmetric instability studies. Moreover, it is the problem of limited amplitude symmetric disturbance under weak nonlinear effect that we are devoted to such that our findings apply only to the case $|\Delta| = \left|S^2 / S_c^2 - 1\right| = \left|\bar{u}_z / (\bar{u}_z)_c - 1\right| \ll 1$. For a moderate $|\Delta|$ no particularly effective access has been found to the solution in the present nonlinear dynamic theories, a problem that awaits further study.

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维向切变流中的非线性对称不稳定

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摘　要　　　P4　A

讨论了维向切变流中的非线性对称不稳定问题。文中采用绝热无粘的非线性对称扰动方程组，利用多尺度摄动方法分析其不稳定波动的有限振幅特性。研究结果表明：不稳定波的有限振幅在强度上呈现出振荡趋势。无论是超临界切变情况，还是次临界切变情况，对称扰动振幅都随时间呈现出周期性的变化，振荡周期的大小不仅与基本场稳定度参数及波的特性有关，而且还与初始扰动的振幅及其增长率有关。

关键词：对称不稳定，非线性，有限振幅