# Application of Linear Thermodynamics to the Atmospheric System. Part I: Linear Phenomenological Relations and Thermodynamic Property of the Atmospheric System

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ABSTRACT

A series of thermodynamic property of the atmospheric system can be deducted, in accordance with restriction of the general thermodynamics theory or other nature principle to saddle on the phenomenological relation. The relationship between the turbulence transport coefficients of K turbulence close theory and the phenomenological coefficients are deduced using the linear thermodynamics of nonequilibrium state. A cross coupling between the heat transportation and the vapor transportation in the atmospheric system is proved. Even a turbulence intensity theorem is demonstrated. The distributional heterogeneity of velocity and potential temperature is the turbulence fountainhead and the turbulence intensity is proportional to the scalar product of velocity and potential temperature gradient in the non-compressed and isotropy turbulence atmosphere. More about an atmospheric vortex theorem is demonstrated. The shear of potential temperature leads to a vortex movement or sundry circumfluence movement and the velocity vorticity equals to the vector product of velocity and potential temperature gradient. An application foreground of the linear thermodynamics is exhibited to the atmosphere system.

**Key words:** Atmospheric system, Linear thermodynamics, Linear phenomenological relation, Turbulent transportation coefficient, Turbulence intensity

## 1. Introduction

The reciprocal relation established by Onsager (1931a,b) and the minimum principle of entropy production established by Prigogine (1945, 1967) in the 1945 are the basis of the linear thermodynamics. It indicates the mature of the linear thermodynamics of nonequilibrium state. Its main results have important applications to many transportation phenomena.

Although the linear phenomenological relation should be regarded as a hypothesis other tnan the thermodynamics, the thermodynamic method can conveniently provide much knowledge about the characteristics of phenomenological coefficients once this hypothesis has been made. Acquirement of these knowledge needs no special dynamic model. Many macroscopic experiment facts, especially the transportation of matter and energy about the irreversible process, prove that it indeed satisfies those linear relations between the thermodynamic force and the thermodynamic flow under certain conditions.

The restrictions that the thermodynamic principle saddles on the phenomenological coefficients are in general: (i) The second thermodynamic law restricts the phenomenological

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coefficients, all being positive definite, with the result that the entropy production is the positive definite. (ii) The spatial symmetry restriction—Curier–Prigogine principle indicates that the thermodynamic forces cannot have more symmetry elements than the coupling thermodynamic flows. The thermodynamic flows and the thermodynamic forces, which have different symmetry characteristics, exist no coupling in the isotropic medium (Onsager 1931a,b; Katchalsky and Curran 1965). (iii) The time symmetry restriction– Onsager reciprocal relation, in practice is a restriction of the microscopic reversibility principle. This restriction condition leads to an important conclusion that the phenomenological coefficients have symmetry. Hence it is called the symmetry principle of phenomenological coefficient. Above restricts of the thermodynamic principle to the phenomenological coefficients have been proved in the statistical thermodynamics (Groot et al. 1962).

Main results of the linear thermodynamics of nonequilibrium state deduced from the thermodynamic principle are of universality. And they should be applicable to the atmosphere system. Also we should indicate that many special conditions must be considered for the atmosphere system: (i) The main characteristics of atmospheric movement are the turbulent flow, so far as to the turbulent transportation far greater than the molecule transportation. (ii) The dynamic processes cannot be neglected in the atmosphere system; therefore atmosphere linear thermodynamics must consider dynamic processes controlled by the gravity field and the Coriolis force of earth rotation. In general, less consideration of these forces has been taken in the system of theoretical physics or chemistry.The classic nonequilibrium state thermodynamics we have analyzed indicate that the dynamic processes, controlled by the gravity field and the Coriolis force of earth rotation are very important for the environment fluid of atmosphere and marine (Hu 2002). Though the linear thermodynamics theory has maturated already, up to now the studies on the application of linear thermodynamics to atmospheric system are seldom conducted. This paper tends to apply the linear thermodynamics theory to the atmosphere system, and to study linear thermodynamic property of the atmosphere system.

## 2. The entropy equilibrium equation and the linear phenomenological relations of atmosphere system

Based on the above physic characteristics of the atmosphere system, the entropy equilibrium equation of atmospheric system can be deduced from the thermodynamics principle of nonequilibrium state as follows (Hu 1999):

$$\frac{\partial}{\partial t}(\rho s) = -\frac{\partial}{\partial x_j}\boldsymbol{J}_{sj} + \sigma + \sigma_g, \tag{1}$$

in which the entropy flux $\boldsymbol{J}_{sj}$ and the entropy production $\sigma$ as well as the dynamic entropy production $\sigma_g$ are respectively

$$\boldsymbol{J}_{sj} = \rho s \boldsymbol{U}_j + \frac{1}{\theta}\boldsymbol{J}_{\theta j} + \frac{\Delta\mu}{T}\boldsymbol{J}_{Vj} - \frac{\boldsymbol{U}_i}{T}\tau_{ij}, \tag{2}$$

$$\sigma = \boldsymbol{J}_{\theta j}\frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right) + \boldsymbol{J}_{Vj}\frac{\partial}{\partial x_j}\left(\frac{\Delta\mu}{T}\right) - \tau_{ij}\frac{\partial}{\partial x_j}\left(\frac{\boldsymbol{U}_i}{T}\right) + \sum_{\alpha=1}^{1}\omega_\alpha\lambda_\alpha, \tag{3}$$

$$\sigma_g = \rho\frac{U_i}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + g\delta_{i3} - f_c\varepsilon_{ij3}U_j\right) \tag{4}$$

Moreover, $\Delta\mu = \mu_d - \mu_v$ is the remainder of chemical potential between the dry air and the moist air. The remainder of chemical potential $\Delta\mu$ is function of the ($T$, $p$ and $q$)

$$\Delta\mu = \Delta c_p[T\ln\frac{T}{T_0} - (T - T_0)] - R_V T\ln q - T\Delta R\ln\frac{p}{p_0}. \tag{5}$$

where $\theta$, $q$, $U_i$ and $T$ are the atmosphere potential temperature, the specific humidity, the wind speed in $i$ direction and the absolute temperature respectively; $p$ is the atmospheric pressure; $J_{\theta j}$, $J_{Vj}$, $\tau_{ij}$ are the heat flux, the vapor flux and the momentum flux of the turbulent transportation respectively; $\rho$, $c_p$ and $R_V$, the atmosphere density, the specific heat at the isopiestic pressure and the vapor gas constant; $\Delta c_p$, $\Delta R$ are the remainder of specific heat at the isopiestic pressure and gas constant between the dry air and the moist air; $\omega_\alpha$, $\lambda_\alpha$ are the rate and the latent heat of phase change.

Each term in the entropy production (3) is the product of thermodynamic flow and thermodynamic force. The thermodynamic forces are the function of macroscopic parameter of the atmosphere system; the thermodynamic flows are turbulent transportation flux, if the molecule viscosity is neglected. Therefore the thermodynamic flows $J_{\theta j}$, $J_{Vj}$, $\tau_{ij}$ and the thermodynamic forces $X_{\theta j}$, $X_{Vj}$, $X_{mij}$ of heat transportation, vapor transportation and momentum transportation are respectively

$$J_{\theta j} = \rho c_p\overline{u'_j\theta'_\theta}; \quad X_{\theta j} = \frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right) \tag{6}$$

$$J_{Vj} = \rho\overline{u'_j q'}; \quad X_{Vj} = \frac{\partial}{\partial x_j}\left(\frac{\Delta\mu}{T}\right)_{p,T} = -\frac{R_V}{q}\frac{\partial q}{\partial x_j}, \tag{7}$$

$$\tau_{ij} = \rho\overline{u'_i u'_j}; \quad X_{mij} = \frac{\partial}{\partial x_j}\left(-\frac{U_i}{T}\right) \tag{8}$$

The thermodynamic force is the driving force of irreversible process in the nonequilibrium state thermodynamics; the thermodynamic flow is the develop rate of irreversible process. The nonequilibrium state is the character related to the equilibrium state: the thermodynamic force and the thermodynamic flow are the characterization of measure deviating from the thermodynamic equilibrium state. The thermodynamic force is the reason why the thermodynamic flow is produced; hence we consider that the thermodynamic flow $J$ is certain function $J(X)$ of the thermodynamic force $X$. Supposing this functional relation exists and tends to be contineous, the thermodynamic flow $J$ can be expanded as a Taylor function of the thermodynamic force $X$ according to the reference state, which is the equilibrium state (it is a state that the force and the flow are zero) (Li 1986). For a single process, we have the Taylor function

$$J = J(X) = J_0(X_0) + \left(\frac{\partial J}{\partial X}\right)_0(X - X_0) + \frac{1}{2}\left(\frac{\partial^2 J}{\partial X^2}\right)_0(X - X_0)^2 + \cdots. \tag{9}$$

Because the force and the flow are zero at the equilibrium state $X_0 = 0$, $J_0 = 0$, we have

$$J(X) = \sum_{n=1}^{k}\frac{1}{n!}L_n X^n, \quad L_n = \frac{\partial^n J(X)}{\partial X^n}. \tag{10}$$

in which $L_n$ is the Taylor coefficient. As the thermodynamic force is very weak, it means the systematic state deviates equilibrium state a little. In the time, the terms of high power of $X$ in the formula (10) are far smaller than the first term. Hence the high power terms can be neglected, and we have

$$J = LX. \tag{11}$$

We can summarize to get some analogous relations, similar to formula (11) through direct observation of the irreversible process phenomena. These relationships are called the phenomenological relation between the thermodynamic force and the thermodynamic flow. The coefficient $L$ of proportion is called the linear phenomenological coefficient. If the linear phenomenological coefficient is relatively smaller to the thermodynamic force, formula (11) represents that the thermodynamic force and the thermodynamic flow satisfy linear relation.

## 3. Turbulent transportation of heat and vapor along with across coupling between them

Supposing only irreversible processes for the heat turbulent transportation and the vapor turbulent transportation exist in the atmosphere system, from formula (3) we can get the entropy production

$$\sigma = J_{\theta_j} X_{\theta_j} + J_{V_j} X_{V_j} = J_{\theta_j} \frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right) + J_{V_j} \frac{\partial}{\partial X_j}\left(\frac{\Delta\mu}{T}\right)_{p,T} \tag{12}$$

If the turbulent fluxes, taken as the thermodynamic flow in the atmosphere system, are expanded as the Taylor series for the thermodynamic force, then we can find the turbulent transportation coefficient is the Taylor series of the gradient of systemic macroscopic parameter. It is supposed that the thermodynamic flow $J_{\theta_i}$ of heat and the thermodynamic flow $J_{V_j}$ of vapor satisfy the linear phenomenological relation, even they are the cross coupling, along with the coupling coefficients satisfying the reciprocal relation. The turbulent fluxes as the thermodynamic flow are expanded as well as the Taylor series for the thermodynamic force only keeping down linear term, we have

$$J_{\theta_i} = L_{\theta\theta} X_{\theta_j} + L_{\theta V} X_{V_j}; \quad J_{V_j} = L_{VV} X_{V_j} + L_{\theta V} X_{\theta_j}. \tag{13}$$

Here $L_{\theta\theta}$, $L_{VV}$, $L_{\theta V} = L_{V\theta}$ are respectively defined as the phenomenological coefficient and the cross coupling phenomenological coefficient of heat and vapor. Put the thermodynamic force (6) and (7) into the above two formulae to get the turbulent flux of heat and vapor as follows:

$$J_{\theta_j} = -\rho c_p K_\theta \frac{\partial\theta}{\partial x_j} - \rho K_{\theta V} \frac{\partial q}{\partial x_j}; \quad J_{V_j} = -\rho K_V \frac{\partial q}{\partial x_j} - \rho c_p K_{V\theta} \frac{\partial\theta}{\partial x_j}. \tag{14}$$

in which the turbulent transport coefficients of heat and vapor, $K_\theta$ and $K_V$, along with the turbulence coupling coefficients of them, $K_{\theta V}$ and $K_{V\theta}$ are respectively

$$K_\theta = \frac{K_{\theta\theta} - K'_{V\theta} D_{\theta V}}{1 - D_{\theta V} D_{V\theta}}, \quad K_{\theta V} = \frac{K_{VV} D_{\theta V}}{1 - D_{\theta V} D_{V\theta}}, \tag{15}$$

$$K_V = \frac{K_{VV}}{1 - D_{\theta V} D_{V\theta}}, \quad K_{V\theta} = \frac{K_{\theta\theta} D_{\theta V} - K'_{V\theta}}{1 - D_{\theta V} D_{V\theta}} \tag{16}$$

Here $K'_{V\theta}$ is

$$K'_{V\theta} = \frac{T}{\theta}\left(\Delta c_p \ln\frac{T_0}{T} - R_V \ln q - \Delta R \ln\frac{p}{p_0} - \frac{\Delta\mu}{T^2}\right)\left(L_{VV} - \frac{L_{\theta V}^2}{L_{\theta\theta}}\right) \tag{17}$$

And $K_{\theta\theta}$ and $K_{VV}$ are respectively

$$K_{\theta\theta} = \frac{1}{\rho c_p \theta^2}\left(L_{\theta\theta} - \frac{L_{\theta V}^2}{L_{VV}}\right); \quad K_{VV} = \frac{R_V T}{\rho q}\left(L_{VV} - \frac{L_{\theta V}^2}{L_{\theta\theta}}\right) \tag{18}$$

The defines of the heat cross coupling coefficient $D_{V\theta}$ and the vapor cross coupling coefficient $D_{\theta V}$ are respectively

$$D_{\theta V} = \frac{L_{\theta V}}{L_{VV}}; \quad D_{V\theta} = \frac{L_{\theta V}}{L_{\theta\theta}}. \tag{19}$$

When there is no cross coupling effect between heat and vapor

$$L_{\theta V} = 0; \quad D_{\theta V} = D_{V\theta} = 0, \tag{20}$$

we have

$$K_\theta = K_{\theta\theta} = \frac{1}{\rho c_p \theta^2} L_{\theta\theta}, \quad K_V = K_{VV} = \frac{R_V T}{\rho q} L_{VV}, \quad K_{V\theta} = K'_{V\theta}. \tag{21}$$

Consequently, formula (14) of the turbulent transport fluxes of heat and vapor has the following form:

$$J_{\theta j} = -\rho c_p K_\theta \frac{\partial \theta}{\partial x_j}, \tag{22}$$

$$J_{Vj} = -\rho K_V \frac{\partial q}{\partial x_j} - \rho c_p K_{V\theta} \frac{\partial \theta}{\partial x_j}. \tag{23}$$

It means that the vapor gradient has no influence on the heat transportation, when the cross coupling between the heat and the vapor does not exist. The heat turbulent flux (22) corresponds with the K turbulence close; therefore the K turbulence close is a special case for formula (14). But the vapor turbulent flux cannot be predigested to its brief form, similar to formula (22), due to that the entropy production formula (12) is not the immediate action of vapor specific humidity, but is the action of the gradient of chemical potential remainder $\Delta\mu$. Formula (5) show that the remainder $\Delta\mu$ of chemical potential is function of the atmospheric temperature, the pressure and the specific humidity. Therefore the potential temperature gradient still has influence on the vapor transportation. We call the coupling effect of the temperature to the vapor flux "thermodynamic coupling".

The cross coupling between the heat transportation and the matter diffuse has been proved in the field of physics and chemistry. And the heat flow caused by the matter concentration gradient is called Dufour effect; the matter flow caused by the temperature gradient is called Sort effect (Li 1986). The coupling phenomenon between the heat turbulent transportation and the vapor turbulent transportation in the atmospheric system has been discussed in research on the turbulent transportation in the surface layer. The literature (Zhang and Hu 1995) shows influence between the heat turbulent flux and the vapor turbulent flux in the atmospheric surface layer. Although some results have given rise to a few controversies (Zhang and Hu 1995; Shashi et al. 1978), at present they mainly deal with the precision restriction of turbulence observations under the technical conditions. Considering Curia principle in the linear thermodynamics theory, the entropy production (12) of atmospheric system shows the thermodynamic flow $J_{\theta j}$ of heat transportation and the thermodynamic flows $J_{V_j}$ of vapor transportation is the vector, hence the cross coupling may exist between them.

## 4. Momentum turbulent transportation and vortex theorem

### 4.1 *Linear phenomenological relation of the momentum transportation*

According to Curier principle of restriction of the spatial symmetry in the linear thermodynamics, the viscosity stress (consisting of the molecule viscosity and the turbulence viscosity) is a tensor. It has no cross coupling with other thermodynamic flows (for instance the heat flux or the vapor flux) that are vector type, and with thermodynamic flows (for instance the phase transition rate) that are scalar quantity type. Considering only the irreversible process to be relative to the momentum transportation owing to the viscosity stress, the entropy production is

$$\sigma_m = \tau_{ij} X_{mij} = -\tau_{ij}\frac{\partial}{\partial x_j}\left(\frac{U_i}{T}\right) \tag{24}$$

Supposing the stress tensor has the symmetry character as following:

$$\tau_{ij} = \tau_{ji} \qquad (i \neq j). \tag{25}$$

Due to that the viscosity stress is a tensor, the linear phenomenological relation should have the following form:

$$\tau_{ij} = L_{ijkl} X_{mkl} \qquad (i,j,k,l = 1,2,3). \tag{26}$$

$L_{ijkl}$ in formula (26) is the linear phenomenological coefficients of momentum transportation, that is a tetradic tensor to have $3 \times 3 \times 3 \times 3 = 81$ components. Because the atmospheric turbulence is anisotropic, the phenomenological coefficient can not be reduced as two components as molecule viscosity. We suppose the momentum transportation is independent in each direction; even various directions are without the cross coupling effect. As a result the phenomenological coefficient $L_{ijkl}$ should be shrinked as the second-order tensor, which remains only $3 \times 3 = 9$ components that is marked by $L_{ij}$. Putting the thermodynamic force (8) into formula (26), we perform some calculations using the definition of potential temperature, formula (26) changes as the following form:

$$\tau_{ij} = -\rho K_{ij}\left(\frac{\partial U_i}{\partial x_j} - \frac{U_i}{\theta}\frac{\partial \theta}{\partial x_j}\right) \quad K_{ij} = \frac{L_{ij}}{\rho T}; \quad K_{ij} = K_{ji}. \tag{27}$$

Here the momentum turbulent coefficient is defined as $K_{ij}$. Above formula shows the momentum turbulent coefficient is a second-order tensor, not a scalar quantity, because the turbulence is non-isotropy. Atmospheric turbulent transportation can be supposed as a symmetrical second-order tensor in general. The turbulent transport coefficient can be just supposed as a scalar quantity, only when the turbulence can be supposed as the isotropy in space. And also above formula shows that the thermodynamic coupling effect exists in the momentum turbulent transportation process. It means that the turbulent transport flux $\tau_{ij}$ of momentum is not only relative to the velocity gradient but also to the potential temperature gradient, because the thermodynamic force (8) $X_{mij}$ in the irreversible process of momentum turbulent transportation is relative to temperature. If the atmospheric system satisfies the following condition

$$\left(\frac{1}{\theta}\frac{\partial\theta}{\partial x_j}\right)\Big/\left(\frac{1}{U_i}\frac{\partial U_i}{\partial x_j}\right)\ll 1. \tag{28}$$

it means the relative gradient of potential temperature is far smaller than the relative gradient of velocity, as a result the thermodynamic coupling effect can be neglected. At this time, formula (27) can be predigested as the following form:

$$\tau_{ij} = -\rho K_{ij}\frac{\partial U_i}{\partial x_j}. \tag{29}$$

It is what we have seen the form of the momentum turbulent transportation to appear customarily in the K close.

4.2 *Turbulence intensity theorem and turbulent momentum flux*

Neglecting the molecule viscosity action, the momentum transport flux is just the turbulent flux, therefore formula (27) can be expanded as the following component form:

$$\tau_{ij} = \rho\begin{vmatrix} \overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\ \overline{w'u'} & \overline{v'^2} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'^2} \end{vmatrix}$$

$$= -\rho\begin{vmatrix} K_{11}\left(\frac{\partial U}{\partial x}-\frac{U}{\theta}\frac{\partial\theta}{\partial x}\right) & K_{12}\left(\frac{\partial U}{\partial y}-\frac{U}{\theta}\frac{\partial\theta}{\partial y}\right) & K_{13}\left(\frac{\partial U}{\partial z}-\frac{U}{\theta}\frac{\partial\theta}{\partial z}\right) \\ K_{21}\left(\frac{\partial V}{\partial x}-\frac{V}{\theta}\frac{\partial\theta}{\partial x}\right) & K_{22}\left(\frac{\partial V}{\partial y}-\frac{V}{\theta}\frac{\partial\theta}{\partial y}\right) & K_{23}\left(\frac{\partial V}{\partial z}-\frac{V}{\theta}\frac{\partial\theta}{\partial z}\right) \\ K_{31}\left(\frac{\partial W}{\partial x}-\frac{W}{\theta}\frac{\partial\theta}{\partial x}\right) & K_{32}\left(\frac{\partial W}{\partial y}-\frac{W}{\theta}\frac{\partial\theta}{\partial y}\right) & K_{33}\left(\frac{\partial W}{\partial z}-\frac{W}{\theta}\frac{\partial\theta}{\partial z}\right) \end{vmatrix}. \tag{30}$$

in which $U_j = (U,V,W)$, $x_j = (x,y,z)$.

Consequently the velocity variance should be

$$\overline{u'^2} = -K_{11}\left(\frac{\partial U}{\partial x}-\frac{U}{\theta}\frac{\partial\theta}{\partial x}\right);\ \overline{v'^2} = -K_{22}\left(\frac{\partial V}{\partial y}-\frac{V}{\theta}\frac{\partial\theta}{\partial y}\right);\ \overline{w'^2} = -K_{33}\left(\frac{\partial W}{\partial z}-\frac{W}{\theta}\frac{\partial\theta}{\partial z}\right) \tag{31}$$

It is obvious that the gradient of velocity and temperature is the physical reason to cause the turbulence variant of velocity. It implies the space heterogeneity of the velocity and the temperature is the fountainhead of turbulence. In general the atmosphere can be supposed as incompressible $\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0$; and the turbulence can be supposed as isotropic, namely $K_{11}=K_{22}=K_{33}=K_m$. To sum up formulae in (31), we can deduce following important property about the atmosphere velocity variant,

$$(\overline{u'^2}+\overline{v'^2}+\overline{w'^2}) = K_{ii}\frac{1}{\theta}U_i\frac{\partial\theta}{\partial x_i} = K_m\frac{1}{\theta}\vec{U}\cdot\nabla\theta. \tag{32}$$

If the left side of formula (32) is regarded as the turbulence intensity, then this formula reveals a material fact that the turbulence intensity is in proportion to the scalar product of velocity and relative gradient of potential temperature. It is called the turbulence intensity theorem in the isotropy atmosphere. We know from this theorem that the turbulence roots in the potential temperature gradient in the isotropy and incompressible fluid.

Considering the symmetry condition (25) of stress tensor, we obtain the covariance, namely the momentum flux is

$$\overline{u'v'}=\overline{v'u'}=-K_{xy}\left(\frac{\partial U}{\partial y}-\frac{U}{\theta}\frac{\partial\theta}{\partial y}\right) \tag{33}$$

$$\overline{w'u'}=\overline{u'w'}=-K_{xz}\left(\frac{\partial U}{\partial z}-\frac{U}{\theta}\frac{\partial\theta}{\partial z}\right) \tag{34}$$

$$\overline{w'v'}=\overline{v'w'}=-K_{yz}\left(\frac{\partial V}{\partial z}-\frac{V}{\theta}\frac{\partial\theta}{\partial z}\right) \tag{35}$$

Here $K_{12}=K_{21}=K_{xy}$, $K_{13}=K_{31}=K_{xz}$, $K_{23}=K_{32}=K_{yz}$. The relationships (33–35) show the turbulent momentum fluxes are not only relative to the velocity gradient, but also to the potential temperature gradient. When the relative gradient of atmospheric potential temperature is far smaller than the relative velocity gradient, i.e. under the condition of relationship (28), the momentum fluxes are form of the K turbulence close.

$$\overline{u'v'}=-K_{xy}\frac{\partial U}{\partial y},\quad \overline{w'u'}=-K_{xz}\frac{\partial U}{\partial z},\quad \overline{w'v'}=-K_{yz}\frac{\partial V}{\partial z}. \tag{36}$$

### 4.3 *Vortex theorem*

As the relative gradient of potential temperature is large enough, the second term in formula (27) cannot be neglected. Using the conditions (25) of tensor symmetry and the symmetry of turbulent transportation coefficient in (27), an important atmosphere relationship can be deduced from the momentum transportation flux (27)

$$\left(\frac{\partial U_i}{\partial x_j}-\frac{U_i}{\theta}\frac{\partial\theta}{\partial x_j}\right)=\left(\frac{\partial U_j}{\partial x_i}-\frac{U_j}{\theta}\frac{\partial\theta}{\partial x_i}\right)\quad i\neq j. \tag{37}$$

It can be written as the tensor form or the intuitionistic vector form

$$(-1)^k\varepsilon_{ijk}\frac{\partial U_i}{\partial x_j}=(-1)^k\varepsilon_{ijk}\frac{U_i}{\theta}\frac{\partial\theta}{\partial x_j};\quad \nabla\times\vec{U}=\frac{1}{\theta}\vec{U}\times\nabla\theta. \tag{38}$$

Formula (38) specifies that the velocity vorticity equals to the vector product of velocity and relative gradient of potential temperature. It is called the vortex theorem. The vortex theorem indicates that the potential temperature gradient forms vortex movement or circulation movement in the atmospheric velocity field. It is a well–known fact that the heterogeneity of atmospheric temperature and pressure causes the vortex movement in the atmospheric velocity field. The vortex theorem is an approximate description of the atmosphere vortex movement in the thermodynamic linear region. To define the velocity vorticity

$$\vec{\Omega}=\nabla\times\vec{U}=(\xi,\eta,\zeta), \tag{39}$$

three components of the velocity vorticity are respectively

$$\xi=\frac{\partial W}{\partial y}-\frac{\partial V}{\partial z}=\frac{1}{\theta}\left(W\frac{\partial\theta}{\partial y}-V\frac{\partial\theta}{\partial z}\right) \tag{40}$$

$$\eta=\frac{\partial U}{\partial z}-\frac{\partial W}{\partial x}=\frac{1}{\theta}\left(U\frac{\partial\theta}{\partial z}-W\frac{\partial\theta}{\partial x}\right) \tag{41}$$

$$\zeta=\frac{\partial V}{\partial x}-\frac{\partial U}{\partial y}=\frac{1}{\theta}\left(V\frac{\partial \theta}{\partial x}-U\frac{\partial \theta}{\partial y}\right) \tag{42}$$

Two horizontal components (40 and 41) represent the vortex movement caused by the vertical circulation in the atmosphere. The vertical circulation is a kind of important circulation movement in the atmosphere, such as the land and sea breeze, the mountain and valley winds, the lake and land breeze, etc.. The formulae (40) and (41) give a concision and intuitive description of mathematics for this kind of vertical circulation. The vertical vorticity (42) describes horizontal vortex movement. The formula shows that the horizontal gradient of potential temperature causes the horizontal vortex movement. It is a mathematical description approximately for the relation between the vertical vorticity and the horizontal gradient of potential temperature in the thermodynamic linear region. The atmospheric cyclone or anticyclone, the typhoon and tornado etc. belong to these movements.

## 5. Discussions

(1) Above results show that the thermodynamic forces and the thermodynamic flows indeed satisfy the linear relation. These relationships characterize the thermodynamic property of linear turbulent exchange coefficient. It proves in the theory that turbulence intensity is relative to the velocity and the potential temperature gradient, and the space heterogeneity of the velocity and the potential temperature is the fountainhead of turbulence. And the turbulence intensity theorem is proved. The turbulence intensity is in proportion to the scalar product of the velocity and the potential temperature gradient in the atmosphere that is the incompressible gas and the turbulent isotropy. Moreover the atmosphere vortex theorem is also proved. The potential temperature shear generates the vortex movement or sundry circulation movements and the velocity vorticity equals to the vector product of the velocity and the relative gradient of potential temperature.

(2) As is well–known, the K turbulent close theory makes an analogy of turbulence with the molecule viscosity using Prandtl theory to get the turbulent exchange coefficient. The linear thermodynamics makes the linear phenomenological coefficient $L$ to expand thermodynamic flow to the Taylor series of the thermodynamic force and to neglect the higher–order term. The linear turbulent relationships respectively representing relation between the turbulent flux of the heat, vapor and momentum with the potential temperature, the specific humidity, and the velocity has been found using the linear phenomenological relation. They are coincident with the K turbulence close theory. The K close theory uses Prandtl approximation, which is a half–experience theory and cannot indicate their scope of application in the theory. But the linear phenomenological theory is a more rigorous theory of the linear thermodynamics of nonequilibrium state to have wider universality. It yet indicates that scope of application is the linear region near the thermodynamic equilibrium state.

(3) The coupling relation between the heat transportation process and the vapor transportation process cannot be generally found in the K theory. The linear phenomenological theory not only indicates the cross coupling effect, that may generate in the atmosphere, between them in the theory, but also indicates the potential temperature gradient may affect the vapor transportation due to the thermodynamic coupling effect caused by the potential temperature gradient. The linear phenomenological theory even proves the momentum turbulent flux is in proportion to the gradient of velocity and potential temperature. As the relative gradient of potential temperature is far smaller than the relative gradient of velocity, the momen-

tum turbulent flux can be predigested the form in the K turbulence close customarily. Furthermore, the momentum turbulent transport coefficient is a tensor, that is just a scalar quantity only to suppose isotropy turbulence.

(4) No special dynamic models are needed for obtaining the above theoretical results. The knowledge about the atmosphere thermodynamic property, that was rarely seen, has been deduced from the general thermodynamic principle. These results have also the universal character of thermodynamics. These theoretic results are easily understood in the physics, part of results need to be proved from the observation experiment. These studies exhibit the application foreground of the linear thermodynamics to the atmosphere system; especially it provides leads for further study of the thermal nonuniform atmosphere.

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# 线性热力学对大气系统的应用(I)
# 大气系统的线性唯象关系和热力学性质

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## 摘 要

从一般的热力学原理或其它自然原理对唯象关系所强加的限制，能够演绎出大气系统的一系列热力学性质。利用非平衡态线性热力学导出了湍流 K 闭合理论中湍流交换系数同唯象系数的关系，从理论上证明大气系统热量湍流输送同水汽之间存在交叉耦合，还导出了湍流强度同速度和位温梯度的关系，从而证明速度和位温空间分布的非均匀性是湍流之源。并证明

湍流强度定理，不可压缩气体和各向同性湍流大气中，湍流强度正比于速度与位温梯度的标积，进而证明大气涡旋定理，位温的切变将导致涡旋运动或各种环流运动，速度涡度等于速度同位温相对梯度的矢积。展现了线性热力学在大气系统的应用前景。

**关键词**: 大气系统，线性热力学，线性唯象关系，湍流输送系数，湍流强度

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Beginning in the second half of 2002, we will create two new sections in *AAS*. The first is "*Letters*", which will allow for the quick publication of creative works. The published works could be preliminary research or prototypes, such as scientific hypotheses, new diagnostic tools, important findings or new atmospheric phenomena that have yet to be introduced in the literature. In this respect, some new ideas— even controversial ones — may lead to the stimulation of discussions, arguments and even critiques that are very much needed in today's research environment. More complete works could be submitted later for publication in AAS. The second section is "*Notes and Correspondence*". The aim of this section is to promote the more active practice of scientific dialogue and to encourage scientific exchange through publications of comments and replies. This section will also serve as a forum for an author who has found faults in his previously published work to tell our readers of such mistakes made. The page limits for both sections are 2–5 pages.

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