Critical Time Span and Nonlinear Action Structure of Climatic Atmosphere and Ocean

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ABSTRACT

This paper studies the critical time span and the approximate nonlinear action structure of climatic atmosphere and ocean. The critical time span of the climatic atmosphere and ocean, which is related to the spatial resolution required, the strength of nonlinear action, and the calculation exactness, may represent the relative temporal scale of predictability. As far as the same characteristic spatial scale is concerned, the minimum critical time span of the ocean is about 9 times of that of atmosphere, several days or more. Usually, the stronger the nonlinear action, the shorter the critical time span with smooth changes of external forces. The approximate structure of nonlinear action of climatic atmosphere and ocean is: the nonlinear action decreases usually with increasing latitude, which is related to the role of the Coriolis force in fluid motion (forming geostrophic current); the nonlinear action changes with the anomalous cyclonic or anticyclonic circulation shear, for instance, when the strength of anomalous eastward zonal circulation is comparable to that of anomalous meridional circulation, the nonlinear action is the strongest; wind stress plus gradient forces enhance the nonlinear action, etc..

Key words: structure of nonlinear action, critical time span, circulation shear, predictability

1. Introduction

The establishment (Zebiak and Cane 1987) and development of dynamical numerical coupling models of ocean–atmosphere make the study and forecast of climate more objective and quantitative. There are many aspects to studies on the development of dynamical climate models, including: studies and analysis of the roles of various physical elements and their mechanisms in the evolution of climate mainly by use of dynamical models (Chen et al. 1994; Wang et al. 1995; Guo et al. 2001; Li et al. 2001), parameterization and sensitivity studies of the parameters and physical elements (Chen et al. 1994, 1999; Huang et al. 2001), introduction of new physical and mathematical methods and techniques into dynamical models (Chen et al. 2000; Huang et al. 2001; Yang et al. 2001), etc.. Studies like these contribute significantly to the development of dynamical climate models. The nonlinear property of the climate system makes the model (that is applied to study and predict this system) fertile for much study. Such a research space seems difficult to limit as newly discovered problems, appear, for example nonlinear computational instability (Ji et al. 2001). As we emphasize

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computational facilities and the temporal and spatial density of observations (National Research Council 2001), it seems natural to ask: Is it true, the higher the resolution of the model, the better, even without considering the increased computation caused by the increasing model resolution? Is there a resolution–matching issue between the atmospheric model and the oceanic model when they are coupled, since there are some significant differences between atmospheric and oceanic fluid?

We make a primary attempt to study such issues in this paper according to the evolving and adjusting properties of the real climate atmosphere and oceanic fluid. The kinetic atmosphere and ocean of the climatic scale synchronously evolve and adjust (as shown in Fig. 1). In *the evolution and adjustment (Zeng 1979), external forces, such as gravitation from the Earth,* the Sun, and the Moon, and wind stress, provide the fluids with energy to develop; friction dissipates the kinetic energy; the Coriolis force does not change the energy but the motion of the fluids; the advection and convection transports, and all forms of gradient forces, contribute to the adjustment that is defined by the difference (i.e., the asymmetry) in the spatial distribution of elements, and the adjustment is strengthened or weakened by the strengthening or weakening of the asymmetry. In the practical evolution and adjustment that is a complicated nonlinear process, the atmosphere and ocean will macroscopically reach a certain state at a given temporal and spatial resolution, and under the control of certain conditions (Guo and Chou 1986). Since it is rather difficult, and even impossible presently, for us to get an analytic solution for a certain state directly with traditional analytic methods (Lorenz 1963), we attempt to find a time span within which the slowly developing climatic system would adjust to a certain state beginning with the approximated constant initial adjustment items (all forms of gradient items) and external forces. We can take the initial adjustment items as the undetermined coefficients and get the solution that includes the undetermined coefficients. Further more, we find the undetermined coefficients by iteration. The solution, including the undetermined coefficients, can be used to diagnose the slowly developing climatic system when the form of the solution does not change in the entire time span.

The spatial resolution of a discrete fluid seems to limit the critical time span (the length of predictability). A lower spatial resolution (say, taking the Earth as one point) permits us to make longer prediction with rather exactness. The critical time span also relates to the

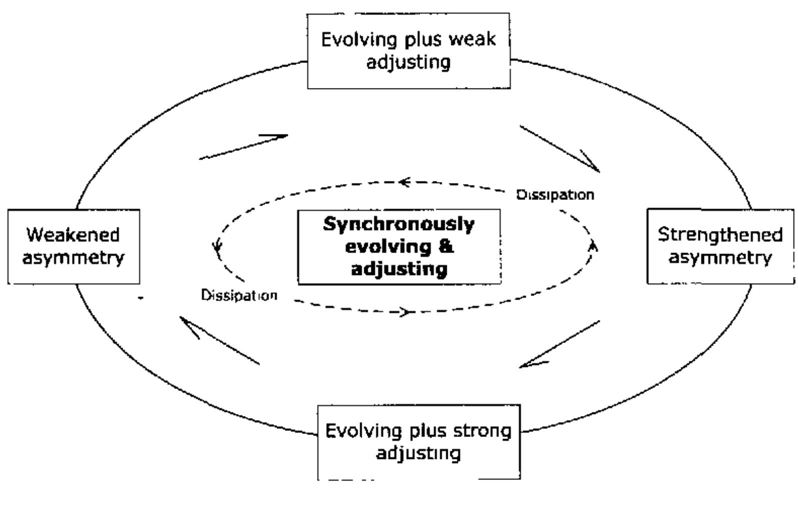

Fig. 1. Synchronously evolving and adjusting kinetic atmosphere and ocean.

nonlinear action structure. This paper will demarcate the strength of the nonlinear action and figure out the approximate nonlinear structure.

2. Simple processing of primary fluid kinetic equations

In a local coordinate system (x, y, z), the primary fluid kinetic equation is

$$\frac{\partial \boldsymbol{V}}{\partial t} + \boldsymbol{V} \cdot \nabla \boldsymbol{V} + (2\boldsymbol{\Omega}_0 + \boldsymbol{\Omega}_r) \times \boldsymbol{V} = -\frac{1}{\rho}\nabla p + \boldsymbol{G}_r + \boldsymbol{F}_{ri} + \boldsymbol{F}_{ro} . \tag{1}$$

Marking $A_{qs} = \frac{\partial q}{\partial s}$ (q is any element, $s = x, y$, or z). We perform the following simplifications: (a) omit the comparatively small curvature force $\boldsymbol{\Omega}_r \times \boldsymbol{V}$ (since $\boldsymbol{\Omega}_r \times \boldsymbol{V} \cdot \boldsymbol{V} \equiv 0$, this omission does not affect the conservation of kinetic energy); (b) assume the studied layer of atmosphere or ocean is dynamically homogeneous in the vertical direction and is not affected by other layers; (c) for low–speed fluids, approximate the intrinsic friction as Rayleigh friction: $|\boldsymbol{F}_{ri}| = k_f|\boldsymbol{V}|$, where k_f is dissipation coefficient; (d) average elements in the space defined by the grids chosen $S_p = \{[x_1, x_2], [y_1, y_2], [z_1, z_2]\}$ (z_1 and z_2 can be the heights of the top and bottom of the atmosphere), i.e., giving element q ($q = u, v, w$ or p) and setting, $\bar{q} = \frac{1}{S_p}\iiint_{S_p} q dS_p$, $q = \bar{q} + q' \approx \bar{q}$ (Such averaging is always done automatically in discrete space). Thus, component forms of Eq.(1) can be written as

$$\frac{du(t)}{dt} + (A_{ux} + k_f)u(t) + (A_{uy} - f)v(t) + f_0 w(t) = F_x(t) , \tag{2}$$

$$\frac{dv(t)}{dt} + (A_{vx} + f)u(t) + (A_{vy} + k_f)v(t) = F_y(t) , \tag{3}$$

$$\frac{dw(t)}{dt} + (A_{wx} - f_0)u(t) + A_{wy}v(t) + k_f w(t) = F_z(t) , \tag{4}$$

where $\boldsymbol{V} = u\boldsymbol{i} + v\boldsymbol{j} + w\boldsymbol{k}$ is the three–dimensional velocity of atmospheric or oceanic fluids; $-\rho^{-1}\nabla P$ the pressure force; $\boldsymbol{G}_r$ the gravitation from the Earth, the Sun and the Moon; $\boldsymbol{F}_{ri}$ the intrinsic friction; $\boldsymbol{F}_{ro}$ the wind stress on ocean (the external friction on atmosphere is omitted); $F_x(t)$, $F_y(t)$ and $F_z(t)$ are the external gradient forces in the x, y and z directions, respectively; $f = 2\Omega\sin(\varphi)$, $f_0 = 2\Omega\cos(\varphi)$, $\Omega = 7.2921 \times 10^{-5}\ \mathrm{s}^{-1}$, φ the latitude angle. The angular velocities of the fluids' motion to the Earth, and the angular velocities of the Earth's rotation, respectively, are

$$\boldsymbol{\Omega}_r = -\frac{v}{r}\boldsymbol{i} + \frac{u}{r}\boldsymbol{j} + \frac{u\mathrm{tg}\varphi}{r}\boldsymbol{k} , \quad \boldsymbol{\Omega}_0 = \Omega\cos\varphi\boldsymbol{j} + \Omega\sin\varphi\boldsymbol{k} .$$

3. The critical time span

3.1 *Definition and meaning of the critical time span*

We define the critical time span as L_t if (a) there exists $L_t > 0$, and L_t is so short that the absolute value of the increment of $A_{qs}(\boldsymbol{V})$ can be omitted since it is much smaller than the absolute value of $A_{qs}(\boldsymbol{V})$ within any interval $[t_i, t_{i+1}]$ whose length is shorter than L_t, and (b) beginning with $A_{qs}(\boldsymbol{V}_i)$ at time t_i, Eqs.(2)–(4) can be adjusted (or converged) to a certain solution.

It is still rather difficult for us to solve Eqs.(2)–(4) directly using the traditional analytic method; whereas the actual properties of evolution and adjustment for climatic atmosphere and ocean (Fig. 1), the presence of critical time span means that: (a) within every interval $[t_i, t_{i+1}]$ whose length is not longer than L_t, we can get the approximate solution of Eqs.(2)–(4) by taking $A_{qs}(V)$ as constant $A_{qs}(V_i)$; (b) since the form of the analytic solutions are the same in every interval $[t_i, t_{i+1}]$ for the given $A_{qs}(V)$ and Eqs.(2)–(4) can be adjusted (or converged) to a certain solution which belongs to the interval $[t_i, t_{i+1}]$, beginning from $A_{qs}(V_i)$ at time t_i, we can further take the initial adjustment items as the undetermined coefficients and approach the real solution by finding the undetermined coefficients through iterations; (c) from the following study, we can know the critical time span is, roughly, in direct proportion to the spatial resolution and in inverse proportion to the nonlinear action of the climatic atmosphere and ocean (E_s). If the nonlinear action of the climatic atmosphere and ocean is very strong and the spatial resolution studied is very high, the critical time span will be very short and, therefore, the corresponding integration step length will be very short. So, related to the spatial resolution required and the strength of nonlinear action, the critical time span of the climatic atmosphere and ocean may represent the relative temporal scale of predictability since the critical time span may be taken as maximum integration step length and the cumulative error is in direct proportion to the number of integration steps. If the spatial resolution in the ocean is required to be 50 kilometers, the corresponding maximum integration step length is about 49.5 hours (Table 1). Provided that the mean error in every integration step is 1% and the total error permitted is 50%, the total effective time length for prediction is about 103 days.

3.2 *Estimation of the critical time span*

Take $DT = \max\limits_{i=1,+\infty}(t_{i+1} - t_i)$, and $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$, $\Delta z = z_2 - z_1$.

According to the above requirements for the time step length, we require

$$DT < \min\left\{\left|\frac{\Delta x}{u}\right|, \left|\frac{\Delta y}{v}\right|, \left|\frac{\Delta z}{w}\right|, L_t\right\} . \quad ①$$

Thus, the differential equations of u, v and w obtained through the simultaneous equations (2)–(4) can be approximately written in the same form as

$$\left(\frac{d^3}{dt^3} + A_1\frac{d^2}{dt^2} + A_2\frac{d}{dt} + A_3\right)U(t) = F_e(t) \tag{5}$$

with every interval $[t_i, t_{i+1}]$ (since A_1, A_2, A_3 are taken as constants).
Here

$$A_1 = 3k_f + D_i, \quad A_2 = k_f(3k_f + 2D_i) + O_m + fR_0 + A_{uv} - f_0 A_{wx} ,$$
$$A_3 = k_f[O_m + k_f(k_f + D_i) + fR_0 + A_{uv} - f_0 A_{wx}] + f_0(f_0 A_{vy} + fA_{wy} + A_{vw}) ,$$
$$U = \{u, v, w\} ,$$
$$F_e = \{F_u(t), F_v(t), F_w(t)\} ,$$
$$F_u(t) = k_f(A_{vy} + k_f)F_x(t) + [A_{wy}f_0 + k_f(f - A_{uy})]F_y(t) - f_0(A_{vy} + k_f)F_z(t)$$
$$+ (A_{vy} + 2k_f)F'_x(t) + (f - A_{uy})F'_y(t) - f_0 F'_z(t) + F''_x(t) ,$$
$$F_v(t) = -k_f(A_{vx} + f)F_x(t) + [(f_0 - A_{wy})f_0 + (k_f + A_{ux})k_f]F_y(t) + f_0(A_{vx} + f)F_z(t)$$

$$- (A_{vx} + f)F'_x(t) + (A_{ux} + 2k_f)F'_y(t) + F''_y(t) ,$$

$$F_w(t) = [A_{vw} + fA_{wy} - k_f A_{wx} + f_0(A_{vy} + k_f)]F_x(t) - [A_{uw} + fA_{wx} + f_0(A_{uy} - f) + k_f A_{wy}]F_y(t) + [A_{uv} + f(f + R_0) + k_f(D_i + k_f)]F_z(t) + (f_0 - A_{wx})F'_x(t) - A_{wy}F'_y(t) + (D_i + 2k_f)F'_z(t) + F''_z(t) ,$$

$D_i = A_{ux} + A_{vy}$, the horizontal divergence

$R_0 = A_{vx} - A_{uy}$, the vertical vortex

$A_{uv} = A_{ux}A_{vy} - A_{uy}A_{vx}$, circle index of u and v in $x-y$ plane

$A_{uw} = A_{ux}A_{wy} - A_{uy}A_{wx}$, circle index of u and w in $x-y$ plane

$A_{vw} = A_{vx}A_{wy} - A_{wx}A_{vy}$, circle index of v and w in $x-y$ plane

Also

$$s_{g1} = -\frac{A_1^2}{9} + \frac{A_2}{3} , \quad s_{g2} = -\frac{A_1^3}{27} + \frac{A_1A_2}{6} - \frac{A_3}{2} , \quad s_g = s_{g1}^3 + s_{g2}^2 .$$

$$A = -\frac{1}{2}[(-\sqrt{s_g} + s_{g2})^{1/3} + (\sqrt{s_g} + s_{g2})^{1/3}] - \frac{A_1}{3}$$

$$B = \frac{\sqrt{3}}{2}[-(-\sqrt{s_g} + s_{g2})^{1/3} + (\sqrt{s_g} + s_{g2})^{1/3}] ,$$

$$C = (3A + A_1)^2 + B^2 .$$

$$D = 2A + A_1 .$$

By the SI unit system, the maximum possible extent of the elements of climatic atmosphere and ocean is (the practical extent is a little smaller):

Atmosphere: $-20 < u, v < 20$, $-5 < w < 5$, $10^{-7} \leqslant k_f \leqslant 3.0 \times 10^{-4}$,
$O(F_x, F_y) \leqslant 10^{-3}$, $O(F_z) \leqslant 10^{-6}$,

Ocean: $-5 < u, v < 5$, $-1 < w < 1$, $6.4 \times 10^{-6} \leqslant k_f \leqslant 6.4 \times 10^{-3}$,
$O(F_x, F_y) \leqslant 10^{-5}$, $O(F_z) \leqslant 10^{-6}$.

$DL = \min(dx, dy)$, the spatial scale or difference step length. $DL > 200$ km.
In this climatic fluid system, the extent of the concerned functions metioned above is:
For atmosphere:

$$s_{g1} \approx 7.086 \times 10^{-9}, \quad -8.470 \times 10^{-22} \leqslant s_{g2} \leqslant 0, \quad s_g \approx 3.5578 \times 10^{-25},$$
$$-1.00 \times 10^{-4} \leqslant A \leqslant -1.00 \times 10^{-7}, \quad B \approx 1.45796 \times 10^{-4},$$
$$C \approx 2.1256 \times 10^{-8}, \quad 1.00 \times 10^{-7} \leqslant D \leqslant 1.00 \times 10^{-4} . \tag{6a}$$

For ocean:

$$s_{g1} \approx 7.086 \times 10^{-9} , \quad 0 \leqslant s_{g2} \leqslant 2.78 \times 10^{-17} , \quad s_g \approx 3.559 \times 10^{-25} ,$$
$$-6.4 \times 10^{-3} \leqslant A \leqslant -6.4 \times 10^{-6} , \quad B \approx 1.458 \times 10^{-4} ,$$
$$C \approx 2.1256 \times 10^{-8} , \quad 6.4 \times 10^{-6} \leqslant D \leqslant 6.4 \times 10^{-3} . \tag{6b}$$

Since,

$$s_g > 0 \tag{②}$$

the solution of Eq.(5) within every interval $[t_i, t_{i+1}]$ is:

$$U = C_1 e^{-Dt} + C_2 e^{At}\cos(Bt) + C_3 e^{At}\sin(Bt) + \frac{e^{-Dt}}{C}\int e^{Dt} F_e(t)dt$$
$$+ \frac{e^{At}}{C}\cos(Bt)\int e^{-At}\left[\cos(Bt) + \frac{3A+A_1}{B}\sin(Bt)\right]F_e(t)dt$$
$$+ \frac{e^{At}}{C}\sin(Bt)\int e^{-At}\left[\frac{3A+A_1}{B}\cos(Bt) - \sin(Bt)\right]F_e dt \ .$$

Here $C_m = \{c_{m1}, c_{m2}, c_{m3}\}$, c_{m1}, c_{m2} and c_{m3} are integral constants, $m = 1, 2, 3$. Since $A < 0, B > 0, C > 0, D > 0$, $C_1 e^{-Dt} + C_2 e^{At}\cos(Bt) + C_3 e^{At}\sin(Bt)$ decreases to zero with time. After a long enough time,

$$U = \frac{e^{At}}{C}\sin(Bt)\int e^{-At}\left[\frac{3A+A_1}{B}\cos(Bt) - \sin(Bt)\right]F_e(t)dt + \frac{e^{-Dt}}{C}\int e^{Dt} F_e(t)dt$$
$$+ \frac{e^{At}}{C}\cos(Bt)\int e^{-At}\left[\cos(Bt) + \frac{3A+A_1}{B}\sin(Bt)\right]F_e(t)dt \ . \qquad (7)$$

Set $U = U_i$ when $t = t_i$. If all the $A_{qs}(U)$ are always constant, what we get from Eq.(7) are just the analytic solutions of equations (2)–(4), approximately. Otherwise, the right side of equation (7) is the function U. Usually, we can further take the initial adjustment items as the undetermined coefficients and approach the real solution, which includes the undetermined coefficients, finding the undetermined coefficients through iteration. Here, we estimate the critical time span by checking the convergence of the iterations. Eq.(7) can be written as

$$U \approx U_i + \frac{e^{-Dt}}{C}\int_{t_i}^{t} e^{D\tau} F_e(\tau)d\tau + \frac{e^{At}}{BC}\cos(Bt)\int_{t_i}^{t} e^{-A\tau}[B\cos(B\tau)$$
$$+ (3A + A_1)\sin(B\tau)]F_e(\tau)d\tau + \frac{e^{At}}{BC}\sin(Bt)\int_{t_i}^{t} e^{-A\tau}[3A$$
$$+ A_1)\cos(B\tau) - B\sin(B\tau)]F_e(\tau)d\tau \ . \qquad (8)$$

In practice, we get U on time t_{i+1} from those values on t_i. Within time span $DT = t_{i+1} - t_i$, we require that $|F_e(t)|$ does not change too abruptly with the given $F_x(t)$, $F_y(t)$ and $F_z(t)$, that is:

$$|F_e(t_{i+1}) - F_e(t_i)| / F_{em} \ll 1 \ , \qquad ③$$

where F_{em} is the maximum absolute value of $|F_e(t)|$ in the entire space studied. If $|F_e(t)| \equiv 0$ in the entire space studied, it naturally meets ③.

By use of the mid–value theorem of integration, Eq.(8) can be further written as

$$U = U_i + \frac{DT}{C}\{F_e(t_{m1}) + \cos(Bt)[\cos(Bt_{m2}) + \frac{3A+A_1}{B}\sin(Bt_{m2})]F_e(t_{m2})$$
$$+ \sin(Bt)[\cos(Bt_{m3}) + \frac{3A+A_1}{B}\sin(Bt_{m3})]F_e(t_{m3})\}$$
$$\approx U_i + \frac{DT}{C}[1 + \cos(2Bt) + \frac{3A+A_1}{B}\sin(2Bt)]F_e(t)$$
$$= U_i + \frac{DT}{C}[1 + \frac{\sqrt{C}}{B}\sin(2Bt + a_c)]F_e(t) \ .$$

Here, $a_c = \mathrm{arctg}[B/(3A + A_1)] \approx$ const., $t_i < t_{mj} < t \leqslant t_{i+1}$ ($j = 1, 2, 3$).

If any element or function W_w resulting from the nth iteration does not remain constant, it is marked as $W_w^{(n)}$. $|W_w|$ is its absolute value. Thus:

$$\left|U^{(n+1)} - U^{(n)}\right| = \left|\frac{DT}{C}[1 + \frac{\sqrt{C}}{B}\sin(2Bt + a_c)][F_e^{(n+1)} - F_e^{(n)}]\right|$$
$$\leqslant \frac{DT}{C}(1 + \frac{\sqrt{C}}{B})\left|F_e^{(n+1)} - F_e^{(n)}\right| .$$

In the numerical processing, if necessary, we should reasonably make some weighted average of elements between two adjacent iterative steps in order to make them approach the limit smoothly, which will reduce the numerical effects. Concretely speaking,

$$F_e^{(n+1)} - F_e^{(n)} \approx \frac{\partial F_e^{(n)}}{\partial U}(U^{(n)} - U^{(n-1)}) . \quad ④$$

Further,

$$\left|U^{(n+1)} - U^{(n)}\right| \leqslant C_{or}^n \left|U^{(1)} - U^{(0)}\right| .$$

The sufficient condition for convergence of the iterative process is,

$$C_{or} < 1 \quad ⑤$$

Here,

$$C_{or} = \max\{\frac{DT}{C}(1 + \frac{\sqrt{C}}{B})\left|\frac{\partial F_e}{\partial U}\right|\} \leqslant \frac{DT}{DL}E_s\frac{1}{C}(1 + \frac{\sqrt{C}}{B})$$
$$\approx 9.41 \times 10^7 E_s \frac{DT}{DL} . \quad (9)$$

And,

$$E_s = \sqrt{E_f} ,$$
$$E_f \approx [(f_0 F_y + fF_z)^2 + 2F_y^2 k_f^2]\cos(A_x)^2 + \{f^2 F_z^2 + (f_0 F_x + F_z k_f)[f_0 F_x + F_z (k_f - 2fS_i)]\}\sin(A_x)^2 - (f_0 F_y + fF_z)(fF_z S_i - f_0 F_x - F_z k_f)\sin(2A_x) , \quad (10)$$

$$S_i = \frac{\mathrm{sign}[\frac{\partial|\sin(A_x)|}{\partial x}]}{\mathrm{sign}[\frac{\partial|\cos(A_x)|}{\partial y}]} = \pm 1 , \quad A_x = \arccos\left(\frac{Du}{|DU|}\right)$$

We can estimate the critical time span as

$$L_t \leqslant DL/(9.41 \times 10^7 E_s) . \quad (11)$$

From the following analysis, we have $E_s \leqslant 2.6 \times 10^{-7}$ (for atmosphere) and 3.0×10^{-9} (for ocean). There exists a minimum critical time span (Table 1) under conditions ① – ⑤, which enables us to obtain the solution of equations (2) to (4) by means of a combination of numerical and analytic methods. For the same spatial resolutions or scales of the atmosphere and ocean, the minimum critical time span of the ocean is about 90 times of that of the atmosphere; as far as the same characteristic spatial scale is concerned, the minimum critical time span or the relative temporal scale of predictability of the ocean is about 9 times of that of atmosphere. So, in a suited coupling model of atmosphere–ocean, the spatial resolution of the ocean model should be much higher than that of the atmosphere.

Table 1. Spatial resolution (scale) DL and the corresponding critical time span L_t

Atmosphere	DL (km)	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500
	L_t (h)	1.13	2.27	3.40	4.54	5.67	6.81	7.94	9.08	10.2	11.3	12.4	13.6	14.7	15.9	17.0
Ocean	DL (km)	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750
	L_t (h)	49.5	99.0	148	198	246	295	344	393	442	492	542	592	639	688	738

4. Approximate structure of the nonlinear action of the climatic atmosphere and ocean

From equation (11), we can see the critical time span is in direct proportion to the spatial resolution studied and in inverse proportion to E_r. We will see that E_s represents the strength of the nonlinear action of the climatic atmosphere and ocean.

4.1 *Expression of the strength of nonlinear action*

R is an element ($R = u, v, w$ or p) to be obtained from the solution of equations, and c is a coordinate (x, y, z or t). The R–equation can be written as

$$R = F[R(c), S(c)] \ . \tag{12}$$

Here,

$$dR = \left(\frac{\partial F}{\partial R}\frac{\partial R}{\partial c} + \frac{\partial F}{\partial S}\frac{\partial S}{\partial c}\right)dc = V_{RS}\, dc \ .$$

where, $\frac{\partial S}{\partial R} = 0$. $S(c)$ can be a function of external force, and $V_{RS} = \frac{\partial F}{\partial R}\frac{\partial R}{\partial c} + \frac{\partial F}{\partial S}\frac{\partial S}{\partial c}$.
If $\frac{\partial V_{RS}}{\partial c} = 0$, Eq.(12) is linear about c; otherwise, Eq.(12) is nonlinear about c. The absolute value of the n–order (about c) derivative of V_{RS} ($n \geqslant 1$) (marked as $E_{RS}^{(n)}$) represents the strength of the n–order (about c) nonlinear action. The first order nonlinear action strength is

$$E_{RS} = E_{RS}^{(1)} \left| \frac{\partial^2 F}{\partial R \partial c}\frac{\partial R}{\partial c} + \frac{\partial F}{\partial R}\frac{\partial^2 R}{\partial c^2} + \frac{\partial^2 F}{\partial S \partial c}\frac{\partial S}{\partial c} + \frac{\partial F}{\partial S}\frac{\partial^2 S}{\partial c^2} \right|$$

$$= \left| \left(\frac{\partial F}{\partial R} + \frac{\partial F}{\partial S}\right)\frac{\partial^2 R}{\partial c^2} + \frac{\partial^2 F}{\partial R^2}\left(\frac{\partial R}{\partial c}\right)^2 + 2\frac{\partial^2 F}{\partial R \partial S}\frac{\partial S}{\partial c}\frac{\partial R}{\partial c} + \frac{\partial^2 F}{\partial S^2}\left(\frac{\partial S}{\partial c}\right)^2 \right| \ .$$

As far as a real geophysical fluid is concerned, at least for the large– or climate–scale geophysical fluid with smoothly changing external forces, there exists the following relationships:

$$\left|\frac{\partial S}{\partial c}\right|,\ \left|\frac{\partial R}{\partial c}\right|,\ \left|\frac{\partial F}{\partial S}\right|,\ \left|\frac{\partial F}{\partial R}\right| \ll 1 .$$

These relationships can be easily derived from any classical geophysical fluid dynamics theory containing scale analysis. For example, $\left|\frac{\partial u}{\partial x}\right| \ll 1$. So,

$$E_{RS} \approx \left|\left(\frac{\partial F}{\partial R}+\frac{\partial F}{\partial S}\right)\frac{\partial^2 R}{\partial c^2}\right| . \tag{13}$$

This means the strength of the first order (about c) nonlinear action is mainly defined by $\left(\frac{\partial F}{\partial R}+\frac{\partial F}{\partial S}\right)\frac{\partial^2 R}{\partial c^2}$. Here, $\left|\frac{\partial F}{\partial R}+\frac{\partial F}{\partial S}\right|$ represents the temporal and spatial structure of the strength of the first order (about c) nonlinear action of the system described by the equation; $\left|\frac{\partial^2 R}{\partial c^2}\right|$ is the temporal and spatial structure of the strength of the first order (about c) nonlinear action of the element R itself.

For a given coordinate c, getting R from Eq.(12) by the iteration process, $S(c)$ stays the same. Mark $R^{(i)}$ as the value of R from the ith iteration, then,

$$\begin{aligned}|R^{(i+1)}-R^{(i)}| &= |F(R^{(i)},S)-F(R^{(i-1)},S)| \approx \left|\frac{\partial F}{\partial R}(R^{(i)}-R^{(i-1)})\right| \\ &\approx \left|\left(\frac{\partial F}{\partial R}\right)^i\right| |R^{(1)}-R^{(0)}| .\end{aligned} \tag{14}$$

The sufficient and necessary condition for iteartion convergence is: $\left|\frac{\partial F}{\partial R}\right| < 1$. The smaller the $\left|\frac{\partial F}{\partial R}\right|$, the faster the convergent rate of the iteration. For smoothly changing external forces, $\left|\frac{\partial F}{\partial R}\right|$ plays the main role in the strength of the first order (about c) nonlinear action of the system described by the equation. So, checking Eqs.(9), (11), (13) and (14), we can see the E_s represents the first–order strength of the nonlinear action of the climatic atmosphere and ocean. At least for the large– or climatic–scale geophysical fluids with smoothly changing external forces, the bigger the E_s, the stronger the first order (about c) strength of the nonlinear action of the system described by the equation. When E_s is zero, the climatic fluid system is linear, and the critical time span can be infinitely large.

4.2 *The approximate structure of nonlinear action*

The following two–dimensional figures were chosen from hundreds of figures to be ergodic after grouping by structure. So, there are no concrete scales on the E_s–axes. The "TOP" values mainly change with A_x. Du and DU are anomalies of u and U in the iteration, respectively. In actual evolving and adjusting kinetic geophysical fluids, Du and DU can be the anomalies of u and U. The A_x is the angle between Du and DU, representing the comparison of strengths between anomalous zonal and meridional circulations. The S_i can be regarded as the properties of the horizontal shear of the anomalous circulation (Fig. 2). $S_i = 1$ means that the anomalous cyclonic or anticyclonic shear of the circulation changes contrarily

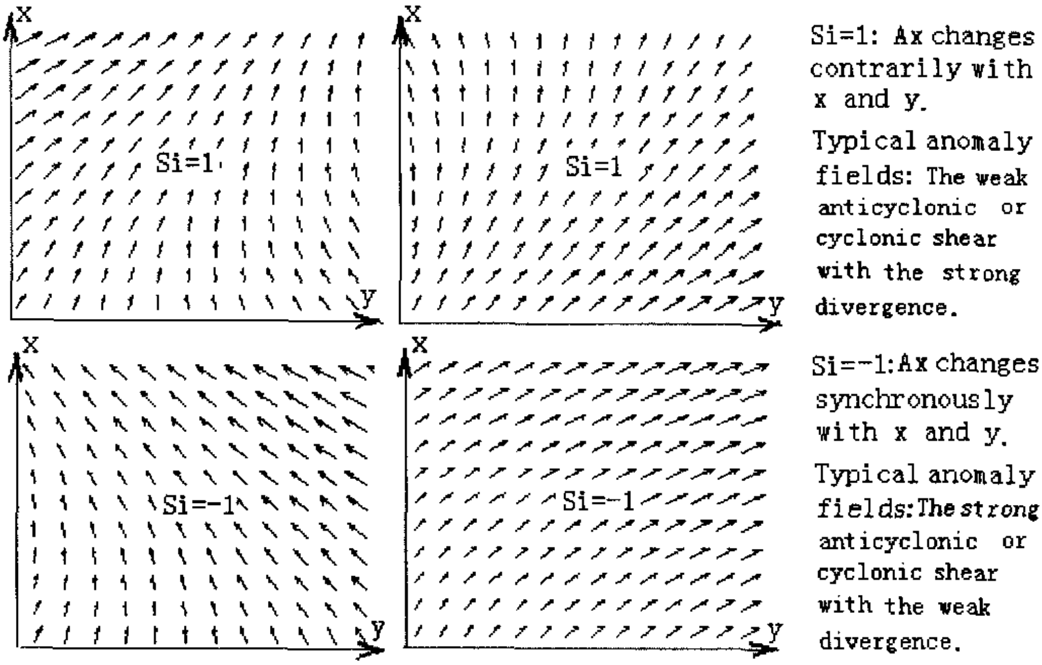

Fig. 2. S_i and the corresponding properties of the anomaly fluid fields.

along x and y, $S_i = -1$ means that the anomalous cyclonic or anticyclonic shear of the circulation changes synchronously along x and y.

4.2.1 *The effect of resultant external force $F_{\tau+g}$, A_x, φ and S_i on E_s (Fig. 3)*

E_s increases with $|F_{\tau+g}|$, i.e., wind stress plus gradient forces enlarge the nonlinear action.

The top value of E_s increases with the decreasing of the anomalous eastward zonal circulation (larger than that of the anomalous meridional circulation) until reaching the maximum value when the size of the anomalous eastward zonal circulation equals to that of the anomalous meridional one ($A_x = \pi/4$). Then, the top value of E_s begins to decrease with the continual decreasing of the anomalous eastward zonal circulation (including the increasing of the westward one) until reaching the minimum value when the size of the anomalous westward zonal circulation equals to that of the anomalous meridional circulation ($A_x = 3\pi/4$). The further increasing of the anomalous westward zonal circulation results in the increasing of the top value of E_s.

Usually, E_s decreases with $|\varphi|$, maximum at the equator and minimum at the poles (Case 1). The Coriolis force is the main E_s–contributing element. Its defining fluid motion (forming the geostrophic current) weakens the nonlinear action. When $A_x = 3\pi/4$, E_s changes little with φ (Case 2). If the dissipation K_f is small enough (less than 10^{-6}), S_i will affect the structure of E_s this way (Case 3 and Case 4): the maximum nonlinear action occurs in middle latitudes but not at the equator, and a westward drive plus $S_i = 1$ (e.g., the weak anomalous cyclonic/anticyclonic circulation with strong anomalous divergence or convergence) or an eastward drive plus $S_i = -1$ (e.g., the strong anomalous cyclonic/anticyclonic circulation with weak anomalous divergence or convergence) will increase the nonlinear action in the Northern Hemisphere (the reverse in the southern one).

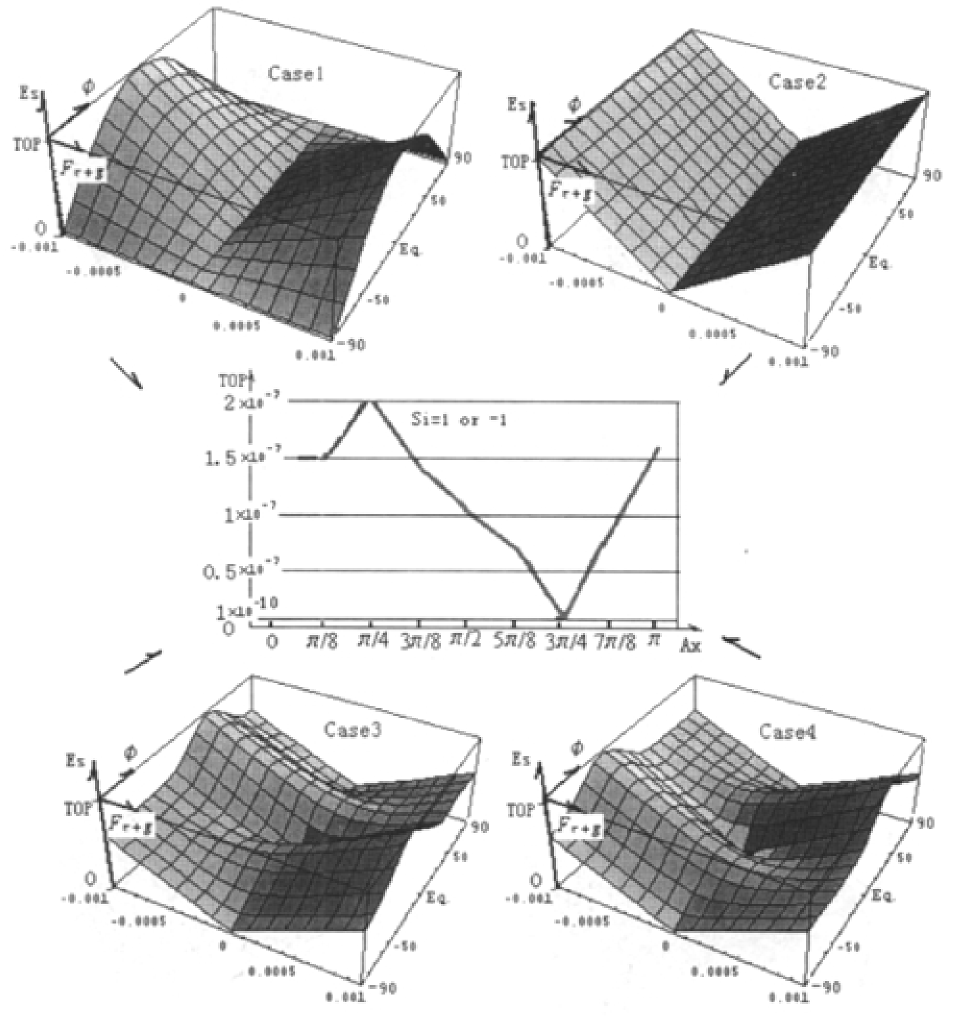

Fig. 3. The spatial and mathematical structure of E_s (TOP is the maximum E_s). Case 1 is the space $\{S_i = 1$ or -1, $A_x = 0$ to π, $k_f = 10^{-7}, 10^{-6}, 10^{-5}$ or 10^{-4}, $\varphi = -\pi/2$ to $\pi/2$, Case 2 $\{A_x = 3\pi/4$, $k_f = 10^{-6}, 10^{-5}$, or $10^{-4}\}$; Case 3 $\{S_i = 1$, $A_x = 3\pi/4$ and $k_f = 10^{-7}\}$; Case 4 $\{S_i = -$, $A_x = 3\pi/4$, $k_f = 10^{-7}\}$. The curve in the middle shows the changing of top values of E_s in the other four figures with A_x.

4.2.2 *The structure of E_s in $t - A_x$ plane (Fig. 4 and Fig. 5)*

Usually, the top value of E_s decreases with $|\varphi|$ (providing the symmetry between the Southern and Northern Hemispheres). Except when $\varphi = \pi/2$, the maximum nonlinear action occurs when the strength of the anomalous eastward zonal circulation equals to that of the anomalous meridional circulation ($A_x = \pi/4$), and the minimum nonlinear action takes place when the strength of the anomalous westward zonal circulation equals to that of the anomalous meridional circulation ($A_x = 3\pi/4$). At the poles, E_s reaches a minimum when

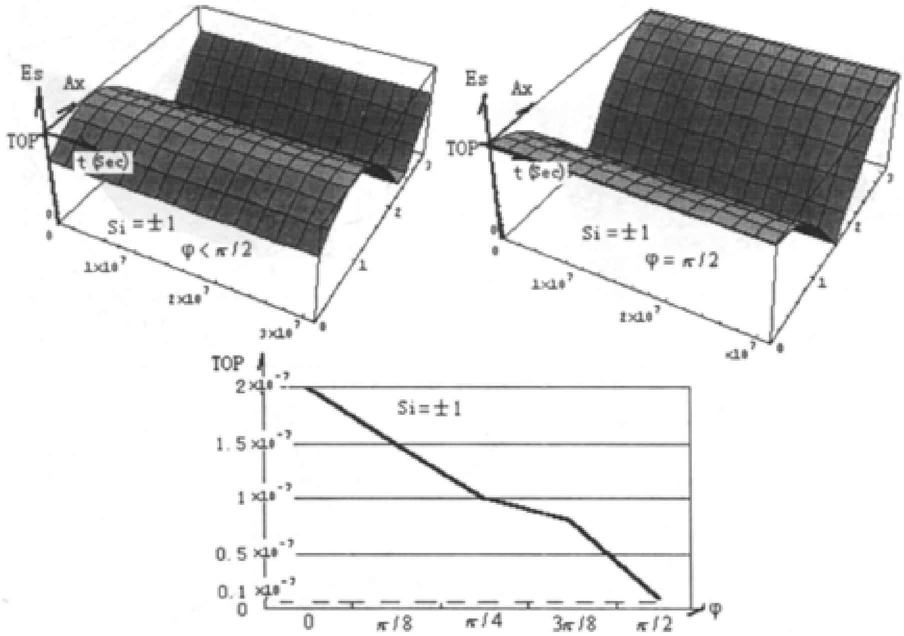

Fig. 4. Atmospheric E_s structure in $t-A_x$ plane with $k_f = 10^{-5}$ and $F_{t+g} = 10^{-3}$.

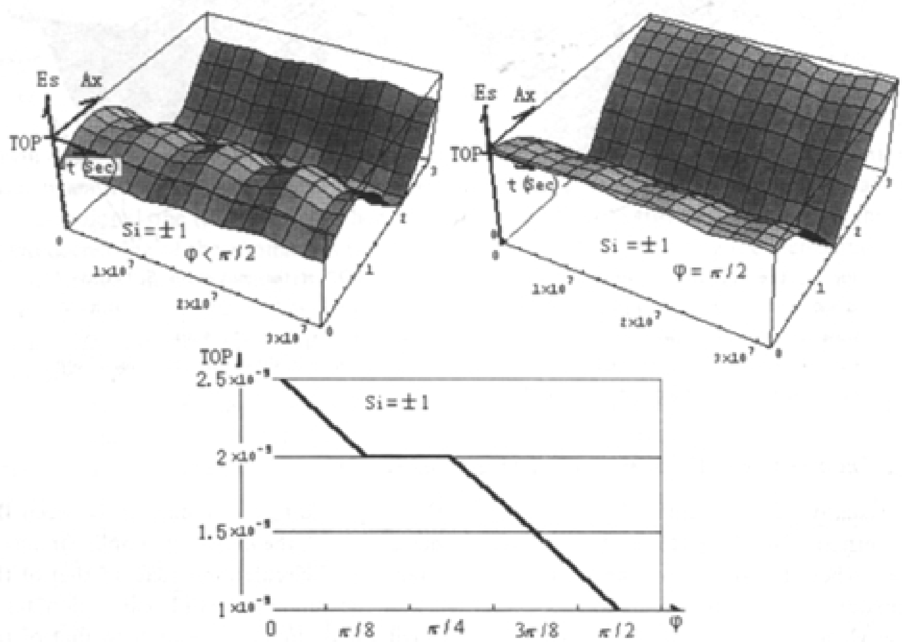

Fig. 5. Oceanic E_s structure in $t-A_x$ plane with $k_f = 10^{-4}$ and $F_{t+g} = 10^{-5}$.

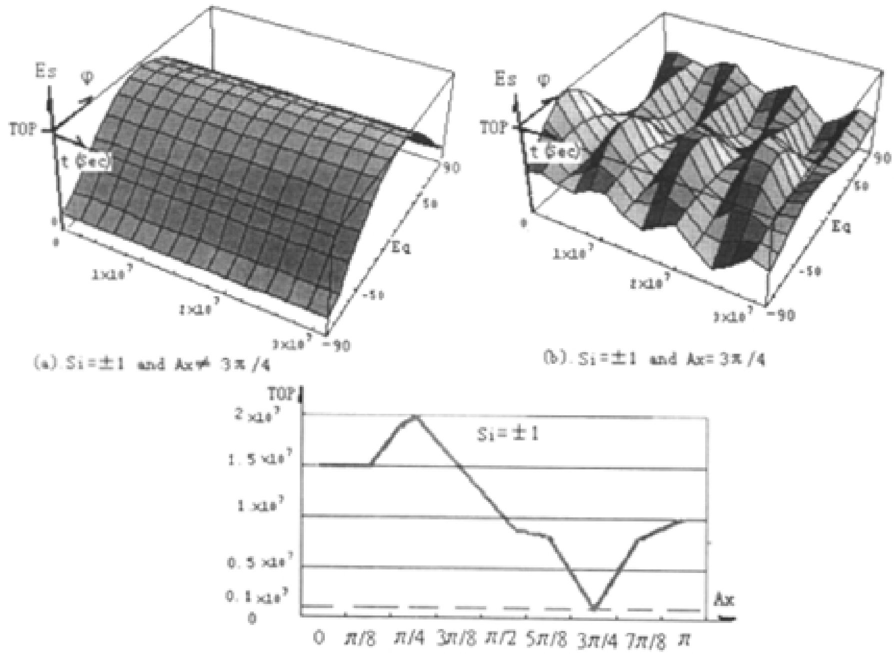

Fig. 6. Atmospheric spatial and temporal structure of E_s with $k_f = 10^{-5}$ and $F_{\tau+g} = 10^{-3}$.

$A_x = \pi/2$ (without anomalous zonal circulation). With time, E_s will change because the driving force changes. Here, we give the maximum possible $F_{\tau+g}$ since E_s always increases with $|F_{\tau+g}|$. The effect of other small forces, for example, the attractions of the Sun and the Moon, can be clearly seen when $|F_{\tau+g}|$ is small enough (for example, in the oceanic circulation).

4.2.3 *The spatial and temporal structure of E_s (Fig. 6 and Fig. 7)*

The general structure of E_s in space $\{A_x, \varphi, t\}$ is the same as what is discussed above. Additionally, when the strength of the anomalous westward zonal circulation equals to that of the anomalous meridional one ($A_x = 3\pi/4$) or when $|F_{\tau+g}|$ is small enough, the effect of the attractions of the Sun and the Moon will show more clearly. $|F_{\tau+g}|$ is always small in oceanic fluid, so the effect of the attractions of the Sun and the Moon will be more significant than in the atmospheric fluids. Therefore, the dynamic effects of the Sun and the Moon should be considered in the theoretical and numerical modeling of the climate for the atmosphere and ocean.

5. Discussion and conclusions

Related to the spatial resolution required and the strength of nonlinear action, the critical time span of the climatic atmosphere and ocean may represent the relative temporal scale of predictability. As far as the same characteristic spatial scale is concerned, the minimum critical time span or the relative temporal scale of predictability of the ocean is about 9 times of that of the atmosphere, several days or more. In the climatic ocean and atmosphere, it is

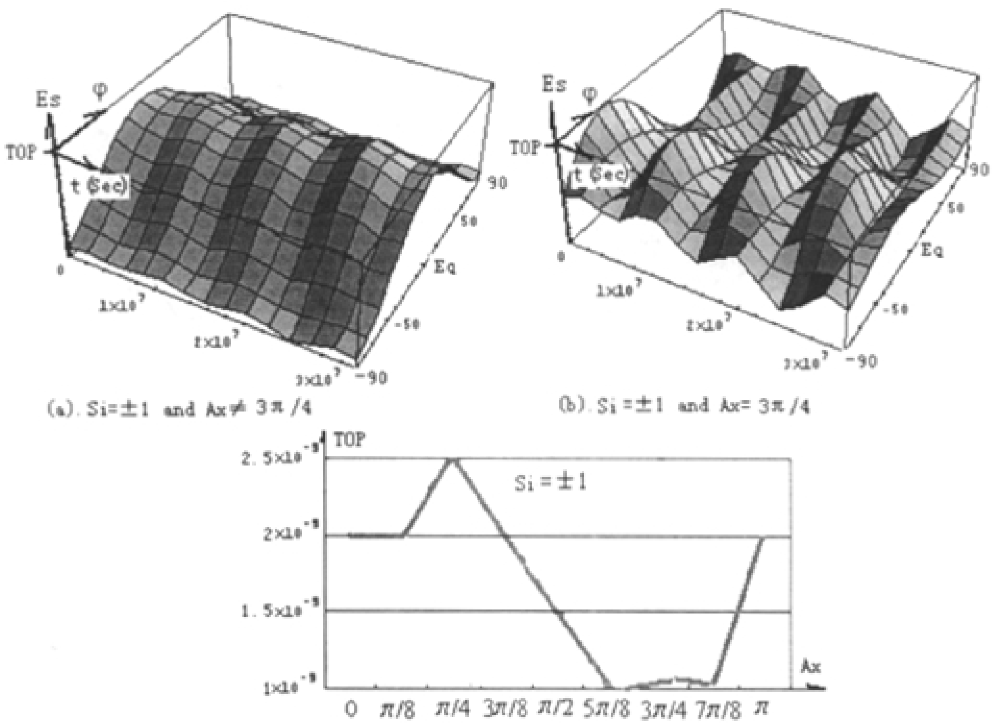

Fig. 7. Oceanic spatial and temporal structure of E_s with $k_f = 10^{-4}$ and $F_{\tau+g} = 10^{-5}$.

conditionally possible for us to obtain the approximate solution of some complicated nonlinear equations by combining analytic and numeric technigues. Our study seems to conduce to theoretical research and numerical modeling by expanding the complexity of the nonlinear mathematical–physical model to include more necessary elements and processes. In a suited coupling model of atmosphere–ocean, the spatial resolution of the ocean model should be much higher than that of the atmosphere, considering the comparison of the minimum critical time span of the atmosphere and that of the ocean.

We also study the approximate structure of nonlinear action of climatic atmosphere–ocean and the relationship among the strengths of nonlinear action, spatial resolution, and the critical time span. At least for the geophysical fluids with large or climatic scales and with smoothly changing external forces, the stronger the nonlinear action or the higher the spatial resolution, the shorter the critical time span. For the linear atmosphere / ocean with certain spatial resolution required, the critical time span can be infinite.

The approximate structure and some properties of nonlinear action of the climatic atmosphere and ocean are as follows. The wind stress plus gradient forces enlarge the nonlinear action; the strength of the nonlinear action of the climate ocean and atmosphere is directly related to the shear structure of the anomalous currents and to the comparison between the strengths of the anomalous meridional and zonal currents. Concretely speaking, the nonlinear action increases with the decreasing strength of the anomalous eastward zonal circulation (larger than that of the anomalous meridional circulation) until reaching a maximum value when the strength of the anomalous eastward zonal circulation equals that of the anomalous meridional one, and then begins to decrease with the continually decreasing strength of the anomalous eastward zonal circulation (or in other words, the increasing of the westward one) until reaching a minimum value when the strength of the anomalous westward zonal circulation equals that of the anomalous meridional circulation. Usually, the nonlinear action decreases as latitude increases, maximum being at the equator and minimum at the poles. The Coriolis force, increasing with latitude weakens the nonlinear action by geostrophic motion. If the nonlinear action is weak or the dissipation is small enough (less than 10^{-6}), the shear structure of the anomalous circulation will significantly affect the structure of the nonlinear action. In other words, the maximum nonlinear action occurs in the mid-latitudes, not at the equator. The westward drive plus the anomalous cyclonic/anticyclonic circulation shear synchronously along x and y will increase nonlinear action in the Northern Hemisphere (with the reverse in the Southern Hemisphere). When the strength of the anomalous westward zonal circulation equals that of the anomalous meridional one or when wind stress plus gradient forces are small enough, the weak external forcings (for example, attractions of the Sun and the Moon) will show more clearly. That seems to say the dynamical effects of the Sun and the Moon should be considered in the theoretical study and numerical modeling of the climate for atmosphere and ocean.

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气候大气和海洋的临界时间跨度与非线性作用结构

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摘　要

研究了气候海洋与大气的临界时间跨度及其非线性作用的大致结构。与预报的空间分辨率及系统的非线性强度相联系，气候大气和海洋的临界时间跨度可反映系统可预报的相对时间尺度。对于具有同样空间特征尺度的大气和海洋，海洋的最小临界时间跨度约是大气的9倍(可达数日至数十日)。一般(外源变化缓慢的)气候海洋与大气的一阶非线性越强，其临界时间跨度越小。气候海洋与大气非线性作用的大致结构是：通常与科里奥利力对流体运动的规范作用(如地转运动)有关，非线性作用随纬度增加而减弱。距平流场的切变结构及其沿经向与纬向上强度的比较直接改变气候大气和海洋的非线性作用(比如，向东的距平环流强度与经向环流强度相当时，非线性作用最强)，较强的外部驱动(风应力和压强梯度力)使非线性作用加大等等。

关键词：非线性作用结构，临界时间跨度，环流切变，可预报性