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Application of Linear Thermodynamics to the Atmospheric System. Part II: Exemplification of Linear Phenomenological Relations in the Atmospheric System

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ABSTRACT

The linear phenomenological relations in the atmospheric boundary layer are proved indirectly using observational facts to combine linear thermodynamic theory and similarity theory in the boundary layer. Furthermore, it is proved that the turbulent transport coefficients are in proportion to the corresponding linear phenomenological coefficients. But the experimental facts show that the linear phenomenological relations are not tenable in the atmospheric mixing layer because the turbulent transport process is an intense non-linear process in the mixing layer. Hence the convection boundary layer is a thermodynamic state in a non-linear region far from the equilibrium state. The geostrophic wind is a special cross-coupling phenomenon between the dynamic process and the thermodynamic process in the atmospheric system. It is a practical exemplification of a cross-coupling phenomenon in the atmospheric system.

Key words: atmospheric system, linear thermodynamics, linear phenomenological relations, turbulent transportation coefficient, atmospheric boundary layer

1. Introduction

The establishment of Onsager's reciprocal relation and Prigogine's principle of minimum entropy production marks the maturity of the theory of linear thermodynamics of the non-equilibrium state. The major results of the theory have important applications to many transportation phenomena (Li 1986). Hu (2002) studied the application of the linear thermodynamics of the non-equilibrium state to the atmospheric system to get a series of important results that are rarely realized. The relations between the turbulent transportation coefficient in the K turbulence closure theory and the linear phenomenological coefficient are deduced; it is proved in the theory that there exists a cross-coupling between the turbulent transportation of heat and vapor. The turbulence intensity theorem is also demonstrated. It shows that the heterogeneity of the spatial distribution of velocity and potential temperature is the cause of turbulence; the turbulence intensity is in proportion to the scalar product of the velocity and the potential temperature in an atmosphere of incompressible gas and isotropic turbulence. Furthermore, the atmospheric vortex theorem is proved; the velocity vorticity equals the vector product of the velocity and the potential temperature gradient. Consequently, the shear of the potential temperature will lead to a vortex movement or

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sundry circulation movements. These theoretic results guide and inspire the application of linear thermodynamics at a non–equilibrium state to the atmospheric system.

The linear phenomenological coefficients must be obtained from the experiments of the linear thermodynamics of the non–equilibrium state. Moreover, the properties of the linear phenomenological coefficients decide the properties of the linear thermodynamic system. But up to now we have not referred to experimental study about the linear phenomenological relations and the linear phenomenological coefficients of the atmospheric system. It is obviously important to develop experimental studies of the atmospheric linear thermodynamics of the non–equilibrium state. But such experiments often require a specialty scheme and demand a higher degree of accuracy for the observations. It is possible at present to prove the linear phenomenological relations of the atmospheric thermodynamics system and to obtain the linear phenomenological coefficients by using results already obtained from current atmospheric study; this is even a convenient way. This study will contribute to the experimental study of the linear thermodynamic phenomena of the atmosphere by studying the linear phenomenological relations and determining the linear phenomenological coefficients of the atmospheric system using observational experiment results in the atmospheric boundary layer. Also, the relation between geostrophic wind and thermal wind is studied from the point of view that the linear thermodynamics of the non–equilibrium state is a widening of the cross–coupling concept.

The entropy equilibrium equation of the atmospheric system can be deduced, using the thermodynamic principle of the non–equilibrium state and the atmospheric physical behaviour (Hu 1999), as follows:

$$\frac{\partial}{\partial t}(\rho s)= -\frac{\partial}{\partial x_j}\boldsymbol{J}_{sj}+\sigma+\sigma_g \ . \tag{1}$$

in which the entropy flow, the entropy production, and the dynamic entropy production are, respectively,

$$\boldsymbol{J}_{sj}=\rho s\boldsymbol{U}_j+\frac{1}{\theta}\boldsymbol{J}_{\theta j}+\frac{\Delta\mu}{T}\boldsymbol{J}_{vj}-\frac{\boldsymbol{U}_i}{T}\tau_{ij} \ , \tag{2}$$

$$\sigma=\boldsymbol{J}_{\theta i}\frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right)+\boldsymbol{J}_{vj}\frac{\partial}{\partial x_j}\left(\frac{\Delta\mu}{T}\right)-\tau_{ij}\frac{\partial}{\partial x_j}\left(\frac{\boldsymbol{U}_i}{T}\right)+\sum_{\gamma=1}^{r}\omega_\gamma\lambda_\gamma \ , \tag{3}$$

$$\sigma_g=\rho\frac{\boldsymbol{U}_i}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij}+g\delta_{i3}-f_r\varepsilon_{ij3}\boldsymbol{U}_j\right)=\rho\boldsymbol{F}_i\left(-\frac{\boldsymbol{U}_i}{T}\right) . \tag{4}$$

According to the hypothesis of the following section, this paper discusses the transportation of heat, vapor, and momentum,which are respectively,

$$\boldsymbol{J}_{\theta j}=\rho c_p\overline{\boldsymbol{u}'_j\theta'_\theta}\,; \quad \boldsymbol{X}_{\theta j}=\frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right) , \tag{5}$$

$$\boldsymbol{J}_{vj}=\rho\overline{\boldsymbol{u}'_j q'}; \quad \boldsymbol{X}_{vj}=\frac{\partial}{\partial x_j}\left(\frac{\Delta\mu}{T}\right)_{p,t}=-\frac{R_v}{q}\frac{\partial q}{\partial x_j} \ , \tag{6}$$

$$\tau_{ij}=\rho\overline{\boldsymbol{u}'_j\boldsymbol{u}'_i}\,; \quad \boldsymbol{X}_{mij}=\frac{\partial}{\partial x_j}\left(-\frac{\boldsymbol{U}_i}{T}\right) . \tag{7}$$

Here θ, q, U_i, T, and p are the potential temperature, specific humidity, wind velocity in the ith direction, and absolute temperature, and atmospheric pressure respectively; $J_{\theta i}$, J_{vi}, and τ_{ij} the atmospheric turbulent transportation flux of heat, vapor, and momentum; ρ, c_p, and R_v the atmospheric density, specific heat at isopiestic pressure, and the gas constant of vapor; and $\Delta\mu = \mu_d - \mu_v$ the chemical potential difference between dry air and moist air. The entropy equilibrium equation (1), the entropy production σ (3), the dynamic entropy production σ_g (4), as well as the thermodynamic flow and the thermodynamic force, (5)–(7), are the base equations in the study of atmospheric linear thermodynamics.

2. The thermodynamic properties of the linear phenomenological coefficient and the linear turbulent transport coefficient in the atmospheric boundary layer

A series of similarity relations have been determined from the many results of observational experiments in the atmospheric boundary layer. We can obtain the relations between the linear phenomenological coefficient and the linear turbulent transport coefficient using these similarity relations and can analyze their thermodynamic properties. In the following discussions, it is supposed that the one and only factor driving the heat transportation is the potential temperature gradient, driving the vapor transportation is the vapor gradient, and driving the momentum transportation is the velocity gradient. This hypothesis should be acceptable in the precision range of the atmospheric observations. Consequently, from the relationships (3) and (5)–(7) we can get the linear phenomenological relations

$$J_{\theta j} = L_\theta X_{\theta j}, \quad J_{vj} = L_v X_{vj}, \quad \tau_{ij} = L_m X_{mij} \ . \tag{8}$$

Here L_θ, L_v and L_m are defined as the linear phenomenological coefficient of heat, vapor, and momentum, respectively. Inserting the potting thermodynamic force (5)–(7) into the above formulas gives the turbulent transportation flux of heat, vapor, and momentum

$$J_{\theta j} = -\rho c_p K_\theta \frac{\partial \theta}{\partial x_j}, \quad J_{vj} = -\rho K_v \frac{\partial q}{\partial x_j}, \quad \tau_{ij} = -\rho K_m \frac{\partial U_i}{\partial x_j} \ . \tag{9}$$

These are just the turbulent transport relations of the K closure. In them, the relationship between the linear phenomenological coefficient and the linear turbulent transportation coefficient of heat, vapor, and momentum, respectively, is

$$K_\theta = \frac{1}{\rho c_p \theta^2} L_\theta, \quad K_v = \frac{R_v T}{\rho q} L_v, \quad K_m = \frac{L_m}{\rho T} \ . \tag{10}$$

So the linear turbulent transport coefficient is in proportion to the linear phenomenological coefficient; the properties of the turbulent transport coefficient are decided entirely by the linear phenomenological coefficient. The relations of K turbulent closure in (9) are an approximation relation to be proved by a great deal of experimental evidence in the atmospheric system. These relations are very well known, and are used extensively in the atmosphere. The experimental facts demonstrate indirectly that the linear phenomenological relations exists in the atmospheric thermodynamics system and the linear phenomenological coefficients can be obtained by the turbulent transport coefficient from formula (10).

2.1 *The linear turbulent transport coefficient and the linear phenomenological coefficient in the surface layer*

In the surface layer, there are the following relations between the turbulent flux and the gradient of the corresponding meteorological element (Stull 1988),

$$\frac{\overline{\theta' w'}}{\theta_* u_*} = -\frac{K_\theta}{\theta_* u_*}\frac{\partial \theta}{\partial z};\quad \frac{\overline{q' w'}}{q_* u_*} = -\frac{K_v}{q_* u_*}\frac{\partial q}{\partial z};\quad \frac{[(\overline{u' w'})^2 + (\overline{v' w'})^2]^{1/2}}{u_*^2} = -\frac{K_m}{u_*^2}\frac{\partial U}{\partial z}. \tag{11}$$

On the basis of dimensional analysis we can obtain the following similarity relations,

$$\varphi_m\left(\frac{z}{L}\right) = \frac{\kappa z}{u_*}\frac{\partial U}{\partial z} = \begin{cases} \left(1 - \alpha_m \frac{z}{L}\right)^{-\frac{1}{4}}, & \frac{z}{L} < 0 \\ 1 + \beta_m \frac{z}{L}, & \frac{z}{L} > 0 \end{cases}, \tag{12}$$

$$\varphi_\theta\left(\frac{z}{L}\right) = \frac{\kappa z}{\theta_*}\frac{\partial \theta}{\partial z} = \begin{cases} \left(1 - \alpha_\theta \frac{z}{L}\right)^{-\frac{1}{2}}, & \frac{z}{L} < 0 \\ 1 + \beta_\theta \frac{z}{L}, & \frac{z}{L} > 0 \end{cases}, \tag{13}$$

$$\varphi_v\left(\frac{z}{L}\right) = \frac{\kappa z}{q_*}\frac{\partial q}{\partial z} = \begin{cases} \left(1 - \alpha_v \frac{z}{L}\right)^{-\frac{1}{2}}, & \frac{z}{L} < 0 \\ 1 + \beta_v \frac{z}{L}, & \frac{z}{L} > 0 \end{cases}, \tag{14}$$

where $u_* = [(\overline{u' w'})^2 + (\overline{v' w'})^2]^{\frac{1}{4}}$; $\theta_* = -\frac{\overline{\theta' w'}}{u_*}$; $q_* = -\frac{\overline{q' w'}}{u_*}$. Here $\varphi_{m,\theta,v}\left(\frac{z}{L}\right)$ are the similarity functions of velocity, potential temperature, and vapor respectively; their corresponding characteristic scalars are u_*, θ_*, and q_* respectively; the Monin–Obukhov length is

$$L = -\frac{u_*^3}{\kappa \frac{g}{\bar{\theta}}(\overline{\theta' w'})}; \tag{15}$$

here, κ is the Karman constant; $\alpha_m, \alpha_\theta, \alpha_v$ and $\beta_m, \beta_\theta, \beta_v$ are experience constants determined from experiment; g is the acceleration due to gravity, $\bar{\theta}$ is the average temperature.

Deducting the gradient of velocity, temperature, humidity, and turbulent transport flux from formulas (12)–(14), besides considering relation (10), we can obtain the relations of the linear turbulent transport coefficient and the corresponding linear phenomenological coefficient which are respectively

$$K_\theta = \frac{\kappa z u_*}{\varphi_\theta},\quad K_v = \frac{\kappa z u_*}{\varphi_v},\quad K_m = \frac{\kappa z u_*}{\varphi_m}; \tag{16}$$

$$L_\theta = \rho c_p K_\theta \theta^2,\quad L_v = \rho K_v \frac{q}{R_v},\quad L_m = \rho K_m T. \tag{17}$$

The above relations show that the linear turbulent transport coefficient $\{K_\theta, K_v, K_m\}$ and the corresponding linear phenomenological coefficient $\{L_\theta, L_v, L_m\}$ are all functions of the atmospheric stability. By all appearances, the linear phenomenological coefficient is not a constant in the atmospheric surface layer. Even though the neutral stratification $z/L = 0$, it is yet a function of the height z.

2.2 *The linear turbulent transport coefficient and the linear phenomenological coefficient in the planetary boundary layer*

In the planetary boundary layer of the atmosphere, the following similarity relations are summarized from a great amount of observational data. The similarity relations of momentum flux and heat flux are (Stull 1988)

$$\frac{\overline{u'w'}}{(\overline{u'w'})_s} = \left[1 - \frac{(\overline{u'w'})_{top}}{(\overline{u'w'})_s}\frac{z}{z_i}\right]^\alpha = \varphi_m; \quad \frac{\overline{\theta' w'}}{(\overline{\theta' w'})_s} = \left[1 - \frac{(\overline{\theta' w'})_{top}}{(\overline{\theta' w'})_s}\frac{z}{z_i}\right]^\alpha = \varphi_\theta . \quad (18)$$

Here, α is the experience constant, $(\overline{u'w'})_s$ and $(\overline{w'\theta'})_s$ are the momentum flux and the heat flux in the surface layer, but the momentum flux and the heat flux at the top of the planetary boundary layer are $(\overline{u'w'})_{top}$ and $(\overline{w'\theta'})_{top}$, the height of the top of the planetary boundary layer is z_i, and the stability of the planetary boundary layer is z/z_i. For the stable and neutral boundary layer, the profile similarity relations of velocity and temperature are

$$\frac{\kappa L_L}{u_{*L}}\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right]^{\frac{1}{2}} = C_m, \quad \frac{\kappa L_L}{\theta_{*L}}\frac{\partial \theta}{\partial z} = C_\theta . \quad (19)$$

in which L_L, u_{*L}, and θ_{*L} are the local Monin–Obukhov length, the local friction velocity, and the characteristic temperature scale respectively. The constants C_m and C_θ have different experience values for the stable boundary layer, but all equal unity when the Monin–Obukhov length is replaced by z for the neutral boundary layer. Deducing them to be similar to formulas (16) and (17), we can obtain the turbulent transport coefficients as

$$K_m = \frac{\kappa L_L u_{*L}}{C_m}, \quad K_\theta = \frac{\kappa L_L u_{*L}}{C_\theta} \quad (20)$$

for the stable boundary layer, and

$$K_m = \kappa z u_{*L}, \quad K_\theta = \kappa z u_{*L} \quad (21)$$

for the neutral boundary layer. Hence the corresponding linear phenomenological coefficients are

$$L_L = \frac{K_m \kappa u_{*L}}{C_m}, \quad L_L = \frac{\kappa K_\theta u_{*L}}{C_\theta} \quad (22)$$

for the stable boundary layer, but they are

$$L_m = \kappa z u_{*L} T, \quad L_\theta = \kappa z u_{*L} \theta^2 \quad (23)$$

for the neutral boundary layer. Formula (23) shows the linear phenomenological coefficients are only a function of height for the neutral boundary layer.

For the convection boundary layer, i.e., the mixing layer, there are the following profile similarity relations

$$\frac{z_i}{w_*}\frac{\partial U}{\partial z}=\frac{z_i}{w_*}\frac{\partial V}{\partial z}=0 \qquad \left(0.1\leqslant\frac{z}{z_i}\leqslant 0.9\right) \tag{24}$$

$$\frac{z_i}{\theta_{*m}}\frac{\partial \theta}{\partial z}=0,1.4 \qquad \left(0.1\leqslant\frac{z}{z_i}\leqslant 0.9\right) \tag{25}$$

$$\frac{z_i}{q_{*m}}\frac{\partial q}{\partial z}=-5 \qquad \left(0.1\leqslant\frac{z}{z_i}\leqslant 1\right) \tag{26}$$

Here the characteristic scale of velocity, temperature, and humidity are respectively

$$w_*=\left[\frac{g}{\theta}(\overline{\theta'w'})_s z_i\right]^{\frac{1}{3}};\quad \theta_{*m}=\frac{(\overline{\theta'w'})_s}{w_*};\quad q_{*m}=\frac{(\overline{q'w'})_s}{w_*} \tag{27}$$

Now we can only deduce by analogy with (20)–(23) from (26) to get the turbulent transport coefficient and the linear phenomenological coefficient

$$K_v=\frac{z_i w_*}{5},\quad L_v=\frac{z_i w_*}{5}\frac{q}{R_v}\ . \tag{28}$$

These are functions of the stability (z_i, w_*).

The relations (24) and (25) indicate that the velocity and the potential temperature are uniformly distributed in the mixing layer, resulting from the uniform mixing of convection development. So

$$\frac{\partial U}{\partial z}=0,\quad \frac{\partial V}{\partial z}=0,\quad \frac{\partial \theta}{\partial z}=0\ . \tag{29}$$

On the other hand, if the characteristic scale $\{w_*, \theta_{*m}\}$ of the mixing layer replaces the corresponding characteristic scale in formula (18), then

$$\frac{\overline{\theta'w'}}{\theta_{*m}w_*}=-\frac{K_\theta}{\theta_{*m}w_*}\frac{\partial\theta}{\partial z},\quad \frac{\overline{u'w'}}{w_*^2}=-\frac{K_m}{w_*^2}\frac{\partial U}{\partial z},\quad \frac{\overline{v'w'}}{w_*^2}=-\frac{K_m}{w_*^2}\frac{\partial V}{\partial z}\ . \tag{30}$$

The heat and momentum flux in relation (30) have a linear distribution with height; they are a non–zero limited value. Relation (29) causes the turbulent transport coefficients $\{K_\theta, K_m\}$ in (30) to go to infinity, and thus lose their physical significance. Therefore there is not the local turbulent transportation of momentum and heat in the mixing layer. But the turbulent transportation still exists in the entire mixing layer to transport heat from the surface layer to the top of the boundary layer and to transport momentum from the top of the boundary layer to the surface layer. It is a holistic turbulent transportation in the mixing layer, i.e. a large eddy turbulent transportation. It is not a linear transport process, but an intense non–linear transportation process. The above facts show that the linear phenomenological relation and the linear transport processes do not exist in the mixing layer, and of course, there is no linear phenomenological coefficient. On the other hand, the strong and holistic non–linear turbulent transport process exits from the top of the boundary layer to the surface layer. Consequently we can conclude that the convection boundary layer is a non–linear thermodynamic state far from the equilibrium state.

3. Cross–coupling between the dynamic process and the thermal process—the relation between geostrophic wind and thermal wind

The relation between the geostrophic wind and the thermal wind is very well known. That is, it is well studied in dynamic meteorology. The geostrophic wind is a dynamic process, but the thermal wind which is caused by the atmospheric thermal stratification is a *thermodynamic process. The relation between the geostrophic wind and the thermal wind is a* cross–coupling phenomenon between a dynamic process and a thermal process in the atmospheric system from the point of view of linear thermodynamics in a non–equilibrium state. The following section concretely demonstrates this cross–coupling phenomenon in the atmospheric system.

A dynamic entropy production (4), which is work done by the Newton force that drives the airflow, is added into the entropy equation. The dynamic equilibrium is a basic state of the atmosphere, as the work done by the Newton force equals zero, $\sigma_g = 0$. Motionlessness in the atmosphere is an impossibility; this means that the airflow ρU_i cannot be zero. Hence the Newton force must be zero, $-F_i = 0$, during the time the atmosphere is in dynamic equilibrium, *i.e. static equilibrium and geostrophic equilibrium. The equilibrium equation of the static* equilibrium and the geostrophic equilibrium are respectively

$$\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + g\delta_{i3} = 0, \quad \frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} - f_c\varepsilon_{ij3}U_j = 0 \ . \tag{31}$$

We know from (31) there is no airflow in the vertical direction in static equilibrium. In addition, to define that the airflow caused by the velocity U_i is the geostrophic airflow ρU_{gj} in the geostrophic equilibrium equation, and to define that the airflow caused by the baric gradient force $\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij}$ is the baric gradient airflow ρU_{pj}, gives

$$\rho U_{gj} = \rho U_i, \quad \rho U_{pj} = \frac{1}{f_c}\frac{\partial p}{\partial x_i}\delta_{ij} \ . \tag{32}$$

The atmospheric baroclinicity leads to the thermal airflow $\rho U_{\theta j}$ in an air layer of unit height in the vertical direction, which is defined as

$$\rho U_{\theta j} = -\frac{\rho g}{f_c \theta}\frac{\partial \theta}{\partial x_j}\delta_{ij}\varepsilon_{ij3} \ . \tag{33}$$

From relation (31) we know the geostrophic wind is exactly the baric gradient wind at geostrophic equilibrium,

$$U_{gj} = \varepsilon_{ij3}U_{pi} = \varepsilon_{ij3}\frac{1}{\rho f_c}\frac{\partial p}{\partial x_j} \ . \tag{34}$$

The Newton force is non–zero if the atmosphere is in non–geostrophic equilibrium and non–static equilibrium. The airflow ρU_i caused by the atmospheric dynamic deviation can be determined approximately from the linear thermodynamic method if the geostrophic deviation and the static deviation are not too large, i.e. the atmosphere is near the dynamic equilibrium state. *The entropy production (3) and the dynamic entropy production (4) indicate the* airflow ρU_i along with the heat flux $J_{\theta j}$ and the vapor flux J_{vj} are vectors. According to the Curie principle, a thermodynamic cross–coupling effect may exist among them (Hu 2002).

For brevity, we consider only the cross–coupling between the geostrophic deviation airflow and the static deviation airflow under the condition of dynamic equilibrium deviation. We therefore write the entropy production and the dynamic entropy production in the following form

$$\sigma + \sigma_g = J_{\theta j}\frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right) + \rho U_i\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + g\delta_{i3} - f_c\varepsilon_{ij3}U_j\right) . \tag{35}$$

Considering the cross–coupling between the static deviation and the geostrophic deviation airflow, and the reciprocal relation, then the linear phenomenological relations are as follows:

$$J_{\theta j} = L_\theta\frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right) + L_{\theta a}\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + g\delta_{i3} - f_c\varepsilon_{ij3}U_j\right) , \tag{36}$$

$$\rho U_i = L_a\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + g\delta_{i3} - f_c\delta_{ij3}U_j\right) + L_{a\theta}\frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right)\delta_{ij}\varepsilon_{ij3} . \tag{37}$$

Here L_θ, L_a, $L_{a\theta}$, and $L_{\theta a}$ are defined as the thermodynamic phenomenological coefficient and the dynamic phenomenological coefficient along with the cross–coupling coefficient. Relation (36) shows that any dynamic equilibrium deviation and the potential temperature gradient will all cause the heat transport flux. This means that the heat transport flux is relative not only to the potential temperature gradient but also to the dynamic equilibrium deviation, due to the cross–coupling between the dynamic process and the thermal process. Relation (37) shows that any dynamic equilibrium deviation and the potential temperature gradient all will cause the airflow. The left–hand side of formula (37) is the airflow caused by the static deviation and the geostrophic deviation. After considering the definition of (31) and (32), they can be written in the vertical and horizontal component form as,

$$\rho W = L_g\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial z} + g\right) - L_{\rho\theta}\frac{1}{\theta^2}\frac{\partial\theta}{\partial z} , \tag{38}$$

$$\rho U_i = \frac{L_p}{T}(U_{pj}\delta_{ij} - U_{gj})f_c\varepsilon_{ij3} + \frac{L_{p\theta}}{\theta}U_{\theta j}f_c\delta_{ij}\varepsilon_{ij3} , \quad (i = 1, 2) . \tag{39}$$

Relation (38) indicates that it will generate a vertical movement so that the atmosphere deviates from the static equilibrium state and neutral stratification, i.e. the existing atmospheric stratification. But relationship (39) indicates that any geostrophic deviation and horizontal gradient of potential temperature will all generate the horizontal airflow (the geostrophic deviation wind) at a dynamic non–equilibrium state. Moreover, their value is the linear combination of the baric gradient wind, the geostrophic wind, and the thermal wind. If the phenomenological coefficients are taken as

$$L_p = \frac{\rho T}{f_c} \text{ and } L_{p\theta} = \frac{\rho}{g} , \tag{40}$$

then the dynamic non–equilibrium wind (geostrophic deviation wind) is the vector sum of the baric wind, the geostrophic wind, and the thermal wind,

$$U_i = (U_{pj} + U_{\theta j})\delta_{ij}\varepsilon_{ij3} - U_{gj}\varepsilon_{ij3} . \tag{41}$$

This is the relation between the geostrophic wind and the thermal wind deduced from the cross–coupling of linear thermodynamics. This relation is in full accord with the one that is deduced by atmospheric dynamics (Yang et al. 1983). The relation between the geostrophic wind and the thermal wind predicates the existence of the cross–coupling between the airflow transportation and the heat transportation.

4. Discussion

Combining the similarity relations which are obtained using a great deal of observational results in the atmospheric boundary layer with linear thermodynamics theory, proves the existence of the linear phenomenological relations. The linear turbulent transport coefficients are in proportion to the corresponding phenomenological coefficients. This result demonstrates that the atmospheric phenomenological coefficients are a function of stratification stability, which cannot be constant in general. The research results prove that the linear phenomenological relations do not exist in the mixing layer with the large eddy convection because the turbulent transport process of large eddy convection is a strong non–linear process. So the convection boundary layer is a thermodynamic state in the non–linear range far from the equilibrium state. From the point of view of the linear thermodynamics of the non–equilibrium state, the relation between the geostrophic wind and the thermal wind is a special cross–coupling phenomenon between the dynamic process and the thermal process in the atmospheric system. It is a known practical illustration of a cross–coupling phenomenon in the atmospheric system.

It must be emphasized that the phenomenological coefficient is often supposed as a constant by way of analytic convenience in theoretical thermodynamics to prove a series of important theorems, such as the minimum principle of entropy production (de Groot and Mazur 1962). By all appearances, this hypothesis is limited for the atmosphere. The above observational facts in the atmospheric boundary layer show that the linear phenomenological coefficient is not a constant, but a function of the atmospheric stratification stability and the space. The hypothesis that the phenomenological coefficient is constant should be applied cautiously and some important theorems in theoretical thermodynamics must also be applied carefully to the atmospheric system.

We can obtain the conclusion from the above facts and observational results that Hu (2002) demonstrates indirectly not only the authenticity of the theoretical conclusions, but also increases the understanding of the physical essence of the atmospheric phenomena. For example, large eddy convection and the relation between the geostrophic wind and the thermal wind are all phenomena of non–equilibrium state thermodynamics, understood from this point of view.

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线性热力学对大气系统的应用(II)
大气系统中线性唯象关系的例证

胡隐樵

摘　要

将边界层相似性理论同线性热力学理论结合，间接地以观测实验事实证明大气边界层内线性唯象关系是成立的，而且线性湍流输送系数同相应的线性唯象系数成正比关系。但实验事实表明，大涡对流的混合层线性唯象关系是不成立的，混合层内湍流输送过程是一种强的非线性过程。所以，对流边界层是一种远离平衡态非线性区的热力学状态。地转风和热成风是一种大气系统特有的动力过程和热力过程的交叉耦合现象，这是大气系统交叉耦合现象的一个实际例证。

关键词：大气系统，线性热力学，线性唯象关系，湍流输送系数，大气边界层