Effects of Turbulent Dispersion on the Wind Speed Profile in the Surface Layer

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ABSTRACT

New Reynolds mean momentum equations including turbulent viscosity and dispersion are used to analyze the vertical profile of wind speed in the surface layer. It is demonstrated that the wind profile of the surface layer including turbulent dispersion has a logarithmic modification on the classical power law. Under the condition of unstable stratification, the effect of dispersion is stronger than under stable stratification. Under neutral stratification, the power law degenerates to the logarithmic law, but the von Karman constant is replaced by $k_1 = (1 + k/4)^{-1}k$, which can also be obtained by similarity theory.

Key words: dispersion, turbulence, wind profile

1. Introduction

It is well known that the unclosed Reynolds′ mean equations can be obtained when averaging is used to describe atmospheric turbulence. The classical Prandtl's mixing length theory analogized turbulent motion with molecular motion. It has well explained the phenomena of turbulent diffusion, viscosity, and thermal conduction. But It is difficult to explain the phenomena of transformation of eddy kinetic energy (turbulent kinetic energy) to mean kinetic energy (basic flow kinetic energy) (Bayly and Yakhot 1986; Yakhot and Sivashinsky 1987). Starr (1966) used " negative viscosity" to explain the phenomenon. Liu and Liu (1995) reconsidered the Prandtl mixing length theory and obtained the modified Reynolds′ averaged momentum equations that contain the turbulent viscosity and dispersion. It is stated that the turbulent dispersion effect is the main reason for the energy inversion described above.

Negelcting the turbulent dispersion, turbulent viscosity is the only factor to be considered in the atmospheric balance motions of the planetary boundary layer (PBL), so the wind profile has a large departure from reality. Once the turbulent dispersion effect is considered, the balance motions in the PBL will change more or less. Liu et al. (1993) proved that the wind profile in the PBL is probably the result of turbulent viscosity and diffusion effects. Based on the theory of our previous papers (Arya and Wyngaard 1975, Businger et al. 1971, Gao 1985), we will further discuss the influence of turbulent dispersion on atmospheric movement in the surface layer.

2. Classic theory of the wind profile in the surface layer

Neglecting the atmospheric density change and only considering the vertical transport of

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turbulent momentum, the averaged horizontal momentum equations of the atmosphere can be written as

$$\begin{aligned} \frac{d\bar{u}}{dt} - f\bar{v} &= -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} - \frac{\partial \overline{u'w'}}{\partial z}, \\ \frac{d\bar{v}}{dt} + f\bar{u} &= -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} - \frac{\partial \overline{v'w'}}{\partial z}. \end{aligned} \tag{1}$$

Here $\bar{p}$ and $\bar{\rho}$ are the mean pressure and density respectively; $\bar{u}$ and $\bar{v}$ are the mean wind speed of latitude and longitude respectively; and u', v', w' are the fluctuating velocities.

The focus of the problem is how to handle the average of the products of fluctuating velocities, which is called the Reynolds' Stress. Taking $-(\partial \overline{u'w'} / \partial z)$ as an example, we use the Prandtl mixing length theory which introduces a mixing length l': a particle retains its properties within l' and exchanges them with other particles after l'. Assuming $\bar{u}$ as $\bar{u}(z_1)$ and $\bar{u}(z)$ at $z = z_1$ and $z = z$, then it produces a deviation u' when the particle at $z = z_1$ reaches $z = z$ and mixes with the neighboring air ($l' = z - z_1$).

According to the traditional method,

$$u' \approx \bar{u}(z_1) - \bar{u}(z) \approx -\frac{\partial \bar{u}}{\partial z}(z - z_1) = -l'\frac{\partial \bar{u}}{\partial z}, \tag{2}$$

so

$$-\overline{u'w'} = v\frac{\partial \bar{u}}{\partial z}, \quad -\frac{\partial \overline{u'w'}}{\partial z} = v\frac{\partial^2 \bar{u}}{\partial z^2}, \tag{3}$$

where

$$v = \overline{l'w'} \tag{4}$$

is the coefficient of the turbulent viscosity, which is considered a constant.

In this way, Eq.(1) can be written as

$$\begin{aligned} \frac{du}{dt} - fv &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial z^2}, \\ \frac{dv}{dt} + fu &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\frac{\partial^2 v}{\partial z^2}, \end{aligned} \tag{5}$$

where the mean symbols are omitted. (In the following, we also omit the mean symbols of u and v). The ordered pair $\left(v\frac{\partial^2 u}{\partial z^2}, v\frac{\partial^2 v}{\partial z^2} \right)$ is the turbulent viscosity force per unit air mass.

In the surface layer, the equation in which the flux density of turbulent momentum is a constant is usually used to analyze the balance motion. From Eq.(3), we have

$$-\rho\overline{u'w'} = \rho v\frac{\partial u}{\partial z} = T_0 = \text{const}. \tag{6}$$

Using the turbulent semi–empirical theory, we assume that

$$w' = l'\frac{\partial u}{\partial z}. \tag{7}$$

In this way, v can be written as

$$\nu = \overline{l'w'} = \overline{l'^2}\frac{\partial u}{\partial z} = l^2\frac{\partial u}{\partial z}, \tag{8}$$

in which $\overline{l'^2} = l^2$; l is also called mixing length.

Substituting Eq.(8) into Eq.(6), we have

$$l\frac{\partial u}{\partial z} = u_*, \tag{9}$$

where

$$u_* = \sqrt{T_0 / \rho} \tag{10}$$

is the friction velocity.

In Eq.(9), the mixing length l is dependent not only on a dynamic factor (lower boundary) but also on a thermodynamic factor (stratification). There are two methods to determine l. One is Prandtl–Laikhtman's mixing length theory, in which it is assumed that

$$l = kz(z / z_0)^{-\varepsilon}, \tag{11}$$

where $k = 0.4$ is the Karman constant, z_0 is the roughness length, ε is a coefficient which is related to stratification, and it satisfies

$$\varepsilon = \begin{cases} > 0 & \text{stable stratification} \\ = 0 & \text{neutral stratification} \\ < 0 & \text{unstable stratification} \end{cases}. \tag{12}$$

Under the neutral stratification, substituting Eq.(11) into Eq.(9), noticing that $u|_{z=z_0} = 0$, we can obtain the logarithmic law

$$u = \frac{u_*}{k}\ln\frac{z}{z_0}. \tag{13}$$

When $\varepsilon \neq 0$, it is easy to bring about the power law

$$u = \frac{u_*}{k\varepsilon}[(z / z_0)^{\varepsilon} - 1]. \tag{14}$$

Another method to determine l is Monin–Obukhov's similarity theory, in which one introduces a Monin–Obukhov length L which is related to static stability

$$L = \frac{u_* \partial u / \partial z}{kN^2}, \tag{15}$$

where N is Brunt–Väisälä frequency. The mixing length can be written as

$$l = \frac{kz}{\Phi(\zeta)}, \tag{16}$$

where $\Phi(\zeta)$ is a universal function, and

$$\zeta \equiv \frac{z}{L}\begin{cases} > 0 & \text{stable stratification} \\ = 0 & \text{neutral stratification} \\ < 0 & \text{unstable stratification} \end{cases}. \tag{17}$$

So Eq.(9) can be written as

$$\frac{\partial u}{\partial z}=\frac{u_*}{kz}\Phi(\zeta). \tag{18}$$

Now, $\Phi(\zeta)$ is often represented as

$$\Phi(\zeta)=\begin{cases}1+B\zeta & \zeta\geqslant 0\\ (1-\gamma\zeta)^{-1/4} & \zeta<0,\end{cases} \tag{19}$$

where β and γ are constants which are often given the value 5 and 16 respectively.

Substituting Eq.(19) into Eq.(18), and using the boundary condition $u|_{z=z_0}=0$, it is easy to obtain that

$$u=\frac{u_*}{k}\left\{\ln\frac{z}{z_0}-\Psi(\zeta)\right\}, \tag{20}$$

where

$$\Psi(\zeta)=\int_{\zeta_0}^{\zeta}\frac{1-\Phi(\zeta)}{\zeta}d\zeta \qquad (\zeta_0=z_0/L). \tag{21}$$

It is not difficult to find that the profile satisfies the logarithmic law under the neutral condition. Under the stable and unstable condition, respectively, we have

$$\Psi(\zeta)=\begin{cases}-\beta(\zeta-\zeta_0) & \zeta>0\\ \ln\frac{1+x^2}{2}+2\ln\frac{1+x}{2}-2\tan^{-1}x+\frac{\pi}{2} & \zeta<0\end{cases}, \tag{22}$$

where

$$x=(1-\gamma\zeta)^{1/4}. \tag{23}$$

The profiles of the two theories are illustrated by Fig.1 and Fig.2 respectively.

Both Prandtl–Laikhtman's and Monin–Obukhov's theories only consider the effect of turbulence viscosity and not of turbulence dispersion, so their conclusions may have some limits. Nevertheless, from the above we can get the following:

(1) Under the neutral condition, the profile satisfies the logarithmic law.

(2) The nearer to the ground, the smaller the effect of stratification is and the closer profile is to that of the neutral stratification.

3. Turbulence dispersion effects

From Eq.(2), we find that the traditional Prandtl's method only includes $\frac{\partial \bar{u}}{\partial z}$. If we add $\frac{\partial^2 \bar{u}}{\partial z^2}$ to Eq.(2), we can get

$$u'=-\frac{\partial \bar{u}}{\partial z}(z-z_1)-\frac{1}{2}\frac{\partial^2 \bar{u}}{\partial z^2}(z-z_1)^2=-l'\frac{\partial \bar{u}}{\partial z}-\frac{1}{2}l'^2\frac{\partial^2 \bar{u}}{\partial z^2}. \tag{24}$$

Then Eq.(3) can be described as

$$-\overline{u'w'}=\nu\frac{\partial \bar{u}}{\partial z}-\gamma\frac{\partial^2 \bar{u}}{\partial z^2}, \quad -\frac{\partial \overline{u'w'}}{\partial z}=\nu\frac{\partial^2 \bar{u}}{\partial z^2}-\gamma\frac{\partial^3 \bar{u}}{\partial z^3}, \tag{25}$$

where

$$\gamma = -\frac{1}{2}\overline{l'^2 w'} \tag{26}$$

is called the turbulent dispersion coefficient.

In a similar way, we have

$$-\overline{v'w'} = \nu\frac{\partial \overline{v}}{\partial z} - \gamma\frac{\partial^2 \overline{v}}{\partial z^2}, \quad -\frac{\partial \overline{v'w'}}{\partial z} = \nu\frac{\partial^2 \overline{v}}{\partial z^2} - \gamma\frac{\partial^3 \overline{v}}{\partial z^3}. \tag{27}$$

In this way Eq.(5) can be written as

$$\begin{aligned} \frac{du}{dt} - fv &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} - \gamma\frac{\partial^3 u}{\partial z^3}, \\ \frac{dv}{dt} + fu &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2} - \gamma\frac{\partial^3 v}{\partial z^3}, \end{aligned} \tag{28}$$

where $\left(-\gamma\frac{\partial^3 u}{\partial z^3}, -\gamma\frac{\partial^3 v}{\partial z^3}\right)$ may be known as the turbulent dispersion force per unit air mass.

In the surface layer, Eq.(6) with the dispersion effect can be written as

$$\rho\left(\nu\frac{\partial u}{\partial z} - \gamma\frac{\partial^2 u}{\partial z^2}\right) = T_0 = \text{const.} \tag{29}$$

Using Eq.(7), Eq.(26) can be reduced to

$$\gamma \equiv -\frac{1}{2}\overline{l'^2 w'} = -\frac{1}{2}\overline{l'^3}\frac{\partial u}{\partial z} = -\frac{1}{2}l^3\frac{\partial u}{\partial z}. \tag{30}$$

Substituting Eq.(8) and (30) into Eq.(29), we have

$$l^2\left[\left(\frac{\partial u}{\partial z}\right)^2 + \frac{l}{2}\frac{\partial u}{\partial z}\left(\frac{\partial^2 u}{\partial z^2}\right)\right] = u_*^2, \tag{31}$$

and this is the equation of balance motion in the surface layer including the turbulent dispersion.

After considering the turbulent dispersion, the motion will have some changes. Based on the momentum transport mechanism, $-\overline{u'w'}$ is the mean value of zonal west wind momentum passing through a unit area perpendicular to the z axis per unit time. Usually, we have $\frac{\partial u}{\partial z} > 0$, $\frac{\partial^2 u}{\partial z^2} < 0$. When $\nu > 0$, the turbulent viscosity makes u transit from a high value to a low value $\left(\nu\frac{\partial u}{\partial z} > 0\right)$. Since the dissipative process cannot run inversely, ν must be always positive. When $\gamma > 0$, the turbulent dispersion also makes u transit from a high value to a low value $\left(-\gamma\frac{\partial^2 u}{\partial z^2} > 0\right)$. But if $\gamma < 0$, momentum may transfer in the opposite direction $\left(-\gamma\frac{\partial^2 u}{\partial z^2} < 0\right)$. When $\left|\gamma\frac{\partial^2 u}{\partial z^2}\right| > \left|\nu\frac{\partial u}{\partial z}\right|$, momentum will transfer from the low value of u to a high value, which is called the energy inversion.

In the following section, we will discuss the wind profile in the surface layer which is described by Eq.(31).

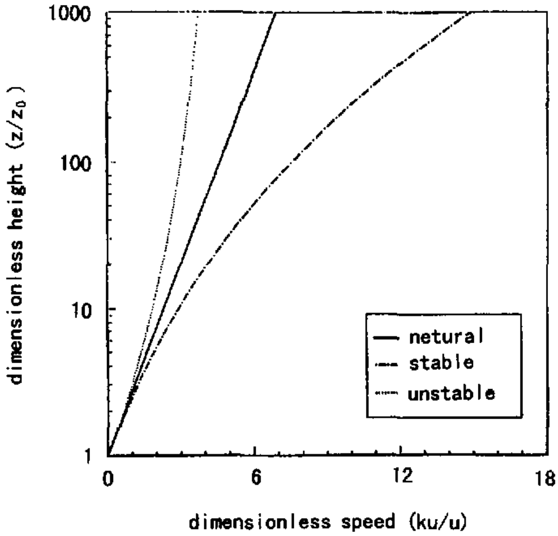

Fig. 1. The wind profile of Prandtl–Laikhtman's theory.

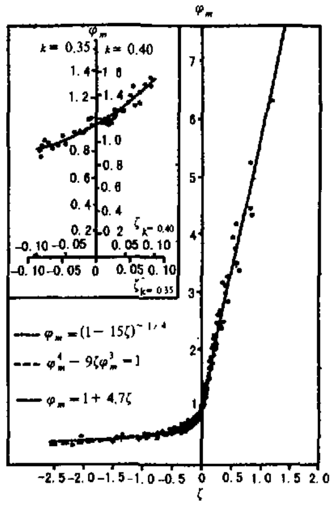

Fig. 2. Comparison of dimensionless wind shear observation with formulas. (Businger et al. 1971).

4. The effect of dispersion on the wind profile in the surface layer

4.1 *Laikhtman's theory*

Eq.(31) can be written as

$$l^2\left[\left(\frac{\partial u}{\partial z}\right)^2+\frac{1}{4}\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^2\right]=u_*^2\,. \tag{32}$$

Substituting Eq.(11) into it, we can obtain

$$\left(\frac{\partial u}{\partial z}\right)^2+\frac{kz(z/z_0)^{-\varepsilon}}{4}\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^2=\frac{u_*^2}{(kz)^2(z/z_0)^{-2\varepsilon}}\quad. \tag{33}$$

Setting

$$u=\frac{u_*}{k}u_1,\qquad z=z_1z_0, \tag{34}$$

we get the non–dimensional form of Eq.(33) as

$$z_1^{3\varepsilon-3}-z_1^{\varepsilon-1}\left(\frac{\partial u_1}{\partial z_1}\right)^2=\frac{k}{4}\frac{\partial}{\partial z_1}\left(\frac{\partial u_1}{\partial z_1}\right)^2. \tag{35}$$

Under the neutral stratification, the wind profile satisfies the logarithmic law,

$$\left.\frac{\partial u_1}{\partial z_1}\right|_{\varepsilon\to 0} \propto z_1^{-1}, \tag{36}$$

and

$$u_1|_{z_1=1} = 0. \tag{37}$$

Since $k = 0.4$, $k/4 = 0.1 \ll 1$, we treat $\alpha = k/4$ as the small parameter, and use the small parameter method to solve Eq.(35); the equation can be reduced to

$$z_1^{3\varepsilon-3} - z_1^{\varepsilon-1}\left(\frac{\partial u_1}{\partial z_1}\right)^2 = \alpha\frac{\partial}{\partial z_1}\left(\frac{\partial u_1}{\partial z_1}\right)^2. \tag{38}$$

Set

$$u_1 = u_1^{(0)} + \alpha u_1^{(1)} + a^2 u_1^{(2)} + \cdots, \tag{39}$$

where $u_1^{(0)}, u_1^{(1)}, u_1^{(2)} \cdots$ are the zero–, first–, and second–$\cdots$ order approximations of u_1 respectively. Substituting Eq.(39) into Eq.(38) then yields

$$z_1^{3\varepsilon-3} - z_1^{\varepsilon-1}\left[\frac{\partial}{\partial z_1}(u_1^{(0)} + \alpha u_1^{(1)} + \alpha^2 u_1^{(2)} + \cdots)\right]^2$$
$$= \alpha\frac{\partial}{\partial z_1}\left[\frac{\partial}{\partial z_1}(u_1^{(0)} + \alpha u_1^{(1)} + \alpha^2 u_1^{(2)} + \cdots)\right]^2. \tag{40}$$

The zero–order approximation of Eq.(40) is given by

$$O(\alpha^0): \qquad z_1^{\varepsilon-1}\left(\frac{\partial u_1^{(0)}}{\partial z_1}\right)^2 = z_1^{3\varepsilon-3} \tag{41}$$

namely

$$\frac{\partial u_1^{(0)}}{\partial z_1} = z_1^{\varepsilon-1}. \tag{42}$$

Integrating the above equation, we have

$$u_1^{(0)} = \frac{z_1^{\varepsilon}}{\varepsilon} + C_1, \tag{43}$$

where C_1 is an arbitrary constant. Using the boundary condition Eq.(37), we can obtain

$$u_1^{(0)}|_{z_1=1} = 0, \tag{44}$$

so $C_1 = -1/\varepsilon$, and

$$u_1^{(0)} = \frac{z_1^{\varepsilon} - 1}{\varepsilon}. \tag{45}$$

This is the famous power law, whose dimensional form is just Eq.(14).

When $\varepsilon \to 0$, we have

$$u_1^{(0)} = \ln z_1. \tag{46}$$

Turning Eq.(46) into the dimensional form, we can get the classic logarithmic law equation which is the same as Eq. (13).

The first–order approximation of Eq.(40) is

$$O(\alpha^1):\qquad 2z_1^{\varepsilon-1}\frac{\partial u_1^{(0)}}{\partial z_1}\frac{\partial u_1^{(1)}}{\partial z_1}+\frac{\partial}{\partial z_1}\left(\frac{\partial u_1^{(0)}}{\partial z_1}\right)^2=0. \tag{47}$$

Substituting Eq.(45) into it, we have

$$\frac{\partial u_1^{(1)}}{\partial z_1}=(1-\varepsilon)z_1^{-1}. \tag{48}$$

Similarly, using the boundary condition

$$u_1^{(1)}\big|_{z_1=1}=0, \tag{49}$$

we obtain

$$u_1^{(1)}=(1-\varepsilon)\ln z_1. \tag{50}$$

Combining Eq.(45) and (50), the solution of Eq.(38) is

$$u_1=\frac{z_1^{\varepsilon}-1}{\varepsilon}+\alpha(1-\varepsilon)\ln z_1=\frac{z_1^{\varepsilon}-1}{\varepsilon}+\frac{k}{4}(1-\varepsilon)\ln z_1. \tag{51}$$

Returning this equation to the dimensional form, we can get

$$u=\frac{u_*}{k}\frac{(z/z_0)^{\varepsilon}-1}{\varepsilon}+\frac{1-\varepsilon}{4}u_*\ln(z/z_0). \tag{52}$$

It should be noted that the first term on the right–hand side is the power law of the classical wind profile, and the second term is the modification of the dispersion effect.

Particularly, under neutral stratification we have $\varepsilon\to 0$, and so Eq.(52) can be written as

$$u=\frac{u_*}{k_1}\ln(z/z_0), \tag{53}$$

where

$$k_1=(1+\frac{k}{4})^{-1}k\approx 0.36. \tag{54}$$

This implies that under the neutral condition, and considering the effect of turbulent dispersion, the profile satisfies a logarithmic law whose coefficient has changed into a new constant which is related to Karman's constant. It is noted that this constant conform to the results that were obtained by Businger et al. (1971) and Liu et al. (1993).

The second–order approximation of Eq.(40) is given by

$$O(\alpha^2):\qquad 2\frac{\partial}{\partial z_1}\left(\frac{\partial u_1^{(0)}}{\partial z_1}\frac{\partial u_1^{(1)}}{\partial z_1}\right)+Z_1^{\varepsilon-1}\left[\left(\frac{\partial u_1^{(1)}}{\partial z_1}\right)^2+2\frac{\partial u_1^{(1)}}{\partial z_1}\frac{\partial u_1^{(2)}}{\partial z_1}\right]=0. \tag{55}$$

Substituting $u_1^{(0)}$ and $u_1^{(1)}$ into it, we have

$$\frac{\partial u_1^{(2)}}{\partial z_1}=\frac{(1-\varepsilon)(3-\varepsilon)}{2}z_1^{-\varepsilon-1}. \tag{56}$$

Using the boundary condition

$$u_1^{(2)}\Big|_{z_1=1}=0, \tag{57}$$

then

$$u_1^{(2)}=\frac{(1-\varepsilon)(3-\varepsilon)}{2\varepsilon}(1-z_1^{-\varepsilon}). \tag{58}$$

The modification is so small that it is neglected.

Figure 3 gives the comparison of different stratifications before and after modification with observation values.

Based on the above, we conclude the following.

(1) After considering the turbulent dispersion, the wind profile in the surface layer satisfies the power + logarithmic law under both stable and unstable conditions, and the traditional power law is its zero–order approximation.

(2) Under the neutral condition, the profile degenerates to the logarithmic law, but the coefficient is changed to a new constant which satisfies Eq.(54).

(3) The dispersion effect is more significant under unstable stratification than under stable stratification.

(4) The effect of dispersion is more significant when z is higher.

In the following we will give a possible explanation for the third and fourth conclusions.

The dispersion effect can be described by the dispersional coefficient γ, which satisfies Eq.(30), and is determined by the mixing length l and wind shear $\frac{\partial u}{\partial z}$. Simply let l satisfy Eq.(11) and u satisfy Eq. (14), then

$$\frac{\partial u}{\partial z}=\frac{u_*}{kz_0}\left(\frac{z}{z_0}\right)^{\varepsilon-1}, \tag{59}$$

so that

$$\begin{aligned}\gamma=-\frac{1}{2}l^3\frac{\partial u}{\partial z}&=-\frac{1}{2}\left[kz\left(\frac{z}{z_0}\right)^{-\varepsilon}\right]^3\times\frac{u_*}{kz_0}\left(\frac{z}{z_0}\right)^{\varepsilon-1}\\&=-\frac{1}{2}u_*k^2z^2\left(\frac{z}{z_0}\right)^{-2\varepsilon},\end{aligned} \tag{60}$$

that is

$$\gamma=-\frac{1}{2}u_*k^2z^{2-2\varepsilon}z_0^{2\varepsilon}. \tag{61}$$

From Eq. (60) we can see that the smaller the stability parameter ε is, the larger $|\gamma|$ is, which is the third conclusion. From Eq. (61), the higher z is, the larger $|\gamma|$ is and the more significant the effect of dispersion is, which is the fourth conclusion.

4.2 *Similarity theory*

Substituting Eq. (16) into Eq. (32), we obtain

$$\left(\frac{\partial u}{\partial z}\right)^2+\frac{kz}{4\Phi(\zeta)}\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^2=\frac{u_*^2}{[kz/\Phi(\zeta)]^2}. \tag{62}$$

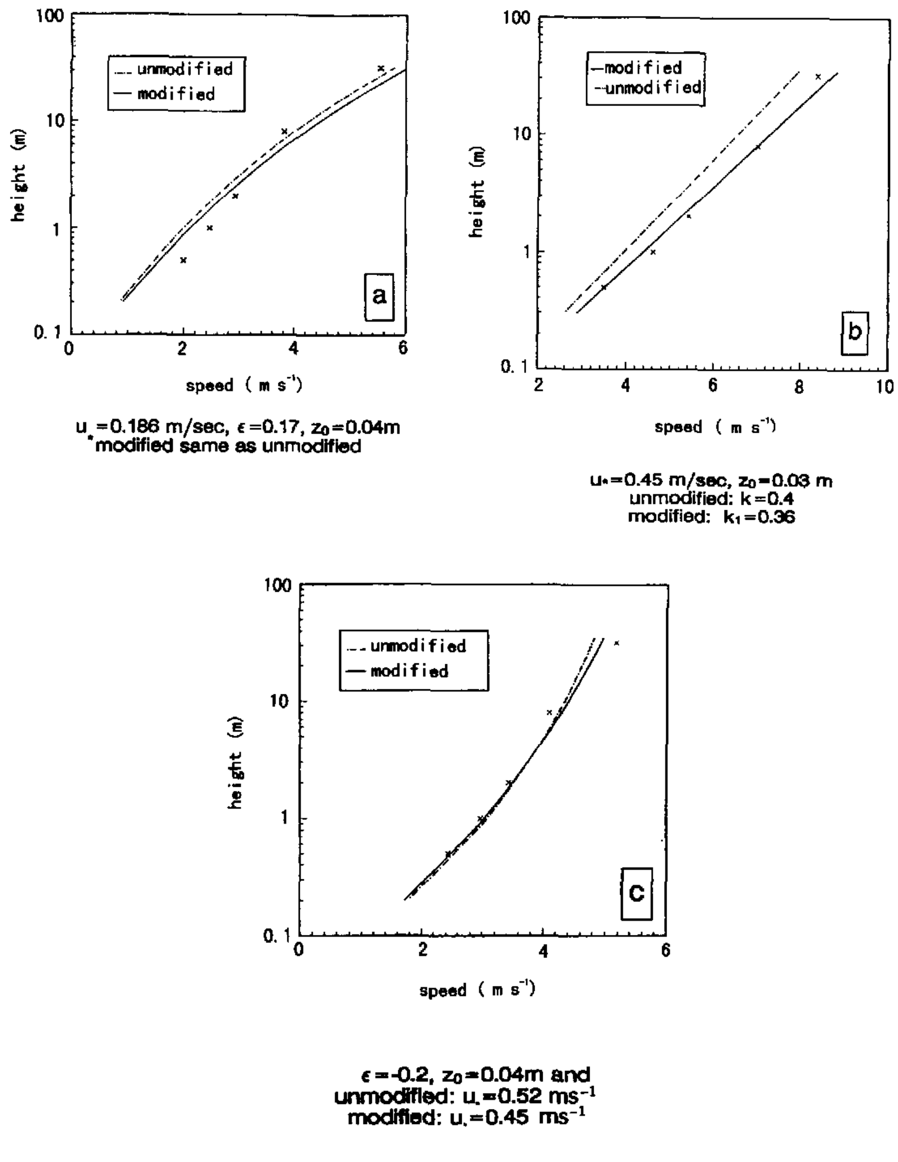

Fig. 3. The comparison of different stratifications before and after modification with observation values.

Set

$$u = \frac{u_*}{k} u_1, \qquad z = L\zeta. \tag{63}$$

Eq. (62) can be reduced to

$$\frac{k}{4}\frac{\partial}{\partial\zeta}\left(\frac{\partial u_1}{\partial\zeta}\right)^2=\Phi_1(\zeta)\left[\Phi_1^2(\zeta)-\frac{\partial u_1}{\partial\zeta}\right]^2, \tag{64}$$

where

$$\Phi_1(\zeta)=\Phi(\zeta)/\zeta. \tag{65}$$

Treat $\alpha=k/4$ as a small parameter and set

$$u_1=u_1^{(0)}+\alpha u_1^{(1)}+\alpha^2u_1^{(2)}+\cdots. \tag{66}$$

Eq.(64) can be written as

$$\Phi_1^3(\zeta)-\Phi_1(\zeta)\left[\frac{\partial}{\partial\zeta}(u_1^{(0)}+\alpha u_1^{(1)}+\alpha^2u_1^{(2)}+\cdots)\right]^2=\alpha\frac{\partial}{\partial\zeta}\left[\frac{\partial}{\partial\zeta}(u_1^{(0)}+\alpha u_1^{(1)}+\alpha^2u_1^{(2)}+\cdots)\right]^2. \tag{67}$$

We can obtain

$$O(\alpha^0):\qquad \left(\frac{\partial u_1^{(0)}}{\partial\zeta}\right)^2=\Phi_1^2(\zeta), \tag{68}$$

$$O(\alpha^2):\qquad -2\Phi_1(\zeta)\frac{\partial u_1^{(0)}}{\partial\zeta}\frac{\partial u_1^{(1)}}{\partial\zeta}=\frac{\partial}{\partial\zeta}\left(\frac{\partial u_1^{(0)}}{\partial\zeta}\right)^2, \tag{69}$$

$$O(\alpha^2):\qquad -2\Phi_1(\zeta)\frac{\partial u_1^{(0)}}{\partial\zeta}\frac{\partial u_1^{(2)}}{\partial\zeta}=-2\frac{\partial}{\partial\zeta}\left(\frac{\partial u_1^{(0)}}{\partial\zeta}\frac{\partial u_1^{(1)}}{\partial\zeta}\right)-\frac{1}{2}\left(\frac{\partial u_1^{(1)}}{\partial\zeta}\right). \tag{70}$$

Eqs. (68), (69) and (70) can be rewritten respectively as

$$\frac{\partial u_1^{(0)}}{\partial\zeta}=\Phi_1(\zeta), \tag{71}$$

$$\frac{\partial u_1^{(1)}}{\partial\zeta}=-\frac{1}{\Phi_1(\zeta)}\frac{\partial^2u_1^{(0)}}{\partial\zeta^2}, \tag{72}$$

$$\frac{\partial u_1^{(0)}}{\partial\zeta}\frac{\partial u_1^{(2)}}{\partial\zeta}=-\frac{1}{\Phi_1(\zeta)}\frac{\partial}{\partial\zeta}\left(\frac{\partial u_1^{(0)}}{\partial\zeta}\frac{\partial u_1^{(1)}}{\partial\zeta}\right)-\frac{1}{2}\left(\frac{\partial u_1^{(1)}}{\partial\zeta}\right)^2. \tag{73}$$

It is not difficult to find that Eq. (71) is the same as Eq. (18). Using the boundary condition $u_1^{(0)}\big|_{\zeta=\zeta_0}=0$ and integrating Eq. (71), we get

$$u_1^{(0)}=\ln\frac{\zeta}{\zeta_0}-\Psi(\zeta), \tag{74}$$

where $\Psi(\zeta)$ satisfies Eq. (21). By substituting Eq.(19) into Eq. (74), it is transformed into Eq. (22). When Eq. (74) is changed into the dimensional form, it is just Eq. (20), so the zero–order approximation of Eq.(64) is just the same as the classical solution of Monin–Obukhov.

Using Eq.(71), Eq.(72) can be reduced to

$$\frac{\partial u_1^{(0)}}{\partial\zeta}=-\frac{\partial\ln\Phi_1(\zeta)}{\partial\zeta}. \tag{75}$$

Integrating the above equation, we can obtain

$$u_1^{(1)}=\ln\Phi_1(\zeta_0)-\ln\Phi_1(\zeta)=\ln\frac{\zeta}{\zeta_0}-\Psi_1(\zeta), \tag{76}$$

where

$$\Psi_1(\zeta)=\ln\Phi(\zeta)-\ln\Phi(\zeta_0). \tag{77}$$

Substituting Eq.(19) into it, then

$$\Psi_1(\zeta)=\begin{cases}\ln(1+\beta\zeta)-\ln(1+\beta\zeta_0) & \zeta\geqslant 0\\ -\ln x & \zeta<0\end{cases}, \tag{78}$$

where x satisfies Eq.(23).

Combining Eq.(74) and (76), we have the solution of Eq.(64)

$$u_1=\left[\ln\frac{\zeta}{\zeta_0}-\Psi(\zeta)\right]+\alpha\left[\ln\frac{\zeta}{\zeta_0}-\Psi_1(\zeta)\right]. \tag{79}$$

Converting it into the dimensional form, we get

$$u=\frac{u_*}{k}\left[\ln\frac{z}{z_0}-\Psi(\zeta)\right]+\frac{u_*}{4}\left[\ln\frac{z}{z_0}-\Psi_1(\zeta)\right]. \tag{80}$$

In the above solution, the first term on the right–hand side is the solution of Monin–Obukhov's theory; the second term is the modification of turbulent dispersion effect. Equation (80) can also be written as

$$u=\frac{u_*}{k_1}\ln\frac{z}{z_0}-\frac{u_*}{k}\Psi(\zeta)-\frac{u_*}{4}\Psi_1(\zeta), \tag{81}$$

where k_1 satisfies Eq.(54).

Under neutral stratification, $\zeta\to 0$, $\zeta_0\to 0$, $\Psi(\zeta)\to 0$, $\Psi_1(\zeta)\to 0$. Then Eq.(81) can be reduced to

$$u=\frac{u_*}{k_1}\ln\frac{z}{z_0}, \tag{82}$$

which is the same as Eq. (53) and is a modified logarithmic law.

Now we find that if we substitute the coefficient of the logarithmic law with k_1, this result conforms to the solution of the modified Laikhtman's theory.

Using Eq.(71) and (72), we can convert the second–order equation (73) into

$$\frac{\partial u_1^{(2)}}{\partial\zeta}=\Phi_1^{-2}(\zeta)\frac{\partial^2\Phi_1(\zeta)}{\partial\zeta^2}-\frac{1}{2}\Phi_1^{-3}(\zeta)\left(\frac{\partial\Phi_1(\zeta)}{\partial\zeta}\right)^2. \tag{83}$$

But we neglect it for its modification to the classic solution is not significant.

5. Conclusions

Applying the modified Prandtl's mixing length theory, this paper studies the influence of turbulent dispersion on the formation of the wind profile in the surface layer. It is found that the effect of turbulent dispersion modifies the classical profile in logarithmic form. This modification is more significant under unstable stratification than stable. In addition, the power law turns into the logarithmic law under neutral stratification, so the modification introduces a new constant k_1 which can also be obtained from similarity theory.

Because of the complexity of the effect of stratification, the modification in this paper is limited. The character of turbulent dispersion and related problems need to be studied further.

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湍流频散对边界层风廓线的影响

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摘 要

应用包括湍流粘性和频散的新的 Reynolds 平均动量方程，分析了边界层的垂直风速廓线，发现包含湍流频散的地面层的风速廓线对经典的风廓线指数规律有一个对数规律的修改；而且在不稳定层结下比在稳定层结下，湍流的频散效应更为显著；在中性条件下，指数规律退化为对数规律并且 Karman 常数被另外一个常数所代替，而这个新常数也可以通过相似理论来获得。

关键词： 频散，湍流，风廓线