Energetics of Geostrophic Adjustment in Rotating Flow

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ABSTRACT

Energetics of geostrophic adjustment in rotating flow is examined in detail with a linear shallow water model. The initial unbalanced flow considered first falls under two classes. The first is similar to that adopted by Gill and is here referred to as a mass imbalance model, for the flow is initially motionless but with a sea surface displacement. The other is the same as that considered by Rossby and is referred to as a momentum imbalance model since there is only a velocity perturbation in the initial field. The significant feature of the energetics of geostrophic adjustment for the above two extreme models is that although the energy conversion ratio has a large case-to-case variability for different initial conditions, its value is bounded below by 0 and above by 1 / 2. Based on the discussion of the above extreme models, the energetics of adjustment for an arbitrary initial condition is investigated. It is found that the characteristics of the energetics of geostrophic adjustment mentioned above are also applicable to adjustment of the general unbalanced flow under the condition that the energy conversion ratio is redefined as the conversion ratio between the change of kinetic energy and potential energy of the deviational fields.

Key words: geostrophic adjustment, energy conversion ratio, mass imbalance, momentum imbalance, kinetic energy, potential energy

1. *Introduction*

The problem of how a fluid, initially not in geostrophic balance, adjusts to that balance is a fundamental problem in the theory of rotating fluids. Blumen (1972) gave the first review in the area and discussed many concepts which have been used since, including potential vorticity conservation and minimum energy principles. Since then, the study of geostrophic adjustment processes has a developing history with many contributors. For example, Kuo (1997) examined the geostrophic adjustment process of the initial unbalanced fields with axis-symmetric perturbation using a numerical model. Lin and Chao (1997) and Chao (2000) discussed the adjustment process in the tropical area and posed the semi-geostrophic adjustment theory. Vallis (1992) and Wu and Fang (2001) found that the final balanced state of geostrophic adjustment need not always be geostrophic balance.

A feature of all geostrophic adjustment problems is that only a fraction of the initial potential (kinetic) energy released, $dE_p(dE_k)$, is converted into kinetic (potential) energy, $dE_k(dE_p)$ of the final geostrophically adjusted state. The energetics of the geostrophic adjustment problem are usually expressed in terms of the energy conversion ratio $\gamma = -dE_k / dE_p$ for the initial mass imbalances and $\gamma = -dE_p / dE_k$ for initial momentum imbalances. Van Heijst (1985) calculated the energy conversion ratio γ for a few cases of two-and

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three–layer fluids and found that in all the cases he examined, γ equals $1/3$ regardless of the model configuration and moreover, this is the same value that Gill (1976) obtained for vastly different problems. This somewhat surprising result prompted him to speculate that there is some general energy conversion property that leads to this constant conversion ratio. For the nonlinear version of the problem considered by Gill (1976, 1982), Boss and Thompson (1995) also showed that $\gamma = 1/3$. However, Killworth (1986) argued the $1/3$ factor is fortuitous. But he was "at a loss to understand" why such a wide range of problems possess answers of $1/3$. Later, Middleton (1987) pointed out that the $1/3$ factor is due to the discontinuity in the initial sea surface and demonstrated that γ smoothly varies from 0 to $1/2$ as the wavelength of the initial disturbance increases from 0 to infinity when the initial surface displacement is sinusoidal.

Nonlinear geostrophic adjustment of a continuously stratified inviscid, incompressible Boussinesq fluid was considered by Ou (1984, 1986), Blumen and Wu (1995) and Wu and Blumen (1995). Ou (1986) and Blumen and Wu (1995) established that $\gamma = 1/2$ for the zero potential vorticity adjustment problem, provided that fronts do not form during adjustment. Blumen and Wu (1995) also extended Ou's study to the uniform potential vorticity adjustment and showed that $\gamma < 1/2$ for particular distributions of the initial density field. The value of γ depends upon the horizontal length scale of the initial density field.

Grimshaw et al. (1998) extended the result that $0 < \gamma < 1/2$, obtained by Middleton (1987) and Blumen and Wu (1995) with the specific initial unbalanced flow, to the initial rest fluid with arbitrary density disturbances. However, some approximations were made during their derivation. In this paper, upper and lower bounds are derived strictly for the energy conversion ratio of geostrophic adjustment for the initial mass imbalance and the initial momentum imbalance problem. In addition to these, the energetics of the geostrophic adjustment of an arbitrary initial velocity and mass field is also discussed in detail.

The model, displayed in section 2, is the linearized shallow–water wave equations. With this model, the energetics of the geostrophic adjustment for three different initial unbalanced flows is examined in section 3. Final remarks concerning the energy conversion ratio are presented in section 4.

2. Formulation of the problem

We consider the linear adjustment of small, local initial disturbances of sea level or velocity in an inviscid barotropic fluid of constant depth H. The initial disturbances are assumed to have horizontal scales much greater than H so that the hydrostatic assumption is valid and the linearized equations of one–dimensional motion may be written as

$$\begin{cases} \dfrac{\partial u}{\partial t} - fv = -g\dfrac{\partial \eta}{\partial x} , \\ \dfrac{\partial v}{\partial t} + fu = 0 , \\ \dfrac{\partial \eta}{\partial t} + H\dfrac{\partial u}{\partial x} = 0 , \end{cases} \tag{1}$$

where $\vec{u} = (u, v)$ denotes velocity and η is the sea surface displacement. All variables may be cast into nondimensional form by means of

$$x = \lambda x' \qquad \eta = H\eta' ,$$

$$t=\frac{1}{f}t' \qquad (u,v)=\sqrt{gH}(u',v') \ , \tag{2}$$

where $\lambda=\sqrt{gH}/f$ is the Rossby radius of deformation. Based on (2), the dimensionless form of (1) is

$$\begin{cases} \dfrac{\partial u}{\partial t}-v=-\dfrac{\partial \eta}{\partial x} \ , \\ \dfrac{\partial v}{\partial t}+u=0 \ , \\ \dfrac{\partial \eta}{\partial t}+\dfrac{\partial u}{\partial x}=0 \ . \end{cases} \tag{1'}$$

The prime notation has been dropped for brevity.

The above equations imply the perturbation potential vorticity to be conserved following a particle so that

$$\frac{\partial v_g}{\partial x}-\eta_g=\frac{\partial v_i}{\partial x}-\eta_i=q_i \ , \tag{3}$$

where q is potential vorticity. The subscript 'i' is used to denote the initial state while 'g' is the adjusted state.

The final state is characterized by geostrophic balance, which may be expressed as

$$v_g=\frac{\partial \eta_g}{\partial x} \ . \tag{4}$$

By using (4), (3) can be expressed alternatively as

$$\frac{\partial^2 \eta_g}{\partial x^2}-\eta_g=\frac{\partial v_i}{\partial x}-\eta_i \ . \tag{5}$$

The above equation describes the relation between the adjusted sea surface displacement and the initial velocity and sea surface displacement. Given the proper boundary conditions, the final adjusted sea surface displacement subject to certain initial conditions can be solved from (5). Then the final geostrophic velocity is obtained and, finally, the energy conversion ratio is determined. This method was first used by Rossby (1937) and then widely adopted in subsequent works. However we will examine the energetics of geostrophic adjustment in an alternative way without the need to solve Eq.(5).

By multiplying both sides by $\frac{1}{2}\eta_g$, Eq.(5) can be formulated as

$$-\frac{1}{2}\left(\frac{\partial \eta_g}{\partial x}\right)^2-\frac{1}{2}\eta_g^2+\frac{1}{2}\frac{\partial}{\partial x}\left(\eta_g\frac{\partial \eta_g}{\partial x}\right)=\frac{1}{2}\left(\frac{\partial v_i}{\partial x}-\eta_i\right)\eta_g \ . \tag{8}$$

Introducing (4) into (8) then integrating the resulting equation over x yields the energy equation

$$\frac{1}{2}\int_{-\infty}^{\infty}(v_g^2+\eta_g^2)dx=-\frac{1}{2}\int_{-\infty}^{\infty}\left(\frac{\partial v_i}{\partial x}-\eta_i\right)\eta_g dx+\frac{1}{2}(\eta_g v_g)|_{-\infty}^{\infty} \ . \tag{9}$$

Obviously, the left–hand side of Eq.(9) is the total energy of the adjusted state.

Introducing Δv and $\Delta\eta$ to denote the difference of the adjusted velocity and sea surface displacement from the initial velocity and sea surface displacement, respectively, then the adjusted velocity and sea surface displacement can be expressed as

$$v_g = v_i + \Delta v\ , \qquad \eta_g = \eta_i + \Delta\eta\ . \tag{10}$$

It is obvious from (3) that $\Delta\eta$ can be expressed by Δv as

$$\frac{\partial \Delta v}{\partial x} = \Delta\eta\ . \tag{11}$$

Substituting (10) into (9) and integrating by parts, (9) is transformed as

$$\frac{1}{2}\int_{-\infty}^{\infty}(v_g^2+\eta_g^2)dx = \frac{1}{2}\int_{-\infty}^{\infty}(v_i^2+\eta_i^2)dx + \frac{1}{2}\int_{-\infty}^{\infty}(v_i\Delta v + \eta_i\Delta\eta)dx + \frac{1}{2}(\eta_g\Delta v)\Big|_{-\infty}^{\infty}\ . \tag{12}$$

After some manipulation, we have

$$E_g - E_i = dE_k + dE_p = \frac{1}{2}\int_{-\infty}^{\infty}(v_i\Delta v + \eta_i\Delta\eta)dx + \frac{1}{2}(\eta_g\Delta v)\Big|_{-\infty}^{\infty}\ , \tag{13}$$

where E_i and E_g designate the total energy of the initial and adjusted state respectively and dE_k and dE_p are the change of kinetic energy and potential energy during geostrophic adjustment, i.e.

$$E_i = \frac{1}{2}\int_{-\infty}^{\infty}(v_i^2+\eta_i^2)dx \qquad E_g = \frac{1}{2}\int_{-\infty}^{\infty}(v_g^2+\eta_g^2)dx\ , \tag{14}$$

$$dE_k = \frac{1}{2}\int_{-\infty}^{\infty}(v_g^2 - v_i^2)dx \qquad dE_p = \frac{1}{2}\int_{-\infty}^{\infty}(\eta_g^2-\eta_i^2)dx\ . \tag{15}$$

It is clear that (13) describes not only the relation between initial total energy and final total energy but also the conversion between kinetic and potential energy during the geostrophic adjustment process. Furthermore, (13) can be rearranged as

$$\begin{aligned} E_g - E_i = dE_k + dE_p &= \frac{1}{2}\int_{-\infty}^{\infty}(v_g\Delta v + \eta_g\Delta\eta)dx - \frac{1}{2}\int_{-\infty}^{\infty}((\Delta v)^2 + (\Delta\eta)^2)dx + \frac{1}{2}(\eta_g\Delta v)\Big|_{-\infty}^{\infty} \\ &= \frac{1}{2}\int_{-\infty}^{\infty}\left(\frac{\partial\eta_g}{\partial x}\Delta v + \eta_g\frac{\partial\Delta v}{\partial x}\right)dx - \frac{1}{2}\int_{-\infty}^{\infty}[(\Delta v)^2 + (\Delta\eta)^2]dx + \frac{1}{2}(\eta_g\Delta v)\Big|_{-\infty}^{\infty} \\ &= -\frac{1}{2}\int_{-\infty}^{\infty}[(\Delta v)^2 + (\Delta\eta)^2]dx + (\eta_g\Delta v)\Big|_{-\infty}^{\infty}\ , \end{aligned} \tag{16}$$

in which (4), (5), and (10) are used. Since the motion in large x, far from the ageostrophic disturbance, will not be influenced by the adjustment process, the flow will keep its initial state, i.e.

$$x \to \pm\infty, \quad \Delta v = 0 \quad \Delta\eta = 0\ . \tag{17}$$

Hence, (16) can be simplified as

$$E_g - E_i = dE_k + dE_p = -\frac{1}{2}\int_{-\infty}^{\infty}[(\Delta v)^2 + (\Delta\eta)^2]dx\ . \tag{16'}$$

Evidently, the left hand side of (16′) is negative (it equals zero only under the condition that both Δv and $\Delta\eta$ are zero). This implies that the adjusted total energy is less than the initial total energy, i.e., $dE_p \neq -dE_k$. In the following section the relation between dE_p and dE_k is to be discussed in detail for three different models.

3. Energetics of geostrophic adjustment

Case I. Mass imbalance model

The initial unbalanced flow is taken as

$$v_i = 0\ ,\quad \eta_i \neq 0\ , \tag{18}$$

which was used by Gill (1976, 1982). Accordingly, (5) is simplified as

$$\frac{\partial^2 \eta_g}{\partial x^2} - \eta_g = -\eta_i\ , \tag{5'}$$

and $\gamma = -dE_k / dE_p$ is taken as the energy conversion ratio because the initial potential energy is released and converted into the adjusted kinetic energy during the adjustment.

By means of (10), (11), (4), the lateral boundary condition (17), and the initial condition (18), the energy conversion ratio can be formulated as

$$\gamma = -\frac{\int_{-L}^{L}(v_g^2 - v_i^2)dx}{\int_{-L}^{L}(\eta_g^2 - \eta_i^2)dx} = -\frac{\int_{-L}^{L}(v_g^2 - v_i^2)dx}{\int_{-L}^{L}[(\eta_g + \eta_i)(\eta_g - \eta_i)]dx} = -\frac{\int_{-L}^{L} v_g^2 dx}{\int_{-L}^{L}[2\eta_g \Delta\eta - (\Delta\eta)^2]dx}$$

$$= -\frac{\int_{-L}^{L} v_g^2 dx}{\int_{-L}^{L}[2\eta_g \frac{\partial \Delta v}{\partial x} - (\Delta\eta)^2]dx} = \frac{\int_{-L}^{L} v_g^2 dx}{\int_{-L}^{L} 2v_g^2 dx + \int_{-L}^{L}(\Delta\eta)^2 dx}\ . \tag{19}$$

It follows immediately that $0 \leqslant \gamma \leqslant \frac{1}{2}$. Thus, during the geostrophic adjustment process there is conversion of potential energy into kinetic energy, but at most one-half of the potential energy released goes into the steady geostrophic mode. The remaining portion is radiated away in the form of inertio-gravity waves. To verify this result, the energy conversion ratio for different initial unbalanced flows is calculated. The results are shown in Fig. 1, in which A1, A2, A3, and B correspond respectively to the initial sea surface displacement as follows

$$\text{A1:}\quad \eta_i(x) = \left[H(x) - \frac{1}{2}\right]e^{-ax^2}\ ,$$

$$\text{A2:}\quad \eta_i(x) = \left[H(x+\frac{\pi}{a}) - H(x-\frac{\pi}{a})\right]\sin ax\ ,$$

$$\text{A3:}\quad \eta_i(x) = -\operatorname{erf}(ax)\ ,$$

$$\text{B:}\quad \eta_i(x) = e^{-ax^2}\ ,$$

where $H(x)$ is the Heaviside function and the reciprocal of parameter a denotes the horizontal scale of the initial disturbance, $a < 1$ ($a > 1$) indicates that the scale of the initial perturbation is larger (smaller) than the Rossby radius of deformation. Three distinguishing features can be extracted from Fig. 1. Firstly, γ increases as the scale of the initial perturbation increases. This is consistent with the results of Grimshaw et al. (1998), Middleton (1987), and Blumen and Wu (1995). Secondly, the varying ranges of γ are different for different initial disturbances. For cases A2 and B, γ ranges from 0 to 1 / 2. However,

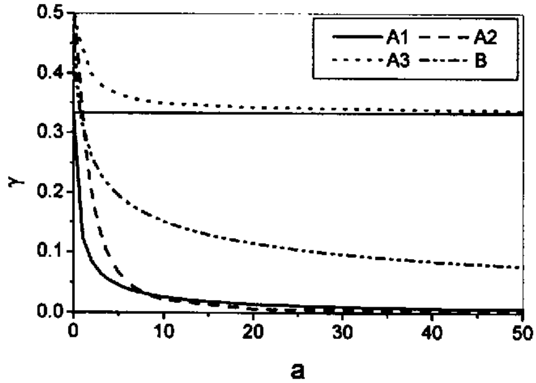

Fig. 1. Energy conversion ratio $\gamma = -dE_k / dE_p$ as a function of the inverse length scale a for the geostrophic adjustment of the mass imbalance problem.

the range of γ is 0–1 / 3 and 1 / 3–1 / 2 for case A1 and A3, respectively. Finally, γ is never larger than 1 / 2 or smaller than zero, although it varies from case to case. This is consistent with the above theoretical analysis.

Case II. Momentum imbalance model

The initial unbalanced flow under consideration in this model is

$$v_i \neq 0, \quad \eta_i = 0 , \tag{20}$$

which was discussed by Rossby (1937). As a result, (5) can be read as

$$\frac{\partial^2 \eta_g}{\partial x^2} - \eta_g = \frac{\partial v_i}{\partial x} . \tag{5''}$$

Since the kinetic energy of the initial flow will be released and go into the adjusted potential energy during the adjustment of this initial unbalanced flow the energy conversion ratio is defined as $\gamma = -dE_p / dE_k$. Using (10), (4), and (11) together, with the lateral boundary condition (17) and the initial condition (18), γ can be formulated as

$$\gamma = -\frac{\int_{-\infty}^{\infty} (\eta_g^2 - \eta_i^2) dx}{\int_{-\infty}^{\infty} (v_g^2 - v_i^2) dx} = -\frac{\int_{-\infty}^{\infty} (\eta_g^2 - \eta_i^2) dx}{\int_{-\infty}^{\infty} (v_g + v_i)(v_g - v_i) dx} = -\frac{\int_{-\infty}^{\infty} \eta_g^2 dx}{\int_{-\infty}^{\infty} [2 v_g \Delta v - (\Delta v)^2] dx}$$

$$= -\frac{\int_{-\infty}^{\infty} \eta_g^2 dx}{\int_{-\infty}^{\infty} \left[2\Delta v \frac{\partial \eta_g}{\partial x} - (\Delta v)^2 \right] dx} = \frac{\int_{-\infty}^{\infty} \eta_g^2 dx}{\int_{-\infty}^{\infty} 2\eta_g^2 dx + \int_{-\infty}^{\infty} (\Delta v)^2 dx} . \tag{21}$$

From (21) it is clear that γ is bounded below by zero and above by 1 / 2 again, i.e., the kinetic energy released is converted into potential energy but at least one–half of it is dispersed by the inertio–gravity wave and only a fraction of it remains in the adjusted state. This result can also be confirmed by numeric calculations. Figure 2 displays the variation of energy conversion

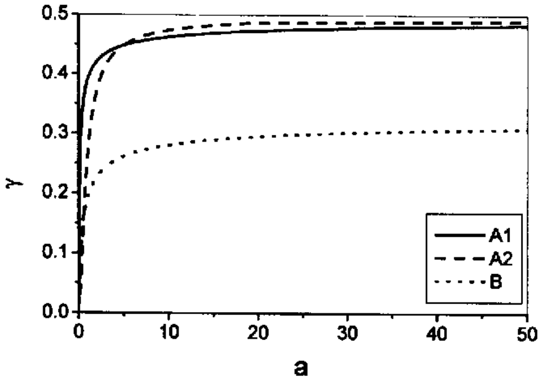

Fig. 2. Energy conversion ratio $\gamma = -dE_k / dE_p$ as a function of the inverse length scale a for the geostrophic adjustment of the momentum imbalance problem.

ratio with the scale of the initial perturbation for three types of initial conditions, in which A1, A2, and B designate the following initial velocity distributions.

$$\text{A1:}\quad v_i(x)=\left[H(x)-\frac{1}{2}\right]e^{-ax^2},$$

$$\text{A2:}\quad v_i(x)=\left[H(x+\frac{\pi}{a})-H(x-\frac{\pi}{a})\right]\sin ax,$$

$$\text{B:}\quad v_i(x)=e^{-ax^2},$$

where $H(x)$ and a are the same as in Case I. Although γ is dependent on the scale and distribution of the initial disturbances, just like in Fig. 1 it is bounded below by 0 and above by 1 / 2.

Case III. Energetics of the general geostrophic adjustment problem

Both the mass imbalance model and momentum imbalance model discussed above are the extreme cases. In the remaining part of this section, the geostrophic adjustment for the general initial unbalanced flow will be considered.

Take the initial unbalanced flow as

$$v_i \neq 0 \quad , \quad \eta_i \neq 0 \ . \tag{22}$$

Since both the initial velocity and sea surface displacement are non–zero, the energetics of the geostrophic adjustment is complicated and the energy conversion ratio cannot reflect the energetics of the geostrophic adjustment properly in this case. Therefore, we will deal with this problem in another way.

For simplicity, assume the geostrophic wind that corresponds to the initial sea surface displacement is v_{ig}, i.e.,

$$v_{ig} = \frac{\partial \eta_i}{\partial x} \ , \tag{23}$$

and take (v_{ig}, η_i) as the basic state. Then the deviation of the initial and adjusted state from this basic state can be expressed as

$$v'_i = v_i - v_{ig} \ , \quad \eta'_i = \eta_i - \eta_i = 0 \ , \quad v'_g = v_g - v_{ig} \ , \quad \eta'_g = \eta_g - \eta_i \ . \tag{24}$$

Clearly, v'_i is the initial geostrophic departure and v'_g and η'_g measure the deviation of the adjusted balanced fields from the initial geostrophic fields. At this stage, the initial condition (22) can be expressed by the deviational variables as

$$v'_i \neq 0 \ , \quad \eta'_i = 0 \ . \tag{25}$$

Substituting (25) into (5) and rearranging it gives

$$\frac{\partial^2 \eta'_g}{\partial x^2} - \eta'_g = \frac{\partial v'_i}{\partial x} \ . \tag{26}$$

In form, (25) and (26) are just the same as (19) and (5″) except the variables are replaced by the deviational variables. Introduce the following variables

$$\begin{cases} \Delta v' = v_g - v_i = v'_g - v'_i \ , \\ \Delta \eta' = \eta_g - \eta_i = \eta'_g - \eta'_i = \eta'_g \ , \\ E'_i = \dfrac{1}{2}\displaystyle\int_{-\infty}^{\infty} (v'^2_i + \eta'^2_i) dx \ , \\ E'_g = \dfrac{1}{2}\displaystyle\int_{-\infty}^{\infty} (v'^2_g + \eta'^2_g) dx \ , \\ dE'_k = \dfrac{1}{2}\displaystyle\int_{-\infty}^{\infty} (v'^2_g - v'^2_i) dx \ , \\ dE'_p = \dfrac{1}{2}\displaystyle\int_{-\infty}^{\infty} (\eta'^2_g - \eta'^2_i) dx = \dfrac{1}{2}\displaystyle\int_{-\infty}^{\infty} \eta'^2_g dx \ , \end{cases} \tag{30}$$

where the definition of $\Delta v'$, $\Delta \eta'$, E'_i, E'_g, dE'_k and dE'_p are similar to Δv, $\Delta \eta$, E_i, E_g, dE_k, and dE_p that we defined in section 2 except that all variables are replaced by the deviational variables. It can be found by comparison that the geostrophic adjustment process described by these deviational variables is similar to the momentum imbalance model discussed in Case II where the initial condition is taken as $v_i \neq 0$, $\eta_i = 0$. The only difference is that the variables are replaced by their deviation from the basic state. So, the varying range of γ we obtained there should be applicable to the energy conversion ratio of the deviational field defined as $\gamma' = -dE'_p / dE'_k$. This implies that during the geostrophic adjustment, the kinetic energy of the deviational field will go into the potential energy of the deviational field, and that γ' increases as the scale of the deviation of the initial fields from the basic state decreases, but it will never be larger than 1 / 2. In fact, the momentum imbalance model in Case II is just one special case of (22) that takes $\eta_i = 0$.

Similarly, if we take the basic state as (v_i, η_{ig}), where η_{ig} is the sea surface displacement that is balanced with v_i, then we can find that the energetics of geostrophic adjustment we discussed in Case I are also applicable here for the energy conversion ratio of the deviational field defined as $\gamma' = -dE'_k / dE'_p$. That is, during the geostrophic adjustment, the deviational potential energy will convert into the deviational kinetic energy and γ' is always smaller than 1 / 2 although it increases as the scale of the deviation of the initial field from the basic state increases. Actually, Case I is an extreme case of (22) with $v_i = 0$.

According to the above analysis, the energetics of geostrophic adjustment are dependent on the basic state that we defined. Generally, for geostrophic adjustment of the large-scale motion, the velocity field adjusts to be in equilibrium with the mass field (Gill, 1982). So, the basic state can be defined as (v_{ig}, η_t). Then, the energetics of the geostrophic adjustment obtained for the momentum imbalance model are applicable by replacing the variables with their deviation from the basic state. On the other hand, for the small-scale motion, the mass field adjusts to be in equlibrium with the velocity field. Therefore, the basic state can be taken as (v_i, η_{ig}). Then, the energetics of the geostrophic adjustment obtained for the mass imbalance model are applicable here under the condition that the variables are replaced by their deviation from the basic state.

4. Conclusions

There is much literature devoted to the problem of the energetics of geostrophic adjustment. However, most of it is subjected to a specific initial unbalanced flow. In this paper, this problem is re-analyzed in a general way.

Two extreme initial unbalanced flows are considered first, namely a mass imbalance model and a momentum imbalance model. Since the initial unbalanced flow of the mass imbalance model is motionless but with a sea surface displacement, potential energy is to be released and converted into kinetic energy during the geostrophic adjustment. On the other hand, in the momentum imbalance model, the initial kinetic energy is released and converted into potential energy because there is only velocity perturbation in the initial unbalanced flow. However, both models share the same feature of the energy conversion ratio. That is, the ratio is never larger than 1 / 2 or smaller than zero, or in other words, at most one-half of the released potential (kinetic) energy can be converted into kinetic (potential) energy and the remaining will be radiated in the form of inertio-gravity waves.

Based on the discussion of the above extreme models, the energetics of adjustment for an arbitrary initial condition is investigated by defining the basic state and the deviation of the initial and final states from this basic state. For geostrophic adjustment of the large-scale motion, in which the basic state can be defined as (v_{ig}, η_t), the varying regulations of the energy conversion ratio defined as $\gamma' = -dE'_p / dE'_k$ are the same as those obtained for the momentum imbalance model by replacing the kinetic and potential energy with the kinetic and potential energy of the deviational field. Similarly, the energetics of the geostrophic adjustment obtained from the mass imbalance model are also applicable to the kinetic energy and potential energy of the deviational field for the small-scale motion, in which the basic state is defined as (v_i, η_{ig}).

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旋转流地转适应过程与能量转换规律

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摘　要

利用一维线性浅水模式从一个比较普遍的角度对地转适应过程中能量转换的特点进行分析。文中首先考虑了两类不同的初始不平衡流的适应问题。一个是 Gill 所采用的质量不平衡模型，即初始场是静止的，只有水面扰动；另一个是 Rossby 中所考虑的动量不平衡模型，其初始不平衡流中只有风场的扰动。对这两个模型的适应过程而言，一个显著的特点就是能量转换率始终不会大于 1／2 或者小于 0，即适应过程中有位(动)能的释放和向动(位)能的转换，但释放出的能量最多只有其中的一半可以保留在最后的平衡场中。另外，本文对任意初始不平衡流适应过程中的能量学特征也进行了分析，指出对于偏差场(相对于一定基本态)的动能和位能而言，上述能量转换关系依然成立。

关键词：地转适应，质量不平衡，动量不平衡，能量转换率，动能，位能