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Elimination of Computational Systematic Errors and Improvements of Weather and Climate System Models in Relation to Baroclinic Primitive Equations

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ABSTRACT

The design of a total energy conserving semi–implicit scheme for the multiple–level baroclinic primitive equation has remained an unsolved problem for a long time. In this work, however, we follow an energy perfect conserving semi–implicit scheme of a European Centre for Medium–Range Weather Forecasts (ECMWF) type sigma–coordinate primitive equation which has recently successfully formulated. Some real–data contrast tests between the model of the new conserving scheme and that of the ECMWF–type of global spectral semi–implicit scheme show that the RMS error of the averaged forecast Height at 850 hPa can be clearly improved after the first integral week. The reduction also reaches 50 percent by the 30th day. Further contrast tests demonstrate that the RMS error of the monthly mean height in the middle and lower troposphere also be largely reduced, and some well–known systematical defects can be greatly improved. More detailed analysis reveals that part of the positive contributions comes from improvements of the extra–long wave components. This indicates that a remarkable improvement of the model climate drift level can be achieved by the actual realizing of a conserving time–difference scheme, which thereby eliminates a corresponding computational systematic error source / sink found in the currently–used traditional type of weather and climate system models in relation to the baroclinic primitive equations.

Key words: fidelity scheme, computational systematical errors, baroclinic primitive equation

1. The physical problem

Baroclinic primitive equation models are widely used in numerical weather prediction and climate simulation. The problem of designing a total energy conserving time difference scheme for a multiple–level baroclinic primitive equation has remained unsolved for a long time, therefore, there exists a false computational systematic error source / sink in the currently–used traditional type of weather and climate system models in relation to the baroclinic primitive equations. Apparently, even though the sources or sinks of corresponding systematic errors are not large or exceedingly small in terms of quantity, their long–term negative accumulative effects are not to be overlooked. For various reasons, except in the trivial first–order conserving schemes and some other very special cases, the basic formulation problem of conserving schemes in the temporal–spatial discrete sense has remained unsolved for a long time although the studies on the formulation of conserving schemes can be traced back to early last century and some instantly conserving schemes (ensuring conservation in space only

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and not in time) have been successfully formulated in the world (Galerkin 1915; Arakawa. 1966; Zeng 1979; Simmons and Burridge 1981; Wang and Ji 1990; Zeng and Zhang 1982). The toughest aspect has been the construction of time discretized conserving schemes. However, an energy perfect conserving semi–implicit scheme of a sigma–coordinate primitive equation has been successfully formulated and realized (Zhong 1994, 1997). By using NCEP/ NCAR 1958–1998 data and this new total energy conserving semi–implicit dynamical model, this paper is to deal with such a question, that is, is the achieving of a high–order total energy conserving scheme significant to the elimination of computational systematical errors and improvements of weather and climate numerical prediction?

In section 2 of this paper, one formulation theorem of time–difference fidelity schemes, capable of retaining the general cubic physical conservation law, will be formulated and proved. Based on this theorem the total energy conserving (semi–implicit) time–difference scheme for global spectral models of baroclinic primitive equations can be constructed. In section 3, by using the NCEP/ NCAR 1958–1998 data, a comparison experiment of thirteen 30–day numerical integrations between a traditional and a new total energy conserving semi–implicit time–difference scheme for global spectral models of baroclinic primitive equations will be designed and conducted. Finally, in section 4, a summary of the present work is given.

2. The mathematical theorem

The operator equation of the evolution problem can be stated as

$$\frac{\partial u}{\partial t} + Au = 0 \ . \tag{1}$$

Based on a general compensation principle and the inverse formulation method of a fidelity scheme (Zhong 1992a, 1992b), a general physical conservation law time–difference fidelity scheme of Eq.(1) can be written as

$$\frac{u^{n+1} - u^n}{\Delta t} + (A^n - A_L^n)u^n + A_L^n u^{n+1} + \Delta t \varepsilon^n B^n u^n = 0 \ . \tag{2}$$

Here, A_L is an auxiliary formulation operator, ε^n an undetermined compensation coefficient, and $B^n u^n$ a compensation operator.

In particular, if operator Eq.(1) yields a general cubic conserving integral property,

$$\int (A_1 u \cdot A_2 u \cdot A_3 Au + A_1 Au \cdot A_2 u \cdot A_3 u + A_1 u \cdot A_2 Au \cdot A_3 u) d\sigma = 0 \ , \tag{3}$$

then the following theorem holds true. Here, A_1, A_2, and A_3 are all bounded space operators independent of u and t; $d\sigma$ is a space integral element.

Theorem: Suppose the compensation coefficient ε^n satisfies

$$\Delta t^4 \gamma_1 (\varepsilon^n)^3 + \Delta t^2 \gamma_2 (\varepsilon^n)^2 + \gamma_3 \varepsilon^n + \gamma_4 = 0 \ , \tag{4}$$

and its order of magnitude is $O(\Delta t^0)$, then scheme (2) is a fidelity scheme with a cubic conserving integral property and is compatible to Eq.(1) where

$$\gamma_1 = \int A_1 LB^n u^n \cdot A_2 LB^n u^n \cdot A_3 LB^n u^n d\sigma \ , \tag{5}$$

$$\begin{aligned}\gamma_2 = -\int (&A_1 LB^n u^n \cdot A_2 LB^n u^n \cdot A_3 Mu^n + A_1 LB^n u^n \cdot A_2 Mu^n \cdot A_3 LB^n u^n \\ &+ A_1 Mu^n \cdot A_2 LB^n u^n \cdot A_3 LB^n u^n) d\sigma \ ,\end{aligned} \tag{6}$$

$$\begin{aligned}\gamma_3 = -\int (&A_1 LB^n u^n \cdot A_2 Mu^n \cdot A_3 Mu^n + A_1 Mu^n \cdot A_2 LB^n u^n \cdot A_3 Mu^n \\ &+ A_1 Mu^n \cdot A_2 Mu^n \cdot A_3 LB^n u^n) d\sigma \ ,\end{aligned} \tag{7}$$

$$\begin{aligned}\gamma_4 = -\int (&A_1 Ku^n \cdot A_2 A^n u^n \cdot A_3 A^n u^n + A_1 A^n u^n \cdot A_2 u^n \cdot A_3 A^n u^n \\ &+ A_1 A^n u^n \cdot A_2 A^n u^n \cdot A_3 u^n + A_1 LA_L^n A^n u^n \cdot A_2 Mu^n \cdot A_3 Mu^n \\ &+ A_1 Ku^n \cdot A_2 Mu^n \cdot A_3 LA_L^n A^n u^n + A_1 Ku^n \cdot A_2 LA_L^n A^n u^n \cdot A_3 Ku^n) d\sigma \ ,\end{aligned} \tag{8}$$

where $K = I - \Delta t A^n$, $M = I - \Delta t A^n + \Delta t^2 LA_L^n A^n$, L is an inverse operator of $I + \Delta t A_L^n$, and I is a unit operator.

Based on this formulation theorem, a total energy conserving semi–implicit time–difference scheme for global spectral–vertical finite–difference models of baroclinic primitive equations has been formulated and realized (Zhong 1994, 1997).

3. Comparative experiments of real data monthly numerical integrations

3.1 *Design of experiments*

(1) By further performing monthly integrations of the total energy conserving semi–implicit scheme using the NCEP / NCAR winter reanalysis data, the present work attempts to test the feasibility of long–term integrations of the scheme under the complex conditions more closely approaching the actual atmosphere, and meanwhile to test the capability of the scheme in preserving high–order total energy conservation in long–term integration operations. In the past, long–time feasibility tests were conducted respectively under idealized conditions using the relatively simplified energy and enstrophy conserving explicit scheme for spectral models of the barotropic vorticity equation and energy, and the enstrophy and angular momentum conserving semi–implicit scheme for spectral models of barotropic primitive equations, although the results turned out to be satisfactory (Zhong 1993, 1995), and only the uninitialized summer FGGE data comparative experiments have been conducted (Zhong 1997).

(2) With identically the CCM3 T42 topography and identical real–data initial conditions, the present work also attempts to further perform comparative experiments with monthly dynamic integrations between the traditional ECMWF–type semi–implicit scheme and the total energy conserving semi–implicit fidelity scheme. The purpose is to test the contribution of the sources or sinks of the systematic errors concerning energy conservation due to the traditional time difference (called type Z systematic errors for short) to the total systematic errors and total errors, under the complex conditions of the coexistence of internal systematic errors with meteorological backgrounds more closely approximate to routine operation. Meanwhile, the paper also aims to test the possibilities and potentials of improving actual monthly, even medium–range forecasts by formulating a total energy conserving (semi–implicit) fidelity scheme and thereby eliminating corresponding types of systematic errors. In the past, comparative experiments have been respectively conducted under idealized conditions between the

relatively simplified energy conserving explicit scheme for a barotropic vorticity equation spectral model, energy conserving semi–implicit scheme for a spectral model of the barotropic primitive equations, and the corresponding traditional scheme. The results revealed marked differences after long time integrations (Zhong 1995), and also only the uninitialized 1979 summer FGGE data comparative experiments have been conducted (Zhong 1997).

Since in the above–described models there is no internal physical process involved in the atmosphere other than the pure dynamic process, and the initial data is used without initialization, implying that the initial value of integrations is far from being perfect, it is, consequently, no surprise that there exist several corresponding systematic error sources or sinks in the model of the total energy conserving fidelity scheme. If the total energy conserving fidelity scheme model is set as a control standard, then it follows that in addition to the systematic error sources in the fidelity scheme, there also exist the sources or sinks of type Z errors in the traditional model due to the failure to retain total energy conservation in the traditional semi–implicit time–difference process.

3.2 *Experiment models and data*

By using the new type of total energy conserving semi–implicit time–difference scheme and the traditional semi–implicit time–difference for a global spectral vertical finite–difference model (T42 / L9) of the baroclinic primitive equations inclusive of the same CCM3 model topography, the present work performs two sets of numerical experiments, each having the same monthly integrations. The initial field of these integrations is the NCEP / NCAR 1958–1998 data without initialization dated 1 January for the respective year. For all the calculations in this paper, the integral time step is $\Delta t = 30$ minutes.

3.3 *Results of experiments*

3.3.1 *Feasibility and improvements of essential properties*

Results of the 30–day numerical integrations of the traditional scheme and the total energy conserving fidelity scheme all suggest that there are notable improvements on the essential properties of total energy and mass conservation of the new type of scheme in contrast with the traditional one. In most of the integral cases, both schemes can integrate smoothly within the whole monthly integral time. The 1998 integral case is given as an example in Fig. 1. For those initiated from 1960, 1961, 1967, 1970, 1977, 1978, 1980, 1987, 1994, and 1996, the nonlinear computational instability of the traditional scheme occurs during the integral time, with the 1996 integral case is given as an example (see Fig. 1). However, for the new scheme, every computational instability can be eliminated completely. Moreover, the new scheme, as the experiments demonstrate, can noticeably modify the deviations from the high–order total energy and mass global integral conservation characteristics of the traditional scheme, although such deviations, increasing monotonously with the growth of the integration time, are not great in quantity on the monthly integral time scale. Take the 30th day of integrations initiated from 1998 for instance; these relative deviations from the conservation variables are all close to some 0.0001. Results of Fig. 1 also demonstrate that without adopting stability methods of artificial time smoothing or filtering, which have side effects, the fidelity scheme can integrate smoothly over a long time. The theorem as formulated in the paper can surely be applied to solving the formulation problems of high–order total energy conserving time–difference schemes for baroclinic primitive equations, and that of the high–order total energy conserving semi–implicit fidelity schemes for global spectral–vertical

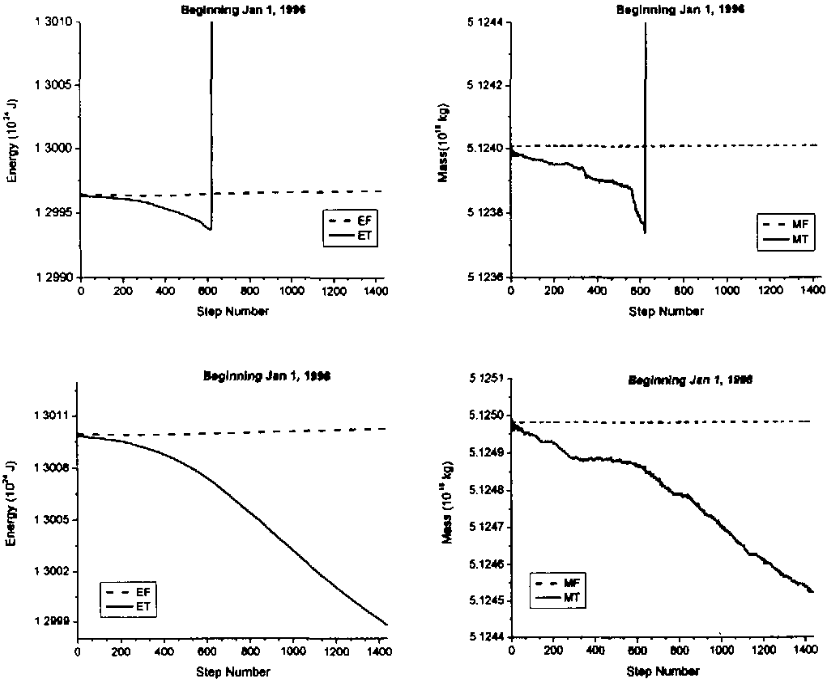

Fig. 1. The 30–day computational variation of the conservative global integration of the total energy and mass of the baroclinic primitive equations in the winter of 1958–1998. EF: computational variation of the total energy using the new fidelity scheme, ET: of the total energy using the traditional scheme, MF: of the mass using the fidelity scheme, MT: of the mass using the traditional scheme.

finite–difference models of baroclinic primitive equations is also applicable to long–term numerical integrations using real data without initialization.

3.3.2 *The contributions of type Z error to the total systematic error of monthly predictions*

Errors derived from the average predictive field of a number of individual integrations represent an important aspect of total errors of the model. The predictive systematic errors in the paper are referred to as errors of the 30–day average predictive field obtained from all the monthly integrations.

Compare the evolution curve of the RMS error of the 850 hPa geopotential height average field between the total energy conserving semi–implicit fidelity scheme and the traditional semi–implicit scheme exclusive of model topography under identical conditions of integration, it can be easily seen that the contribution of the type Z systematic error (forecast RMS error of the traditional scheme with that of the fidelity scheme deducted) to the total RMS error can approximately reach one–fourth of the total RMS systematic error at the end of the second week of integration and about half of its total amount four weeks afterwards (see Fig. 2). Further analysis reveals that a large part of the positive contributions comes from

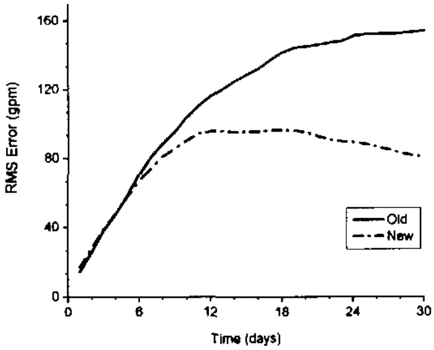

Fig. 2. The 30–day computational variation of the RMS error of the average of the 850hPa height initiated from 1 January 1958–1998 deducted 1960, 1961, 1967, 1970, 1977, 1978, 1980, 1987, 1994, and 1996. Old: the traditional scheme, New: the new fidelity scheme.

improvements of the extra–long wave components (see Fig. 4).

If the type of errors in extending the forecasting period can in a sense reflect the model climate systematic errors associated with climate drift, then it is by no means impossible to contribute much to the solving of climate drift by way of formulating new types of high–order total energy conserving schemes and thereby eliminating corresponding type Z systematic errors in the traditional schemes.

3.3.3 *The average improvement of total errors of monthly integrations*

The average of predictive errors derived from a number of numerical integrations of a model represent the average level of the total predictive errors of the model.

With the daily average value of the RMS error of the 850 hPa geopotential height of the 1958–1998 30–day integrations using the traditional semi–implicit scheme inclusive of CCM3 topography (called Scheme A for short) as a reference, it can be seen that with the coexistence of multiple error sources or sinks, the average improvement of total RMS error obtained by formulating a total energy conserving fidelity scheme (called Scheme B for short) and thereby eliminating the corresponding type Z errors in Scheme A, can reach approximately one–fifth of the total RMS errors at the end of the second week of integration and even about two–fifths of the total four weeks afterwards (see Fig. 3). Further analysis also reveals that a large part of the positive contributions comes from improvements of the extra–long wave components (see Fig. 5). This demonstrates well the great potentials of Scheme B. It also suggests that the advantages of Scheme B primarily focus on long–term numerical integrations.

In the same way, the improvements of predictions can be clearly confirmed by comparison among the geopotential height fields with the corresponding observation field deducted. Figure 6 indicates, averagely speaking, for the deviations in Scheme A, Scheme B reduces them over 60 percent at 500 hPa and about 50 percent at 850 hPa in the largest negative error center (also the maximum global error center in view of absolute value) at the Arctic Pole; in the sublargest negative error center over the Antarctic Pole, Scheme B reduces them one–third at 500 hPa and one–fourth at 850 hPa. Moreover, in the largest positive error center at the mid–latitudes of Northern Hemisphere, Scheme B also clearly reduces them at both 500 hPa and 850 hPa. The striking differences between the two schemes in framework and

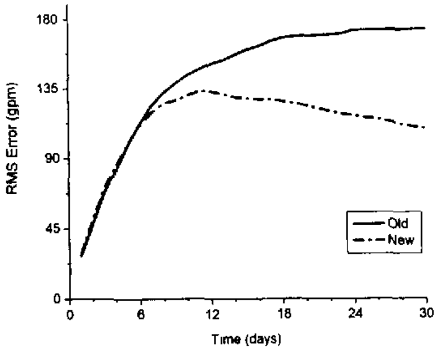

Fig. 3. As in Fig. 2. for the average of the RMS error of the 850 hPa height.

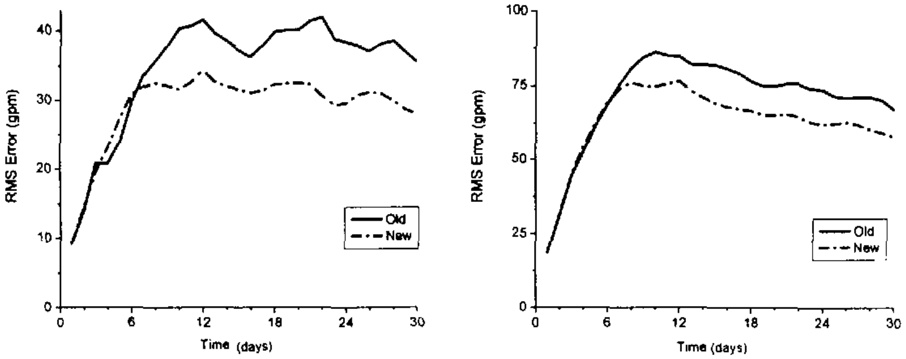

Fig. 4. As in Fig. 2. for the 1–3 waves (extra–long wave components) RMS error of the average of the 850 hPa height.

Fig. 5. As in Fig. 2. for the the average of the 1–3 waves (extra–long wave components) RMS error of the 850 hPa height.

subtleties also reasonably depict the entirely different nature of the two types of errors. In general, on the 30th day of integration, Scheme B certainly has better global performance than Scheme A in the middle and lower troposphere. This is consistent with the calculated results of RMS errors (see Figs. 2–5).

4. Summary

A total energy conserving semi–implicit time–difference scheme of baroclinic primitive equations can be realized by the theorem and the nonlinear computational instability of the traditional scheme can be eliminated completely without adopting stability methods of artificial time smoothing or filtering (see Fig. 1).

The RMS error of averaged forecast height in the lower troposphere, for example, at 850 hPa, can be improved clearly after the first integral week and the reduction can reach 50 percent of systematic error and about 40 percent of total error within monthly integration (see Figs. 2 and 3).

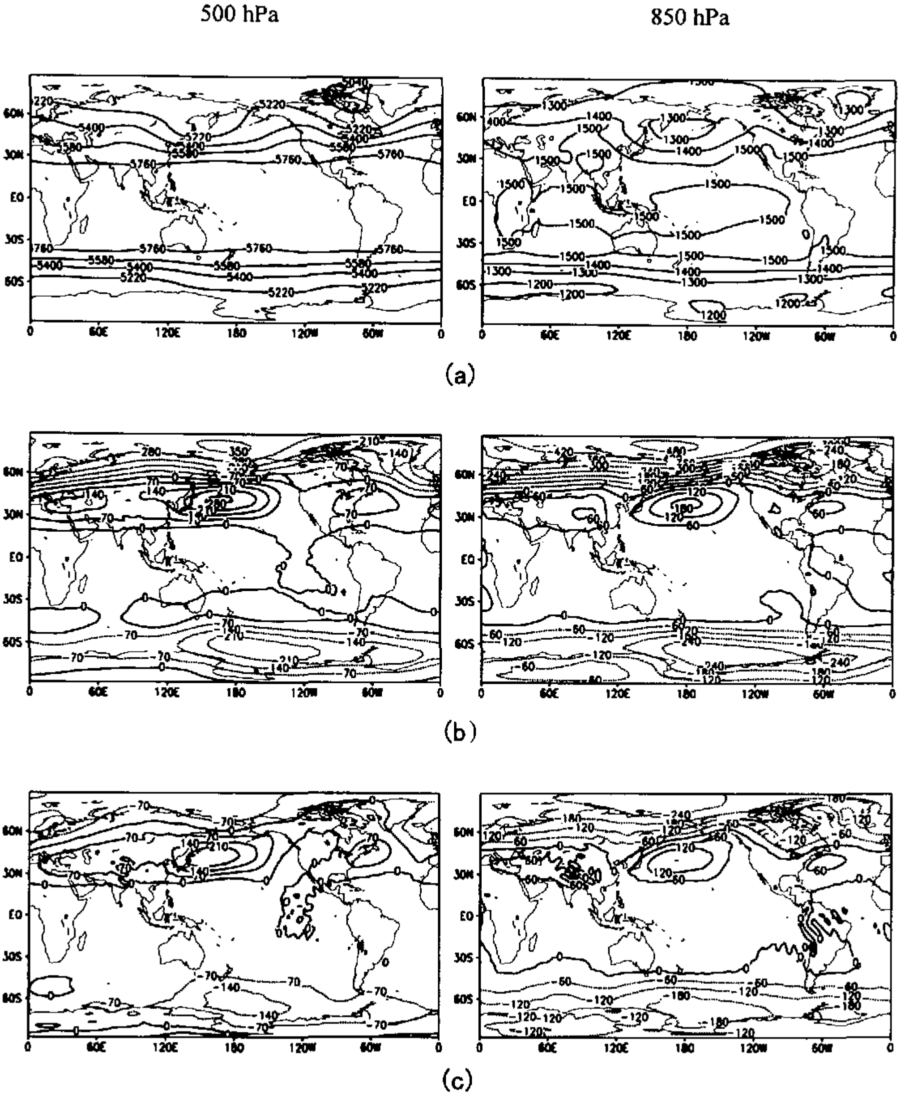

Fig. 6. The 30-day averaged height global contour chart of the thirty-one integrations initiated from 1 January 1958-1959, 1962-1966, 1968-1969, 1971-1976, 1979, 1981-1986, 1988-1993, 1995, and 1997-1998. (a) Observational 500 / 850 hPa height; (b) Forecast height using the traditional scheme with corresponding observational height deducted; (c) Forecast height using the fidelity scheme with the corresponding observational height deducted.

The error of monthly mean height in the middle and lower troposphere at the second, third, and even fourth integral week can also be largely reduced and some well-known systematical defects can be greatly improved, for example, like these at 500 hPa (see Fig. 6).

A large part of the positive contributions comes from improvements of the extra–long wave components (see Figs. 4 and 5).

It may be stated that functions of revising small digit internal deviation from the energy conservation characteristic is by no means small, and the achieving of a high–order total energy conserving scheme is of significance to the elimination of computational systematical errors and improvements of either weather or climate numerical predictions.

Besides these, the results concerning the improvements of the upper model atmosphere will be presented in a separate paper as it concerns fidelity computations of physical dissipation and more complex interactions between the physical conservation laws and the physical dissipation and others, and is therefore beyond conservative system modeling.

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一类计算性系统误差消除与斜压原始方程天气气候模式改进

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摘　要

斜压原始方程半隐式全能量守恒格式的构造问题长期没有解决。本研究在成功地构造实现其全能量完全守恒的半隐式方案基础上，进行了此守恒方案与欧洲中期天气预报中心(ECMWF)的σ-坐标原始方程全球谱模式半隐式方案间的实际资料对比实验。实验表明，850 hPa 平均预报高度场 RMS 误差在积分一周以后得到明显改进，到第 30 天其预报误差降低达到了 50%．进一步的对比实验表明，对流层中部和下部的月预报平均高度场 RMS 误差也显著降低，而且一些明显的系统性误差也得到大幅度改进。更加详细的分析显示，这些收益的很大一部分是从超长波成分的改进中得到的。这说明，通过构造守恒性时间差分方案消除了响应的计算性系统误差源汇，进而能够使模式气候漂移得到显著改进，而这种误差源汇存在于传统的、现仍被普遍采用的斜压原始方程天气气候模式中。

关键词: 保真方案，计算性系统误差，斜压原始方程