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The Influence of Convergence Movement on Turbulent Transportation in the Atmospheric Boundary Layer

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ABSTRACT

Classical turbulent K closure theory of the atmospheric boundary layer assumes that the vertical turbulent transport flux of any macroscopic quantity is equivalent to that quantity's vertical gradient transport flux. But a cross coupling between the thermodynamic processes and the dynamic processes in the atmospheric system is demonstrated based on the Curier-Prigogine principle of cross coupling of linear thermodynamics. The vertical turbulent transportation of energy and substance in the atmospheric boundary layer is related not only to their macroscopic gradient but also to the convergence and the divergence movement. The transportation of the convergence or divergence movement is important for the atmospheric boundary layer of the heterogeneous underlying surface and the convection boundary layer. Based on this, the turbulent transportation in the atmospheric boundary layer, the energy budget of the heterogeneous underlying surface and the convection boundary layer, and the boundary layer parameterization of land surface processes over the heterogeneous underlying surface are studied. This research offers clues not only for establishing the atmospheric boundary layer theory about the heterogeneous underlying surface, but also for overcoming the difficulties encountered recently in the application of the atmospheric boundary layer theory.

Key words: linear thermodynamics, turbulent transportation, cross coupling, atmospheric boundary layer, heterogeneous underlying surface

1. Introduction

Hu (2002a) analyzed the difficulties of applying linear thermodynamics to the atmosphere to indicate the necessity of introducing the dynamic processes and the turbulent transportation into the entropy equilibrium equation of nonequilibrium thermodynamics in the environment fluid. Wherefore the classical entropy equilibrium equation of nonequilibrium thermodynamics was modified. The theorem of minimum entropy production was proved and the Lyapounov thermodynamic stability function of the ideal fluid has been obtained based on the modified entropy equilibrium equation (Hu, 2002b). Further, the entropy equilibrium equation that can apply to the atmospheric system has been established (Hu, 1999; Hu, 2002c). Sequentially based on these, the linear thermodynamics of the atmospheric system has been established (Hu, 2002d). The application of linear nonequilibrium thermodynamics to the atmospheric system is studied to gain a series of important theoretic results that are rare. It has been demonstrated that there exist indeed the linear phenomenological relations based on a great number of observational facts of the atmospheric boundary layer to obtain the relationships between the linear phenomenological coefficient and the turbulent transport coefficient using those observational facts, (Hu, 2002e).

The cross coupling is an important phenomenon of the linear thermodynamics (De Groot and Mazur, 1962). Hu (2002d) has proved the cross coupling between the heat turbulent transportation and the vapor turbulent transportation in the atmospheric system in the theorization. And it has been proved also that the relation between the geostrophic wind and the thermal wind in the atmospheric system is a special cross cou-

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pling phenomenon between the dynamic processes and the thermodynamic processes (Hu, 2002e). This article seeks to prove another cross coupling phenomenon between the dynamic processes and the thermodynamic processes in the atmospheric system, namely how the movement of divergence or convergence affects the turbulent transportation in the atmosphere. This theoretical result of linear thermodynamics may offer a key to deal with the subject of turbulent transportation under the condition of the heterogeneous underlying surface.

2. Principle of cross coupling between the dynamic processes and thermodynamic processes along with its influence on the turbulent transportation

Based on the Curier-Prigogine principle of linear thermodynamics, there exists a cross coupling between the airflow and the heat flux, because they are entirely vector quantities. Considering the Onsager reciprocal relation, the cross coupling relations between the airflow and the heat flux are as follows (Hu, 2002e):

$$\begin{aligned}\mathbf{J}_{\theta j} =& L_\theta \frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right) \\ &+ L_{\theta\mathrm{a}}\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_j + \mathbf{g}\delta_{i3} - f_c\varepsilon_{ij3}\mathbf{U}_j\right) , \quad (1)\end{aligned}$$

$$\begin{aligned}\rho\mathbf{U}_i =& L_\mathrm{a}\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + \mathbf{g}\delta_{i3} - f_c\varepsilon_{ij3}\mathbf{U_j}\right) \\ &+ L_{\theta\mathrm{a}}\frac{\partial}{\partial x_j}\left(\frac{1}{\theta}\right)\delta_{ij}\varepsilon_{ij3} . \quad (2)\end{aligned}$$

Here, all symbols are as in Hu (2002e). Relation (1) shows that the heat turbulent transport flux is related not only to the potential temperature gradient but also to the departure from the dynamic balance, owing to the cross coupling between the dynamic processes and the thermodynamic processes. In the same reasoning, relation (2) shows that the airflow is related not only to the departure from the dynamic balance but also to the potential temperature gradient.

The left side in relation (2) is the airflow. Its vertical component can be written in the following form

$$\rho W = L_g \frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial z} + g\right) - L_{\theta p}\frac{1}{\theta^2}\frac{\partial \theta}{\partial z} . \quad (3)$$

Here, W is the average vertical velocity; L_g and $L_\theta p$ are the relevant phenomenological coefficient and the cross coupling coefficient, respectively. Relation (3) shows that any atmospheric departure from the static balance or from neutral stratification causes vertical velocity. On the other hand the atmosphere is supposed as an incompressible fluid, so the continuity equation is

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0 . \quad (4)$$

The vertical component of relation (1) can be written

$$\begin{aligned}J_{\theta z} = H\,|_z = \rho c_p \overline{w'\theta'} &= -\rho c_p K'_\theta \frac{\partial\theta}{\partial z} \\ &+ L_{\theta\mathrm{a}}\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial z} + g\right) , \\ K'_\theta = \frac{L_\theta}{\rho c_p \theta^2} , & \quad (5)\end{aligned}$$

in which the turbulent transport coefficient K'_θ is a linear function of the phenomenological coefficient L_θ (Hu, 2002e). Using relation (3), we can eliminate the second term on the right side of relation (5)

$$J_{\theta z} = H\,|_z = \rho c_p \overline{w'\theta'} = -\rho c_p K_\theta \frac{\partial\theta}{\partial z} + \rho\frac{L_{\theta\mathrm{a}}}{L_g}W . \quad (6)$$

Then using relation (4), we can eliminate the vertical velocity on the right side of relation (6) to obtain the vertical component of heat turbulent flux

$$\begin{aligned}J_{\theta z} = H\,|_z = \rho c_p \overline{w'\theta'} &= -\rho c_p K_\theta \frac{\partial\theta}{\partial z} \\ &- \rho c_p K_{\theta w}\int_0^z (\nabla\,|_\mathrm{h} \cdot \mathbf{V})dz , \\ \nabla\,|_\mathrm{h} \cdot \mathbf{V} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} . & \quad (7)\end{aligned}$$

In relations (6) and (7), the turbulent transport coefficient K_θ of vertical heat transportation caused by the potential temperature gradient and the relevant cross coupling coefficient $K_{\theta w}$ are defined, respectively, as

$$\begin{aligned}K_\theta &= \left(L_\theta - \frac{L_{\theta\mathrm{a}}L_{\theta p}}{L_g}\right)\frac{1}{\rho c_p \theta^2} , \\ K_{\theta w} &= \frac{L_{\theta\mathrm{a}}}{c_p L_g} . \quad (8)\end{aligned}$$

The turbulent transport coefficient and the cross coupling coefficient must be determined by an observation experiment. In formulae (6) and (7), the vertical component of heat turbulent flux at height z is $H\,|_z$. Relation (6) shows that the vertical component of heat turbulent flux is relative to the vertical potential temperature gradient and the vertical velocity. And, relation (7) predicates that the vertical component of heat turbulent flux is composed of both the transportation of the vertical potential temperature gradient and the horizontal convergence or divergence movement.

Analogously, the airflow and the vapor transport flux are moreover entirely vector quantities; so based on the Curier-Prigogine principle there exists also the cross coupling relation

$$\mathbf{J}_{vj} = L_{\mathrm{v}}\frac{\partial}{\partial x_j}\left(\frac{\triangle\mu}{T}\right)_{p,T} + L_{\mathrm{va}}\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial x_j}\delta_{ij} + \mathbf{g}\delta_{i3} - f_{\mathrm{c}}\varepsilon_{ij3}\mathbf{U}_j\right) , \quad (9)$$

where L_{v} and L_{va} are the phenomenological coefficient of vapor transportation and the relevant cross coupling coefficient, respectively. Moreover the chemical potential difference between the dry air and the vapor is $\triangle\mu = \mu_{\mathrm{d}} - \mu_{\mathrm{v}}$, and the relationship between $\triangle\mu$ and the specific humidity q is (Hu, 1999)

$$\triangle\mu = \triangle c_p\left[T\ln\frac{T}{T_0} - (T - T_0)\right] - R_{\mathrm{V}}T\ln q - T\triangle R\ln\frac{p}{p_0} . \quad (10)$$

Here, $R_{\mathrm{v}}, \triangle c_p$, and $\triangle R$ are the vapor gas constant, the difference of the specific heat at constant pressure, and the gas constant difference between the dry air and the vapor, respectively.

Similar to the deduction of formula (7), and considering relation (10), the vertical component of relation (9) is written as

$$\mathbf{J}_{\mathrm{v}z} = -\rho K_{\mathrm{v}}\frac{\partial q}{\partial x_j} + L_{\mathrm{va}}\frac{1}{T}\left(\frac{1}{\rho}\frac{\partial p}{\partial z} + g\right) , \qquad K_{\mathrm{v}} = \frac{L_{\mathrm{v}}R_{\mathrm{v}}}{\rho q} , \quad (11)$$

in which the turbulent transport coefficient K_{v} is yet a linear function of the phenomenological coefficient L_{v}. Using relation (3) eliminates the second term on the right side of the above relation to give the vertical component of vapor turbulent flux

$$\mathbf{J}_{\mathrm{v}z} = -\rho K_{\mathrm{v}}\frac{\partial q}{\partial x_j} + \rho\frac{L_{\mathrm{va}}W}{L_g} + \frac{L_{\theta p}}{L_g}\frac{1}{\theta^2}\frac{\partial\theta}{\partial z} . \quad (12)$$

Then using relation (4), the vertical velocity on the right side of relation (12) is eliminated to obtain the vertical component of vapor turbulent flux

$$\mathbf{J}_{\mathrm{v}z} = E\mid_z = \rho\overline{w'q'} = -\rho K_{\mathrm{v}}\frac{\partial q}{\partial x_j} - \rho K_{\mathrm{v}\theta}\frac{\partial\theta}{\partial z} - \rho K_{\mathrm{v}w}\int_0^z(\nabla\mid_{\mathrm{h}}\cdot\mathbf{V})dz$$

$$K_{\mathrm{v}\theta} = -\frac{L_{\mathrm{va}}L_{\theta p}}{\rho L_g}\frac{1}{\theta^2} ; \quad K_{\mathrm{v}w} = \frac{L_{\mathrm{va}}}{L_g} . \quad (13)$$

In the above formula, $E\mid_z$ is the vertical component of vapor turbulent transportation flux at height z. K_{v} is the turbulent transportation coefficient caused by the vertical vapor gradient. Moreover $K_{\mathrm{v}\theta}$ and $K_{\mathrm{v}w}$ are the cross coupling coefficients that the potential temperature gradient influences on the vapor turbulent transportation and that the velocity divergence influences on it, respectively. Therefore the vertical component of vapor turbulent transport flux is constituted by three parts of the vapor vertical gradient transportation, the cross coupling effect between the potential temperature gradient transportation and the vapor transportation, and the cross coupling effect caused by the velocity horizontal convergence or divergence. For briefness, the cross coupling effect between the potential temperature gradient and the vapor transportation is neglected, so

$$J_{\mathrm{v}z} = E\mid_z = \rho\overline{w'q'} \approx -\rho K_{\mathrm{v}}\frac{\partial q}{\partial x_j} - \rho K_{\mathrm{v}w}\int_0^z(\nabla\mid_{\mathrm{h}}\cdot\mathbf{V})dz . \quad (14)$$

Formulae (7) and (14) show that the vertical heat turbulent flux and the vertical vapor turbulent flux both must include the vertical transportation caused by the convergence or the divergence movement, namely the vertical velocity of flow fields, except the gradient transport flux that is conversant. The convergence and the divergence of flow fields are a dynamic reversible process, but the turbulent transportation is a thermodynamic irreversible process. Therefore it is a cross coupling effect between the dynamic processes and the thermodynamic processes, and it is called the cross coupling principle between the dynamic processes and the thermodynamic processes.

It must be pointed out that the momentum flux is a tensor, but the heat flux and the vapor flux are entirely vector quantities. Based on the Curier-Prigogine principle, there exists no cross coupling between the momentum flux and the heat flux or the vapor flux. Namely, the momentum transportation is still of the classical form

$$\boldsymbol{\tau}_{ij} = -\rho\mathbf{K}_{ij}\frac{\partial \boldsymbol{U}_i}{\partial x_j} . \quad (15)$$

3. The turbulent transportation in the atmospheric boundary layer

If the convection of the atmospheric boundary layer develops sufficiently to the mixture layer, we have (Stull, 1988),

$$\frac{\partial\theta}{\partial z} = 0 ; \quad \frac{\partial U}{\partial z} = 0 ; \quad \frac{\partial V}{\partial z} = 0 . \quad (16)$$

So from formulae (7), (14), and (15), we obtain transportation flux of the heat, the vapor, and the momentum in the mixture layer, respectively,

$$J_{\theta z} = \rho c_p\overline{w'\theta'} = -\rho c_p K_{\theta w}\int_0^z(\nabla\mid_{\mathrm{h}}\cdot\mathbf{V})dz , \quad (17)$$

$$J_{vz} = \rho\overline{w'q'} = -\rho K_v \frac{\partial q}{\partial z} - \rho K_{vw} \int_0^z (\nabla\,|_h \cdot \mathbf{V}) dz\ , \tag{18}$$

$$\boldsymbol{\tau}_{13} = \rho\overline{w'u'} = 0\ ;\ \ \boldsymbol{\tau}_{23} = \rho\overline{w'v'} = 0\ . \tag{19}$$

Obviously, the heat transportation in the mixture layer is in direct ratio to the integral of the velocity divergence in the air column. The vapor transportation should include the contribution of the convergence or the divergence, except the vapor gradient transportation. But the momentum transportation equals zero in the mixture layer.

Formulae (7) and (16) assert that the heat turbulent flux, $J_{\theta j}$, should equal zero in the mixture layer, if the movement of convergence and divergence is not considered as being the same as in the classical theory of the atmospheric boundary layer. But the observed facts in the atmospheric boundary layer indicate that the heat turbulent flux, $J_{\theta j}$, is a limited value being degressive with height (Stull, 1998), and the potential temperature gradient equals zero, $\partial\theta/\partial z = 0$, to cause the turbulent transport coefficients to go to infinity and thus lose their physical significance. But formula (17) shows that the movement of convergence and divergence leads to the vertical heat transportation, because large eddy convection causes the convergence and divergence movement without fail under the condition of the powerful convection development. Sequentially it follows logically from the physical reason that the potential temperature gradient is equal to zero, but the heat turbulent transportation flux is also a non-zero-limited value.

The energy budget on the ground surface is a continuity condition of energy exchange at the ground surface. We use the formulae (7) and (14) to get

$$R_n - G = H + \lambda_v E + \rho c_p K_w \int_0^z (\nabla\,|_h \cdot \mathbf{V}) dz\ ,$$
$$K_w = K_{\theta w} + \frac{\lambda_v}{c_p} K_{vw}\ . \tag{20}$$

K_w in the above formula is the energy transport coefficient of movement of convergence or divergence. The energy budget equation (20) indicates that the available energy ($R_n - G$) equals the sum of the sensitive heat, the latent heat measured at height z, and the energy transportation effect of convergence movement at the height measuring the turbulent fluxes H and λE. Alone, as the energy transportation effect of convergence movement can be neglected, the available energy on the ground surface is equal just to the sum of the sensitive heat and the latent heat measured at height z which is the same as the conclusion of the classical theory of the atmospheric boundary layer.

The boundary layer parameterization of the land surface processes can start out from formulae (7), (14), and (15). If considering the energy transportation effect of the convergence movement, the boundary layer parameterization of the land surface processes should be rewritten as

$$H = -\rho c_p C_H (U - U_0)(T - T_0) - \rho c_p C_{Hw} \int_0^{z_i} (\nabla\,|_h \cdot \mathbf{V}) dz\ , \tag{21}$$

$$E = -\rho C_v (U - U_0)(q - q_0) - \rho C_{vw} \int_0^{z_i} (\nabla\,|_h \cdot \mathbf{V}) dz\ , \tag{22}$$

$$\boldsymbol{\tau} = -\rho C_D (U - U_0)^2\ . \tag{23}$$

In the above formulae, C_D, C_H, and C_v are the drag coefficients used customarily. C_{Hw} and C_{vw} are the cross coupling coefficients that the movement of convergence or divergence causes transportation of the heat and the vapor. Their values must be determined by an observation experiment, the same as for the drag coefficients.

The above formulae show that the parameterizations of heat flux and vapor flux in the atmospheric boundary layer must also add a term that is in direct ratio to the integral of velocity divergence in the air column that is from the ground surface to the top of the boundary layer. It is important that the influence of convergence and divergence movement on vertical transportation of the heat and the vapor is under the condition of the thermal heterogeneous underlying surface and in the convection boundary layer. In general, the air velocity field at the lower boundary of the free atmosphere is known for the parameterization of land surface processes in the meso-scale numerical models or general circulation numerical models. Thus we can estimate the velocity divergence values at the lower boundary of the free atmosphere. Consequently, so long as the cross coupling coefficients of the convergence and divergence movement are determined, we can estimate the influence of the convergence or divergence movement on vertical transportation of the heat and the vapor. The boundary layer parameterization scheme with formulae (21) and (22) may offer a more brief and reasonable scheme, which is a physical disposal scheme, of the boundary layer parameterization of land surface processes under the condition of the heterogeneous underlying surface.

4. Conclusion

The classical turbulent K closure theory of the atmospheric boundary layer assumes that the vertical turbulent transport flux of any macroscopic quantity is equivalent to that quantity's vertical gradient transport flux. But linear thermodynamics demonstrates that the atmospheric movement of convergence or divergence should influence the turbulent transportation. This is a special phenomenon of the cross coupling between the dynamic processes and the thermodynamic processes in the atmospheric system. It results in that the turbulent transportation of energy and substance in the atmospheric boundary layer should include the energy and substance transportation caused by the movement of convergence or divergence, except the transportation caused by the macroscopic quantity gradient.

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REFERENCES

De Groot, G. S. R., and P. Mazur, 1962: *Non-equilibrium Thermodynamics*. Chapter 11, North-Holland Publishing Company-Amsterdam,

Hu Yinqiao, 1999: Research on atmosphere thermodynamics in non-equilibrium state. *Plateau Meteorology*, **18**, 306–320. (in Chinese)

Hu Yinqiao, 2002a: Entropy equilibrium equation and dynamics entropy production in environment liquid. *Progress in Natural Science*, **12**(3), 180–184.

Hu Yinqiao, 2002b: Principle of minimum entropy production and thermodynamic stability in the thermodynamic nonlinear region. *Progress in Natural Science*, **12**(10), 1087–1089. (in Chinese)

Hu Yinqiao, 2002c: *Introduction to Atmospheric Thermodynamic-Dynamics*. Geology Press, Beijing, 406pp. (in Chinese)

Hu Yinqiao, 2002d: Application of the linear thermodynamics to the atmospheric system. Part I: Linear phenomenological relations and thermodynamic property of the atmosphere system. *Advances in Atmospheric Sciences*, **19**(3), 448–458.

Hu Yinqiao, 2002e: Application of the linear thermodynamics to the atmospheric system. Part II: Exemplification of the linear phenomenological relations in the atmosphere system. *Advances in Atmospheric Sciences*, **19**(5), 767–776.

Stull, R. B., 1988: *An Introduction to Boundary Layer Meteorology*. Kluwer Academic Publishers, Netherlands，666pp.