Estimating Heat Fluxes by Merging Profile Formulae and the Energy Budget with a Variational Technique

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ABSTRACT

A variational technique (VT) is applied to estimate surface sensible and latent heat fluxes based on observations of air temperature, wind speed, and humidity, respectively, at three heights (1 m, 4 m, and 10 m), and the surface energy and radiation budgets by the surface energy and radiation system (SERBS). The method fully uses all information provided by the measurements of air temperature, wind, and humidity profiles, the surface energy budget, and the similarity profile formulae as well. Data collected at Feixi experiment station installed by the China Heavy Rain Experiment and Study (HeRES) Program are used to test the method. Results show that the proposed technique can overcome the well-known unstablility problem that occurs when the Bowen method becomes singular; in comparison with the profile method, it reduces both the sensitivities of latent heat fluxes to observational errors in humidity and those of sensible heat fluxes to observational errors in temperature, while the estimated heat fluxes approximately satisfy the surface energy budget. Therefore, the variational technique is more reliable and stable than the two conventional methods in estimating surface sensible and latent heat fluxes.

Key words: heat flux, Bowen ratio, profile method, variational technique

1. Introduction

Surface sensible and latent heat fluxes are conventionally estimated by the Bowen ratio energy balance (BREB) method (Fritschen and Simpson, 1989), and the profile method (Panofsky and Dutton, 1984). The Bowen ratio method uses vertical gradients of temperature and water vapor pressure, and surface energy budget measured by the surface energy and radiation balance system (SERBS), but it does not use information provided by the similarity law and measurements of wind speed in the surface layer. It is well known that the Bowen ratio method becomes computationally unstable and results in spurious large values in the computed fluxes when the Bowen ratio is in the vicinity of −1. On the contrary, the profile method uses the formulae of the similarity law to compute surface heat fluxes, but it does not use information provided by surface energy budget measurements, so the computed fluxes often deviate from the surface energy balance. To fully take the advantages of the two conventional methods, Stewart and Thom (1973) and Thom et al. (1975) combined the two methods and proposed the energy budget determination of heat fluxes. In this method, the heat fluxes computed by profile-gradient formulae are corrected by a stability factor derived from surface energy balance and appropriate profile-gradient data. Later, Hu and Qi (1991) named the energy budget estimation of fluxes as a combined method, and also extended the method so that stability functions of the Monin-Obukhov similarity law can be retrieved. In the combined method, all variables are considered as deterministic quantities and model errors are also not taken into consideration, so it can be concluded that the combined method is a kind of deterministic method. A more realistic approach would be to consider observations as random variables. This is particularly necessary in atmospheric studies in which the observations represent some physical phenomenon that could never be perfectly known, and there will always be some unpredictable fluctuations that can only be represented by stochastic variables. In addition,

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both the profile formulae and the surface energy balance equation cannot be free of stochastic errors even if we assume that they are unbiased, so we should not forget their influence on the accuracy of the estimation of heat fluxes. Taking into consideration the errors of observations and models as well, Xu and Qiu (1997) first proposed a variational method (henceforth referred to as XQ's method) to estimate surface heat fluxes. It has been shown that XQ's method is more reliable and stable than the two conventional methods, however, the computed heat fluxes are still relatively sensitive to measurement errors, especially the errors in the observations of air temperature. Therefore, the method requires accurate measurements of temperatures at different heights in order to obtain reliable estimates of temperature gradients.

Based on the above considerations, a new cost function is formulated with a variational technique (VT) to tackle the above problems. In the new cost function, physical constraints are introduced in a very reasonable way to ensure that the method is more stable and reliable than the two conventional methods and XQ's method as well. Data collected by HeRES (China Heavy Rain Experiment and Study) Program during 11–30 June 2001 at Feixi experiment station are used to test the method. To achieve the goal, this paper is organized as follows: data and weather conditions are described in section 2; model equations and formulations are presented in section 3; the method is tested with data collected at Feixi experiment station, and results are compared with those computed by the profile method, the Bowen ratio method, and XQ's method in section 4; conclusions follow in section 5.

2. Data and weather conditions

The observational data used in this study were collected by the surface Bowen ratio energy balance system (SERBS) supported by the HeRES Program during 11–30 June 2001 at Feixi experiment station (31.41°N, 117.08°E) in the Huaihe River Basin. The ground surface around the station was flat and covered with short sparse grasses. Data were collected every 10 min (averaged), so there are 2880 time-level observations during selected 20-day periods. The SERBS included following measurements: 3-level horizontal wind speeds, temperatures, and water vapor pressures, respectively, at heights of 1 m, 4 m, and 10 m, net radiation flux R (near the surface), and soil heat flux G. Accuracies of measurements were $\pm 0.3 + 0.03\,\mathrm{m\,s^{-1}}$ for wind speed; $\pm 0.2^{\circ}$C for temperature; $\pm 1\%$ for relative humidity; and $\pm 5\%$ of the observed fluxes.

Weather during the observation period was relatively calm without severe thunderstorms or heavy rain. During 20 days there were 6 days (11, 12, 13, 17, 18, and 22 June) with recorded rainfalls ranging from 0.1 mm to 8.9 mm. After 22 June, there were no recorded rainfalls and the soil experienced a gradual drying period until the end of the month.

3. Model equations and method

Based on Monin-Obukhov similarity theory (Businger et al., 1971), nine independent residual differences between the model's estimates and corresponding observations can be described by the following equations:

$$\delta u_i = \frac{u_*}{k}\left[\ln\left(\frac{z_i}{z_{0\mathrm{m}}}\right) - \psi_{\mathrm{m}}\left(\frac{z_i}{L}\right) + \psi_{\mathrm{m}}\left(\frac{z_{0\mathrm{m}}}{L}\right)\right] - u_i\,, \qquad i = 1,2,3 \tag{1}$$

$$\delta T_i = T_{\mathrm{s}} + \frac{T_*}{k}\left[\ln\left(\frac{z_i}{z_{0\mathrm{h}}}\right) - \psi_{\mathrm{h}}\left(\frac{z_i}{L}\right) + \psi_{\mathrm{h}}\left(\frac{z_{0\mathrm{h}}}{L}\right)\right] - T_i\,, \qquad i = 1,2,3 \tag{2}$$

$$\delta q_i = q_{\mathrm{s}} + \frac{q_*}{k}\left[\ln\left(\frac{z_i}{z_{0\mathrm{q}}}\right) - \psi_q\left(\frac{z_i}{L}\right) + \psi_q\left(\frac{z_{0\mathrm{q}}}{L}\right) - q_i\,. \qquad i = 1,2,3 \tag{3}$$

Here, u_* is the frictional velocity defined by $u_* = \tau/\rho$ in association with wind stress τ and air density $\rho; T_*$ and q_* are flux temperature and humidity scale, respectively; T_{s} is surface skin temperature; (u_1, u_2, u_3), (T_1, T_2, T_3), and (q_1, q_2, q_3) are observed wind speeds, temperatures, and specific humidities, respectively, at heights of $z_1 = 1$ m, $z_2 = 4$ m, $z_3 = 10$ m; $z_{0\mathrm{m}}, z_{0\mathrm{h}}$ and $z_{0\mathrm{q}}$ are roughness lengths for momentum, heat, and humidity, respectively; $L = [u_*^2(T_1+T_2+T_3)/3]/kgT_*$ is Obuklov length; $k \approx 0.4$ is the Von Kámán constant; g is the gravitational acceleration; and $\psi_{\mathrm{m}}, \psi_{\mathrm{h}}$ and ψ_q are the stability functions. For the unstable case ($T_* < 0$ or $L < 0$), based on Paulson (1970), Dyer and Hicks (1970), and Hicks (1976), the stability functions ψ_{m} and ψ_{h} are taken as

$$\psi_{\mathrm{m}} = 2\ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2\arctan(x) + \frac{\pi}{2}\,, \tag{4}$$

$$\psi_{\mathrm{h},q} = 2\ln\left(\frac{1+x^2}{2}\right)\,, \tag{5}$$

where $x = (1-16z/L)^{1/4}$. For the stable case ($T_* > 0$, or $L > 0$), based on Holslag and DeBruin (1988) and

Beljaars and Holtslag (1991), the stability functions $\psi_{\rm m}$ and $\psi_{\rm h}$ are given by

$$-\psi_{\rm m} = a\frac{z}{L} + b\left(\frac{z}{L} - \frac{c}{d}\right)\exp\left(-d\frac{z}{L}\right) + \frac{bc}{d}\ , \quad (6)$$

$$-\psi_{{\rm h},q} = \left(1 + \frac{2az}{3L}\right)^{3/2} + b\left(\frac{z}{L} - \frac{c}{d}\right)\exp\left(-d\frac{z}{L}\right) + \frac{bc}{d} - 1\ , \quad (7)$$

here, $a = 1, b = 0.667, c = 5, d = 0.35$. The stability function $\psi_q = \psi_{\rm h}$ is assumed.

Residual error in the surface energy balance equation is denoted by

$$\delta E = R - G - F_{\rm SH} - F_{\rm LH}\ , \quad (8)$$

where R and G are respectively known observations of net radiation flux and soil heat flux. Sensible and latent heat fluxes are computed, respectively, by

$$F_{\rm SH} = -\rho c_p u_* T_*\ , \quad (9)$$

and

$$F_{\rm LH} = -\rho\lambda u_* q_*\ , \quad (10)$$

where C_p is the specific heat at constant pressure and λ is the latent heat of evaporation. Ideally, the residual error should be zero in Eq. (8). However, since the terms on right-hand side of Eq. (8) are measured or calculated with some errors and energy absorption by photosynthesis and respiration is neglected also, the right side of Eq. (8) is not generally equal to zero. Hence, these errors as a whole are represented by the residual error on the left side of Eq. (8).

Our problem is to find five optimal parameters $(u_*, T_*, q_*, T_{\rm s}, q_{\rm s})$ so that values of wind speed, potential temperature, and specific humidity computed by the profile formulae approximate the corresponding observations at three heights, and in addition, the computed fluxes should approximately satisfy the surface energy budget. As we know that the profile formulae are semi-empirical functions and are imperfect, all observations are not free of errors and it is most important that the number of observations is larger than that of the estimated parameters (overdeterminacy), so a deterministic algorithm like the combined method is not suitable for the situation. On the contrary, optimal statistical estimation theory can effectively cope with the above problems. If the probability distribution functions (PDFs) for the models and observations are estimatable, the maximum likelihood estimation will be the best choice (Lorenc, 1986). When the PDFs are multidimensional Gaussian functions, the maximum likelihood estimation changes into the variational technique with L_2 norms (Sasaki, 1970; Lorenc, 1986). Based on the above considerations, we adopt the variational approach to estimate the above five parameters by minimizing the following cost function:

$$J(u_*, T_*, q_*, T_{\rm s}) = \frac{1}{2}\left(\alpha_1\sum_{i=1}^{3}\delta u_i^2 + \alpha_2\sum_{i=1}^{3}\delta T_i^2 + \alpha_3\sum_{i=1}^{3}\delta q_i^2 + \alpha_4\delta E^2\right)\ . \quad (11)$$

Here, the first three terms on the right side of Eq. (11) measure the fits to the observed wind speeds, temperatures, and specific humidity, respectively, at three heights; the last term measures the fit to the surface energy balance. According to maximum likelihood theory, weights $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ in Eq. (11) should be inversely proportional to variances of the corresponding model's errors plus observational errors (Lorenc, 1986), but it is very difficult to compute the variances of models. So we estimate the weights only according to observational errors determined by the accuracy of the instruments. According to the accuracies of the instruments listed in section 2, we set $\alpha_1 = 0.5^{-2} = 4\ {\rm m^{-2}\ s^{-2}}, \alpha_2 = 0.2^{-2} = 25\ {\rm K^{-2}}$, $\alpha_3 = (2.2\times10^{-4})^{-2} = 2.066\times10^7$, and $\alpha_4 = 15^{-2} = 4.4\times10^{-3}\ {\rm W^{-2}\ m^4}$. Although the above weights cannot be accurate, fortunately, computations are found not to be sensitive to the weights in the vicinity (within the same orders of magnitudes) of these values.

The quasi-Newton algorithm is introduced to minimize the cost function J in Eq. (11). The iterative procedures consist of the following basic steps:

(1) Set the initial guesses of the unknowns, say, $u_* = 0.1\ {\rm m\ s^{-1}}, T_* = 0.01\ {\rm K}$, $q_* = 0, T_{\rm s} = 293.0\ {\rm K}$, and $q_{\rm s} = 0$.

(2) Calculate all intermediate parameters such as Monin-Obukhov length L, sensible heat flux $F_{\rm SH}$ and latent heat flux $F_{\rm LH}$, then compute $\delta u_i, \delta T_i$, and δq_i from (1)–(3), δE from (8), and the cost function J from Eq. (11).

(3) Calculate five gradient components of the cost function J with respect to $(u_*, T_*, q_*, T_{\rm s}, q_{\rm s})$.

(4) If the convergence criterion is satisfied ($|\nabla J| \leqslant 10^{-4}$), expected values $(u_*, T_*, q_*, T_{\rm s}, q_{\rm s})$ approximate the estimates at the minimum of the cost function J; otherwise, find a new search direction based on the gradients and the search direction of previous iterations.

(5) Determine the search step size and find the minimum of J along the search direction, obtain new estimates of $(u_*, T_*, q_*, T_{\rm s}, q_{\rm s})$, and return to step 2.

In this algorithm, step size is approximated by fitting a cubic curve through a number of points along the line of the search. To improve the convergence rate of iterations, $(u_*, T_*, q_*, T_s, q_s)$ should be properly scaled to make the super surface of the cost function J more spherical, or, say, less elliptical (Moore, 1991; Xu et al., 1994). In this paper, $(u_*, T_*, q_*, T_s, q_s)$ are scaled by (1.0 m s^{-1}, 1.0 K, 5×10^{-4}, 1.0 K, 5×10^{-4}). The convergence criterion is $|\nabla J| \leqslant 10^{-4}$. With the criterion, the minimization procedure is found to converge within no more than 20 iterative steps.

4. Test results and discussion

In this section, the two conventional methods, XQ's method, and the variational technique (VT) are all tested with data collected at Feixi station during 11–30 June 2001. Detailed results are examined and compared in the following subsections.

4.1 *The roughness length*

The surface momentum roughness length $z_{0\mathrm{m}}$ is an important parameter for the profile method. The local roughness length is related to the roughness characteristics of the surface. When the surface is non-smooth terrain covered with different types of vegetation, it is difficult to estimate the surface momentum roughness length (Xu and Qiu, 1997). In this paper, we use XQ's method to calculate it. We assume that $z_{0\mathrm{m}}$ does not change during the selected periods (20 days). For a selected value $z_{0\mathrm{m}}$, sensible and latent heat fluxes can be estimated by the profile method. Substituting the estimated sensible and latent heat fluxes, together with the observed values R and G, into (8) gives the residual value δE. For a total of $N = 2880$ time-level observations during the selected periods, the rms value of E is computed by

$$\varepsilon = \left(N^{-1} \sum_{n=1}^{N} \delta E_n^2 \right)^{1/2} . \quad (12)$$

It has been found that when $z_{0\mathrm{m}}$ is in the vicinity of 3 cm, ε reaches the minimum value, which suggests that $z_{0\mathrm{m}} = 3$ cm be used as the surface roughness length in order to closely satisfy the surface energy budget. The roughness length $z_{0\mathrm{h}}$ for heat flux can be related to $z_{0\mathrm{m}}$ by following the empirical formula (Thom, 1972):

$$z_{0\mathrm{h}} = z_{0\mathrm{m}} \exp(-kB^{-1}) . \quad (13)$$

Here, $B^{-1} = (T_s - T_0)/T_*$ is the Stanton number, T_s is surface skin temperature as in Eq. (2), and T_0 is aerodynamic temperature at the height of the momentum roughness length $z_{0\mathrm{m}}$. In the limit approaching a smooth surface, Eq. (13) results in $z_{0\mathrm{m}} \to 0, T_s \to T_0$, and $z_{0\mathrm{h}} \to z_{0\mathrm{m}}$. When the surface is not very smooth, T_s can be significantly different from T_0, and thus $z_{0\mathrm{h}}$ can be significantly different from $z_{0\mathrm{m}}$. Huband and Monteith (1986a, b) measured T_s and T_0 under nearly neutral condition and found that $T_s - T_0$ was about 1°C. According to Garratt and Hicks (1973) and Brutsaert (1982), when the atmospheric stratification changed from very unstable to stable conditions over flat grassland, T_* could decrease from 0.5°C to 0.5°C and $T_s - T_0$ could decrease from 3°C to –3°C. This yields B^{-1}=6, or $z_{0\mathrm{h}} \approx 0.1 z_{0\mathrm{m}}$ by using (13), and this value is used in this paper.

4.2 *Comparisons between the fluxes estimated by different methods*

Since direct eddy correlation measurements of sensible and latent heat fluxes are not available, the estimated fluxes will be first compared with the fluxes obtained with XQ's method which has been proved to be a precise method (Xu and Qiu, 1997; Zhou and Xu, 1999). Because discrepancies between the heat fluxes estimated by the VT and XQ's method are too small to tell their differences by observing the figures (not shown), the rms differences between them are computed. The rms difference for sensible heat flux is 2.6 W m^{-2} with the relative rms error 0.03% while for latent heat fluxes it is 3.0 W m^{-2} with the relative rms error 0.02%. On the other hand, for a total of $N = 2880$ time-level observations during 20 days, the rms value of δE in Eq. (12) is about 12 W m^{-2}, so the heat fluxes estimated by the VT approximately satisfy the surface energy balance. Zhu et al. (2003) reported a good consistency between the available energy $(R - G)$ and the sum of sensible and latent heat fluxes calculated by the eddy covariance method during the Huaihe River Basin Energy and Water Cycle Experiment (HUBEX)/IOP in 1998 and 1999. Therefore, based on the above two points, although we cannot conclude that each heat flux is accurately estimated, at least we know their sum is correct.

Secondly, sensible and latent heat fluxes are compared with those computed by the two conventional methods. A comparison between the fluxes computed by the profile method and the VT is plotted in Fig. 1a for sensible heat flux (F_{SH}) and in Fig. 1b for latent heat flux (F_{LH}). Similarly, a comparison between the fluxes computed by the Bowen ratio method and the VT is plotted in Fig. 2a for sensible heat flux and in Fig. 2b for latent heat flux. Figures 1a–b show that the two sensible heat fluxes are much closer to each other than the two latent heat fluxes, but relatively large discrepancies appeared around the midday on 14 and 19 June. Before the two days there were some rainfalls, which may be responsible for the discrepancies. In contrast with the Bowen ratio method (see Figs. 2a–b), the VT can effectively eliminate the spurious spikes in the heat fluxes when the Bowen ratio is close to –1. If the spurious spikes are not included, the fluxes computed

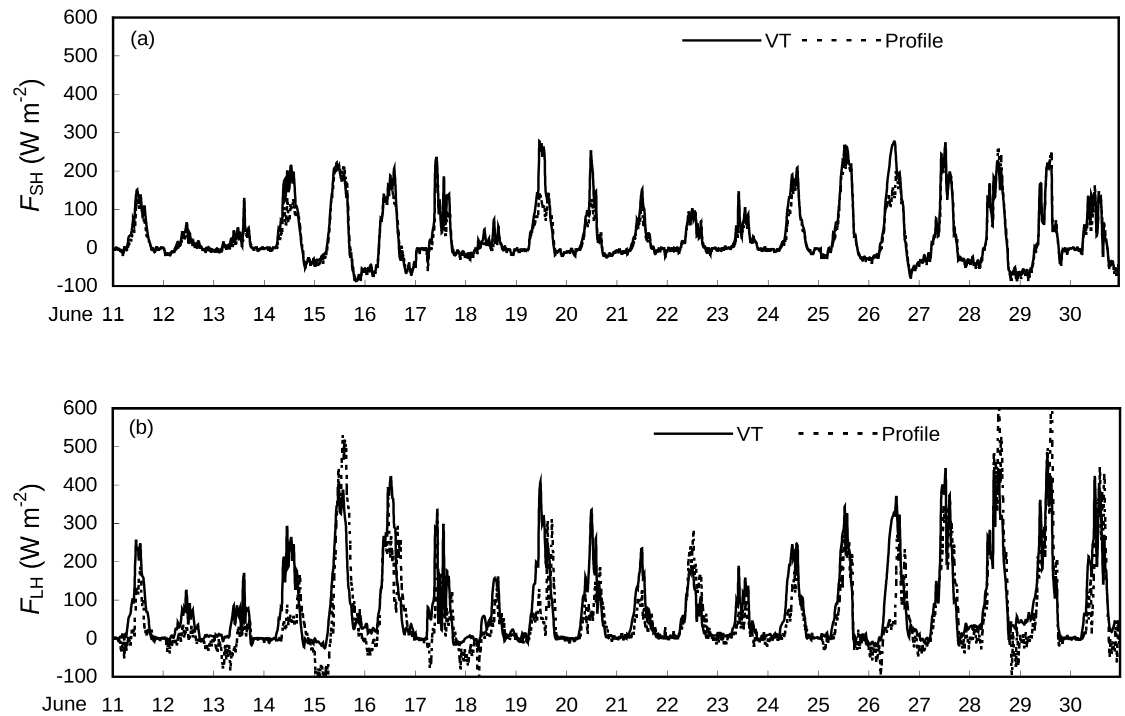

Fig. 1. Comparisons between the fluxes estimated by the variational technique (solid) and those computed by the profile method (dashed) for sensible heat (a) and latent heat (b).

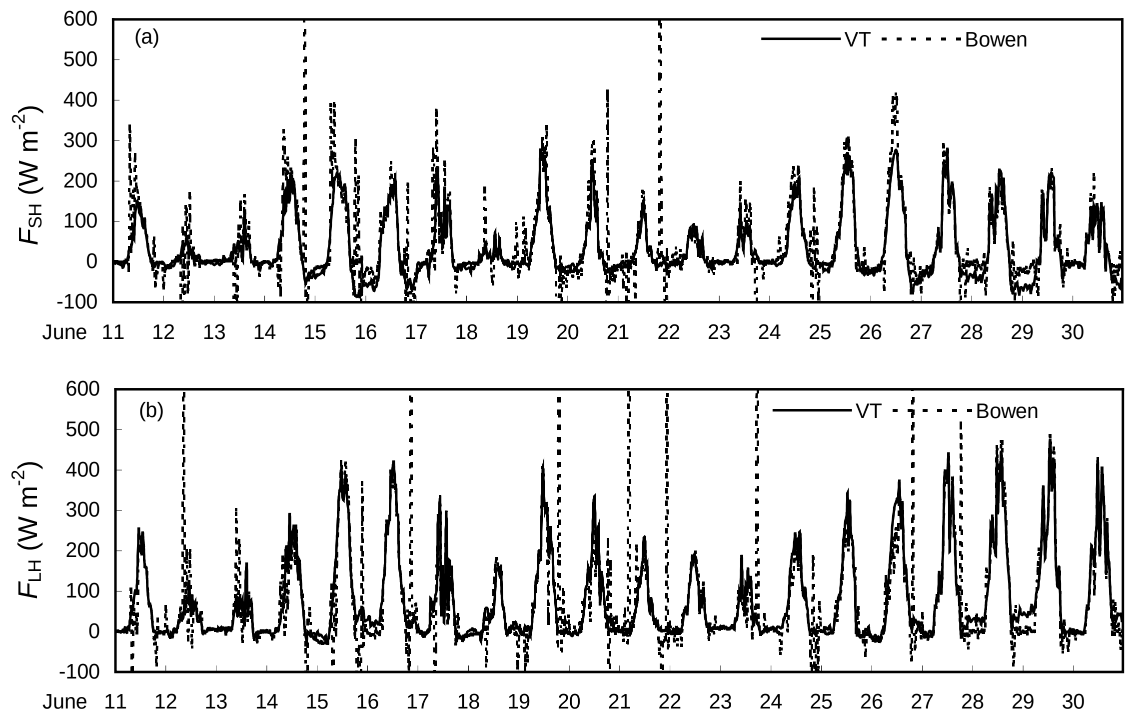

Fig. 2. Comparisons between the fluxes estimated by the variational technique (solid) and those computed by the Bowen ratio method (dashed) for sensible heat (a) and latent heat (b).

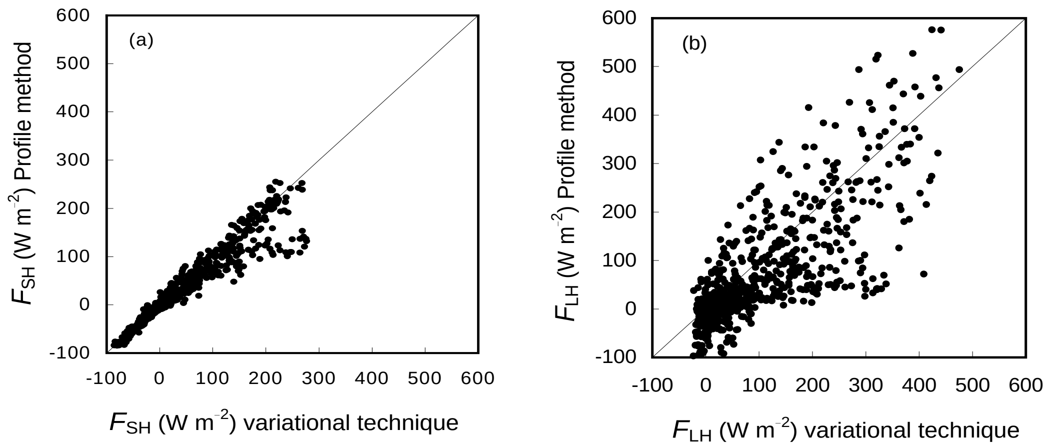

Fig. 3. Correlation diagrams between the fluxes estimated by the variational technique and those computed by the profile method for sensible heat (a) and latent heat (b).

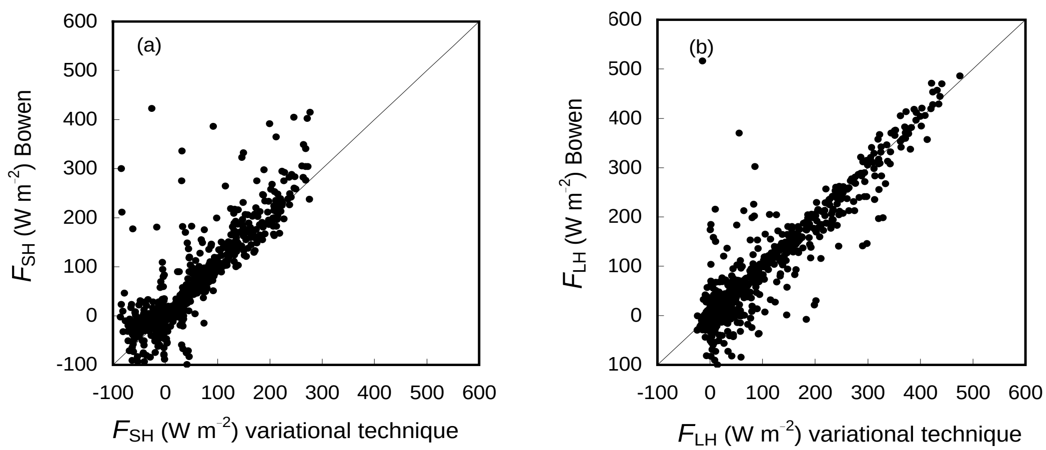

Fig. 4. Correlation diagrams between the fluxes estimated by the variational technique and those computed by the Bowen ratio method for sensible heat (a) and latent heat (b).

by the two methods are in good agreement. Smith et al. (1992) compared the fluxes obtained by the Bowen radio method with the direct eddy correlation measurements collected during the First International Satellite Land Surface Climatology Projects (ISLSCP) Field Experiment (FIFE), and found no serious problem for the consistency of heat fluxes obtained by the two methods (except for the heat fluxes when the Bowen ratio was in the vicinity of –1). Zhu et al. (2003) also found no large discrepancy between the Bowen ratio method and eddy covariance method in the Huaihe River Basin.

To show the overall comparisons, each pair of flux values computed by the two different methods are plotted as a point in their respective correlation diagrams (Figs. 3a–b and Figs. 4a–b). In Figs. 3a–b, the fluxes computed by the profile method and the VT are compared. As shown in Fig. 3a, the correlation points are mostly distributed around the diagonal line when sensible heat flux is small. In contrast with sensible heat fluxes, latent heat fluxes present larger deviations (see Fig. 3b). The rms value of δE is larger than 80 W m^{-2} by using the profile method, which means there is a large deviation from surface energy balance, while it is only 12 W m^{-2} by using the VT. Therefore, from the point of view of energy balance, the profile method gives very poor estimates of heat fluxes. In Figs. 4a–b the Bowen ratio method is compared with the VT and the overall agreements are quite good if we do not include the deviating points appearing when the Bowen ratio is in the vicinity of –1, but the VT can successfully overcome the unstable phenomenon by "absorbing" the similarity law.

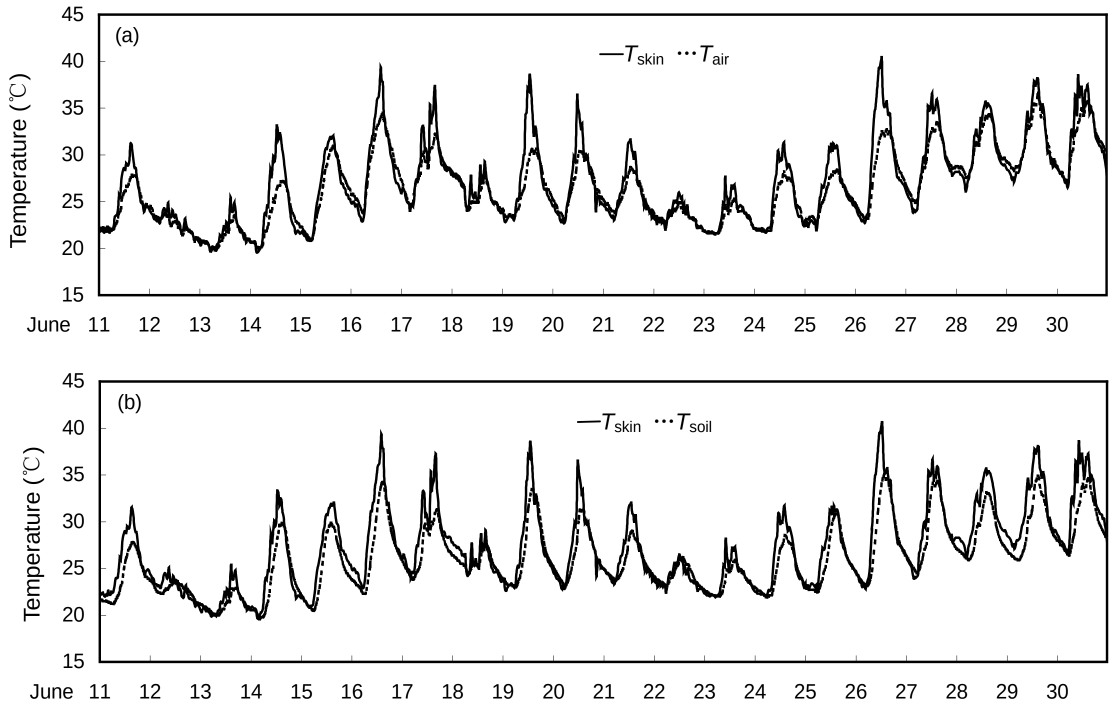

Fig. 5. Comparisons of the estimated surface skin temperatures, denoted by $T_{\rm skin}$, with measurements of air temperature at a height of 1 m, denoted by $T_{\rm air}$ (a), and soil temperatures at a depth of 0 cm, denoted by $T_{\rm soil}$ (b).

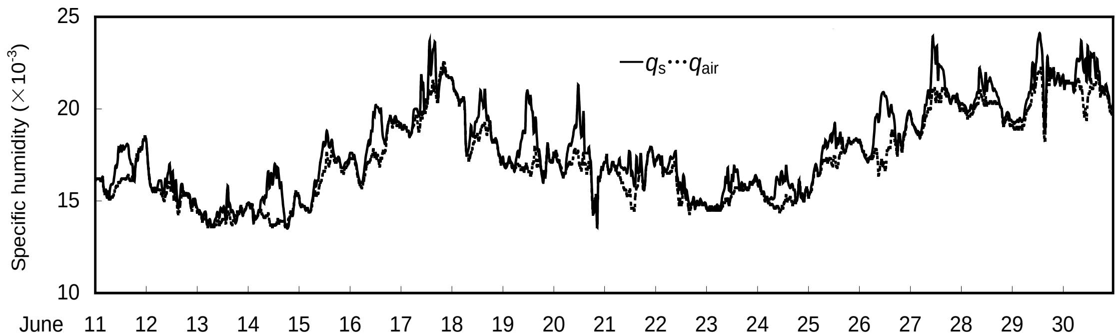

Fig. 6. Comparison between the estimated specific humidity at the height of $z_{\rm oh}$, denoted by $q_{\rm s}$, and measurements of specific humidity at a height of 1 m, denoted by $q_{\rm air}$.

4.3 *Surface skin temperature and specific humidity*

Surface skin temperature, i.e. surface radiative temperature, is the temperature at which the surface emits infrared radiation. Aerodynamically, skin temperature is sometimes regarded as the temperature at the heat-losing level $z_{0\rm h}$. As an important parameter in a surface-atmosphere system, surface skin temperature determines the loss or gain of sensible heat from the surface and relates surface characteristics to fluxes through energy balance near the surface. Although it can be either estimated by an infrared radiometer or derived from profiles of wind and temperature near the surface (Huban and Monteith, 1986a, b), it is difficult to measure in the presence of vegetation. In this paper, surface skin temperature is a retrieved parameter.

A comparison between the retrieved surface skin temperatures and air temperatures at a height of 1 m is shown in Fig. 5a, while a comparison between the retrieved surface skin temperatures and soil temperatures at a depth of 0 cm is plotted in Fig. 5b. The maximum discrepancy between the retrieved surface skin temperatures and air temperatures appeared around noncloudy midday, while the rest of the time, or on cloudy days or on rainy days, the differences are very small. Similar results are obtained for discrepancies between the retrieved surface skin temperatures and near-surface soil temperatures during daytime. At night, the retrieved surface skin temperatures are slightly higher than the near-surface soil temperatures, but their differences are too small to be detected.

In this paper, the specific humidity at $z_{0\mathrm{h}}$ is a retrieved parameter as well. A comparison between the retrieved specific humidity at $z_{0\mathrm{h}}$ and the observational specific humidity at a height of 1 m is plotted in Fig. 6. Differences between the two values are very small and the maximum difference during the day often appeared around midday. The relatively large temperature gradients at that time are responsible for the maximum difference.

4.4 *Sensitivity tests*

To examine sensitivities of the estimated fluxes to observational errors in wind, temperature, and humidity profile, artificial errors are added to the observed wind speed at 10 m, temperature at 1 m, specific humidity at 1 m, and the observed available energy $R-G$, respectively, for the $N=2880$ time levels. According to the reported accuracy of each observational variable, 0.2°C is added onto the observed temperature, 0.5 m s^{-1} is added onto the observed wind speed, 2.2×10^{-4} (about 1% relative humidity at 28°C) is added onto the specific humidity, and 15 W m^{-2} is added onto the observed available energy. Fluxes computed from original data and error-contaminated data are respectively denoted by F and F'. The rms error for the estimated heat fluxes can be evaluated by

$$\mathrm{rms}=[N^{-1}\sum(F'-F)^2]^{1/2}\,, \qquad (14)$$

where the summation is made for all the 20×144 time levels (10 min apart for the 20 days from 11 to 30 June 2001) and $N=2880$. The computed rms errors with the three different methods are listed in Table 1. Among the three methods, the Bowen ratio method is the most sensitive to the observational errors except for errors in wind speed. For the profile method, the estimated latent heat fluxes are very sensitive to the errors in air humidity and moderately sensitive to the errors in wind speed (see the sixth column in Table 1), while sensible heat fluxes are very sensitive to the errors in temperature (see the third column in Table 1). Since the surface energy balance equation is not used in the profile method, its errors have no influence on the results. In contrast with the Bowen ratio and profile methods, the heat fluxes estimated by the variational technique are not very sensitive to any of the observational errors, so they are more stable and reliable. As explained in the introduction, the VT fully uses all available information provided by SERBS measurements, surface energy budget, and the similarity profile formulae. Besides the physical constraints, the use of an optimal estimation is believed to be another reason for the improved performance of the variational technique (Daley, 1991; Cohn, 1997).

To further evaluate the advantages of the VT, we carried out similar sensitivity tests by using XQ's method. It was found that adding an error of 0.2°C to air temperature caused an rms error of 23.5 W m^{-2} in the computed sensible heat flux and an rms error of 21.5 W m^{-2} in the computed latent heat flux, which are all larger than those by using the VT. As for sensitivities to the observational errors in the remaining variables, differences between the VT and XQ's method are small so they are not presented here. Although XQ's method can effectively overcome the problems in the Bowen ratio method and the profile method, it is still relatively sensitive to errors in temperature. The VT retains all advantages of XQ's method and moreover, it is relatively more stable than XQ's method.

Table 1. Sensitivities of the estimated fluxes to observational errors, respectively, by using the profile method, Bowen radio method, and the variational technique (VT).

		Rms error for F_{SH}			Rms error for F_{LH}		
Variable	Error	Profile	Bowen	VT	Profile	Bowen	VT
T	+0.2°C	24.6	259.8	14.7	13.9	259.8	14.3
	–0.2°C	22.4	399.4	14.1	12.5	399.4	13.8
u	+0.5 m s^{-1}	5.6	0.0	4.0	13.5	0.0	3.9
	–0.5 m s^{-1}	5.4	0.0	3.9	12.6	0.0	3.8
q	$+2.2\times10^{-4}$	4.1	843.6	9.4	50.5	843.6	10.5
	-2.2×10^{-4}	2.8	1011.6	8.5	49.7	1011.6	9.7
$R-G$	+15 W m^{-2}	0.0	137.9	4.6	0.0	137.9	7.1
	–15 W m^{-2}	0.0	137.9	4.7	0.0	137.9	4.0

5. Conclusions

In this paper, a variational technique (VT) is proposed to compute surface sensible and latent heat fluxes. In the VT, nine differences between values of temperature, wind speed, and specific humidity predicted by the profile formulae and the corresponding observations, respectively, at three heights (1 m, 4 m, 10 m), are integrated into the cost function together with the weak constraint of surface energy budget, while the frictional velocity, the flux temperature scale, the flux humidity scale, skin temperature, and specific humidity at the height of the heat roughness are five retrieved parameters. By using the quasi-Newton algorithm to minimize the cost function, an optimal estimation of the five parameters can be obtained, and thus the heat fluxes can be calculated. Because the variational technique can merge all useful information provided by the measurements, the similarity law, and surface energy budget with consideration of the observational errors, it has many advantages over the profile method and the Bowen ratio method.

The VT is tested with data collected by the surface energy and radiation balance systems at Feixi experiment station (31.41°N, 117.08°E) installed by the HeRES Program during 11–30 June 2001. The estimated sensible and latent heat fluxes are compared with the fluxes computed by the profile method and Bowen ratio method. The spurious spikes in the fluxes computed by the Bowen ration method are eliminated. In contrast with the profile method, both sensitivities of latent heat fluxes to the errors in humidity and sensible heat fluxes to the errors in temperature are reduced; at the same time, the estimated heat fluxes approximately satisfy the surface energy budget. Therefore, the VT is more reliable and stable than the profile method and the Bowen radio method. Since direct measurements of surface sensible and latent heat fluxes are not available, the estimated fluxes are checked against the fluxes by XQ's method—one of the precise methods. They are in good agreement, and the rms difference between the sensible heat fluxes is 2.6 W m^{-2} and that between the latent heat fluxes is 3.0 W m^{-2}. At the same time, the VT has some advantages over XQ's method because it is less sensitive to the errors in the observational temperatures.

With the VT, surface skin temperature and specific humidity can be retrieved too. The maximum difference between the retrieved surface skin temperature and the observed air temperature at a height of 1 m appeared around non-cloudy midday, while at the other times, or on cloudy days or on rainy days, discrepancies between the two temperatures are very small. Discrepancies between the retrieved surface skin temperature and the observed soil temperature at a depth of 0 cm are less than the former differences, and the maximum appeared around noncloudy midday.

Finally, it should be pointed out that the VT does not necessarily need all the observations as measured at Feixi station (wind speed, temperature, and humidity, respectively, at three heights); observations of wind speed, temperature, and humidity at one height, an additional observation of one of the above three variables at a different height, and the surface energy budget as well are enough to estimate the five parameters (u_*, T_*, q_*, T_s, q_s). Nevertheless, the VT can merge more observations in an effective way, and thus make the estimation more accurate and stable.

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