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Study of the Optimal Precursors for Blocking Events

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ABSTRACT

The precursors of dipole blocking are obtained by a numerical approach based upon a quasi-geostrophic barotropic planetary- to synoptic-scale interaction model without topography and with a localized synoptic-scale wave-maker. The optimization problem related to the precursors of blocking is formulated and the nonlinear optimization method is used to examine the optimal synoptic-scale initial field successfully. The results show that the prominent characteristics of the optimal synoptic-scale initial field are that the synoptic-scale wave train structures exist upstream of the incipient blocking. In addition, the large-scale low/high eddy-forcing pattern upstream of the incipient blocking is an essential precondition for the onset of dipole blocking.

Key words: nonlinear optimization method, dipole blocking, pre-condition, eddy-forcing

1. Introduction

Blocking is an important part of the low-frequency variability of the atmospheric circulation in the mid-high latitudes and can cause anomalous weather conditions over large extratropical regions. Therefore, it has been an interesting, challenging question to meteorologists for decades (Rex, 1950a, b; Chaney and Devore, 1979; McWilliams, 1980; Shutts, 1983; Luo, 1999, 2000, 2005a, b). So far, the maintenance of blocking has been fairly well investigated, while its onset is still very poorly understood. Whether it is predicted successfully or not has been a standard for assessing medium-range prediction models (Tibaldi and Molteni, 1990). Transition to blocking is a problem of both theoretical interest and practical importance for weather prediction.

Recently, new progress has been made on the precursors of blocking. Nakamura et al. (1997) showed that a quasi-stationary wave train across the Atlantic is evident during the blocking amplification over Europe, while no counterpart is found to the west of the amplifying blocking ridge over the North Pacific through composite analysis. Michelangeli and Vautard (1998) found two synchronous antecedent conditions over Europe-Atlantic blocking: planetary-scale waves moving to the west and strong baroclinic synoptic-scale wave trains. Li et al. (1999) proposed that the structural modification of eddies in the wave train leads to the planetary structures that become associated with block onset using adjoint sensitivity perturbations. Both data analysis and diagnosis studies have emphasized the importance of synoptic-scale waves in block onset.

Frederiksen (1997) proposed the concept of the finite-time normal mode in which the basic state changes with time, and he applied it to the study of initial perturbations of blocking, vhereas it had been found that there are optimal perturbations which develop faster than the normal modes in the linear range (Lacarra and Talagrand, 1988; Farrel, 1990). Buizza and Molteni (1996) applied linear instability analysis to study the role of barotropic dynamics in the evolution of blocking and pointed out that the fastest growing perturbations upstream in the ridge are essential to the formation of localized dipole blocking. However, it is well known that the motion of the atmosphere or the ocean is governed by nonlinear systems, and

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the validity of the linear approximation should be investigated. Oortwijn and Barkmeijer (1995) extended the fast-growing perturbation to the nonlinear regime with an iterative procedure and showed that the optimal perturbations lying upstream of the blocking are stronger in the nonlinear range than those in the linear range.

Recently, the nonlinear optimization method has found quite a few applications in atmosphere and ocean studies. To study the nonlinear amplification of initial perturbations, Mu (2000) extended singular vectors (SVs) and singular values (SVAs) to nonlinear theories and proposed a novel concept of nonlinear singular vectors (NSVs) and nonlinear singular values (NSVAs). These concepts were applied to study the first kind of predictability in a two-dimensional quasi-geostrophic model (Mu and Wang, 2001). In Mu et al. (2003), the concept of the conditional nonlinear optimal perturbation (CNOP) was proposed, and then this method was applied to study the "spring predictability barrier" in ENSO (Mu and Duan, 2003) and the optimal precursors for ENSO events (Duan et al., 2004). Besides these, it has also been applied to the study of the stability of the ocean's thermohaline circulation (Mu et al., 2004). The concepts of NSVs and CNOPs are essentially nonlinear problems. Furthermore, Mu et al. (2002) and Xu et al. (2004) applied the nonlinear optimization method to sensitivity analysis of a numerical model. From the above, it is known that the nonlinear optimization method has great potential in applications.

The purpose of this paper is to explore what the characteristics are of the synoptic-scale waves for the onset of blocking in the nonlinear framework. By solving a nonlinear optimization problem, the optimal synoptic-scale waves are captured numerically. The theoretical model used here is a quasi-geostrophic barotropic planetary-to synoptic-scale interaction model without topography and with a localized synoptic-scale wave-maker (Luo, 2005a, b) and its corresponding adjoint model. The envelope Rossby solitons theory not only simulates the life cycle of blocking successfully, but also it is able to reflect the mutual feedback of planetary-scale waves and the synoptic-scale waves (Luo, 1999, 2000, 2005a, b). What is more, it is able to describe the multi-eddy structures of blocking appearing in the daily synoptic chart, while many previous theories can only describe the mature stage of blocking (Butchart et al., 1989; Malguzzi and Malanotte-Rizzoli, 1984).

This paper is organized as follows. In section 2, a quasi-geostrophic barotropic planetary- to synoptic-scale interaction model without topography and with a localized synoptic-scale wave-maker is described. Section 3 presents a nonlinear optimization framework. The optimal synoptic-scale field prior to block onset is found in section 4. Section 5 provides discussion and conclusions.

2. The model details

The dimensionless quasi-geostrophic barotropic planetary- to synoptic-scale interaction model without topography and with a localized synoptic-scale wave-maker on a beta-plane channel, used in this paper, can be written as (Luo, 1999, 2005a, b)

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)(\nabla^2\psi_{\rm p}-F\psi_{\rm p})+J(\psi_{\rm p},\nabla^2\psi_{\rm p})_{\rm p}$$
$$+(\beta+F\overline{u})\frac{\partial\psi_{\rm p}}{\partial x}=-J\left(\psi_{\rm s},\nabla^2\psi_{\rm s}\right)_{\rm p}-J\left(\psi_{\rm s},\nabla^2\psi_{\rm p}\right)_{\rm p}$$
$$-J\left(\psi_{\rm p},\nabla^2\psi_{\rm s}\right)_{\rm p}-\gamma\nabla^2\psi_{\rm p}\quad \text{in }\Omega\times[0,T]\,, \tag{1a}$$
$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)(\nabla^2\psi_{\rm s}-F\psi_{\rm s})+J\left(\psi_{\rm s},\nabla^2\psi_{\rm p}\right)_{\rm s}$$
$$+J\left(\psi_{\rm p},\nabla^2\psi_{\rm s}\right)_{\rm s}+J\left(\psi_{\rm s},\nabla^2\psi_{\rm s}\right)_{\rm s}$$
$$+(\beta+F\overline{u})\frac{\partial\psi_{\rm s}}{\partial x}=F_{\rm s}-\gamma\nabla^2\psi_{\rm s}\quad \text{in }\Omega\times[0,T]\,. \tag{1b}$$

The total stream function

$$\psi_{\rm tol}=-\overline{u}y+\psi_{\rm p}+\psi_{\rm s}$$

has been divided into three parts: the basic field $-\overline{u}y$ for a constant background westerly ($\bar{u}=0.7$), the planetary-scale part $\psi_{\rm p}$ and the synoptic-scale part $\psi_{\rm s}$; $F=(L/R_d)^2$ and $\beta=\beta_0L^2/U$, in which $R_{\rm d}$ is the radius of Rossby deformation, β_0 is the meridional gradient of the Coriolis parameter, both L=10^6 m and U=10 m s^{-1} are the horizontal length and velocity scales respectively. $\gamma=5\times10^{-3}$ is the dissipation coefficient. ∇^2 is the horizontal Laplacian operator. For simplicity, space domain $\Omega=[-6,6]\times[0,5]$ with a zonal periodic boundary condition. $J(a,b)=(\partial a/\partial x)(\partial b/\partial y)-(\partial a/\partial y)(\partial b/\partial x)$ is the Jacobian operator, for example, the synoptic- and synoptic-scale nonlinear interaction $-J(\psi_{\rm s},\nabla^2\psi_{\rm s})$, is split into synoptic-scale part $-J(\psi_{\rm s},\nabla^2\psi_{\rm s})_{\rm s}$ and planetary-scale part $-J(\psi_{\rm s},\nabla^2\psi_{\rm s})_{\rm p}$, in addition, planetary- and planetary-scale nonlinear interaction $-J(\psi_{\rm p},\nabla^2\psi_{\rm p})$, and planetary- and synoptic-scale nonlinear interaction $-[J(\psi_{\rm s},\nabla^2\psi_{\rm p})+J(\psi_{\rm p},\nabla^2\psi_{\rm s})]$ is also split into two parts as the above.

The planetary-scale stream function and synoptic-scale stream function satisfy the following boundary conditions respectively:

$$\frac{\partial\psi_{\rm n}}{\partial x}=0,\ \ \frac{\partial^2\overline{\psi}_{\rm n}}{\partial t\partial y}=0,\ \ y=0,L_y,\ \ ({\rm n}={\rm p,s}) \tag{2}$$

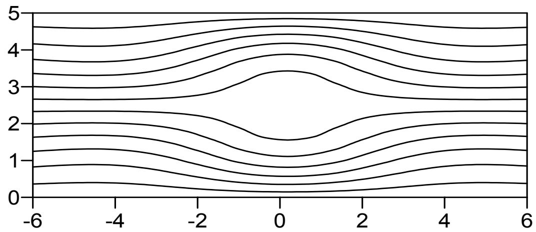

Fig. 1. Planetary-scale initial streamfunction field (Day 0) (A_0=0.45), in which the x-axis represents the zonal direction, and the y-axis represents the meridional direction, and the value is dimensionless length. (The following figures are the same as Fig. 1). The contour interval (CI) is 0.28.

where L_y is the width of the beta-plane channel ($L_y = 5$), and $\overline{\psi}_{\rm n} = \overline{\psi}_{\rm n}(y,t)$ represents a zonal average of the planetary-scale stream function $\psi_{\rm p}$ and synoptic-scale stream function $\psi_{\rm s}$ respectively.

The planetary-scale initial field is assumed to be (Luo, 1999):

$$\psi_{\rm p0} = 2A_0\sqrt{\frac{2}{L_y}}\,\mathrm{sech}(0.85A_0x)\cos(kx)\sin(my)\ , \quad (3)$$

where A_0 is a constant representing the amplitude of initial blocking. $A_0 = 0.45$ is chosen in this paper so that there are no closed centers and only a little ridge of high pressure (as shown in Fig. 1).

$$\mathrm{sech}\,x = \frac{2}{e^x + e^{-x}},\quad k = \frac{2}{6.371\cos(\phi_0)},$$
$$m = -\frac{2\pi}{L_y}\ ,$$

and ϕ_0 is the central latitude of blocking, which as usual is ϕ_0=55°.

In the barotropic model, in order to generate baroclinic waves, a wave-maker is put upstream of an incipient blocking, which is an ideal perturbing source with time. This approach is used by Shutts (1983). The wave-maker used in this paper is as follows:

$$F_{\rm s} = 2C_0\varepsilon[-2\gamma(x+x_0)]e^{-\alpha\varepsilon^2(x+x_0)^2}[a_1\cos(k_1x-\omega_1t) - a_2\cos(k_2x-\omega_2t)]\sin(my/2)\ , \quad (4)$$

where

$$a_1 = \overline{u}(3k_1^2 + m^2/4) - \beta,\quad a_2 = \overline{u}(3k_2^2 + m^2/4) - \beta,$$
$$k_1 = \frac{9}{6.371\cos(\phi_0)},\quad k_2 = \frac{11}{6.371\cos(\phi_0)},$$
$$\omega_1 = \overline{u}k_1 - \frac{(\beta + F\overline{u})k_1}{(k_1^2 + m^2/4 + F)},$$
$$\omega_2 = \overline{u}k_2 - \frac{(\beta + F\overline{u})k_2}{(k_2^2 + m^2/4 + F)},$$

C_0 is the intensity of the perturbing source and x_0 is the position of the source. In this paper, we choose $C_0 = 0.04, \varepsilon = 0.24$ and $\alpha = 1.8$.

In our numerical approach, the temporal discretization is the Adams-Bashforth scheme, where stream functions $\psi_{\rm p}$ and $\psi_{\rm s}$ are treated as unknown terms. The Arakawa finite difference scheme is used to discretize the Jacobian operator, in which subscript p represents 0–4 waves through a space Fourier filtering, and s represents 8–16 waves through the space Fourier filtering. The five-point difference scheme is used to discretize the Laplacian operator, in which the grid spacing $\Delta x = 0.36$ and $\Delta y = 0.33$ correspond to dimensional length of 36 km and 33 km respectively, and the time step $\Delta t = 0.012$ corresponds $\Delta t = 20$ min in the domensional form.

3. The formulation of the nonlinear optimization problem

In the theory of blocking proposed by Luo (2005a, b), the planetary-scale initial field may develop into a dipole blocking pattern under the eddy source upstream of the blocking even in the absence of topography. The numerical experiment of this theory also obtains similar results (Jiang et al., 2005). However, whether the planetary-scale initial field can develop into a typical dipole block pattern depends, to a large extent, on the structure of the initial synoptic-scale initial field. So, in this paper, the answer to the problem needs to be sought, namely to answer: what synoptic-scale initial field can make the planetary-scale initial field $\psi_{\rm p0}$ develop into a typical dipole blocking pattern $\psi_{\rm p,\,obs}$ at a given time? The following $\psi_{\rm p,\,obs}$ can be obtained based upon (3)

$$\psi_{\rm obs} = -\overline{u}y + 2B_0\sqrt{\frac{2}{L_y}}\,\mathrm{sech}(0.85B_0x)\cos(kx)\sin(my)\ , \quad (5)$$

where $\psi_{\rm p,\,obs}$ is 0–4 waves of $\psi_{\rm obs}$ through the space Fourier filtering and B_0 is the amplitude of the typical dipole blocking.

For fixed $T > 0$ and initial condition

$$\psi_{\rm p}|_{t=0} = \psi_{\rm p0},\quad \psi'_{\rm s}|_{t=0} = \psi'_{\rm s0}\ ,$$

the propagators $M_{{\rm p},T}$ and $M_{{\rm s},T}$ are well defined, i.e.,

$$\psi_{\rm p}(x,y,T) = M_{{\rm p},T}(\psi_{\rm p0},\psi'_{\rm s0})\ ,$$
$$\psi_{\rm s}(x,y,T) = M_{{\rm s},T}(\psi_{\rm p0},\psi'_{\rm s0})\ ,$$

are the solution of (1.1) at time T.

In this paper, the Euler norm is employed, which is defined as,

$$\|\psi\|^2 = \int_\Omega \psi^2 dxdy\ . \quad (6)$$

Now we define the objective function,

$$J(\psi_{s0}^{'*}) = \min_{\psi'_{s0}} J(\psi'_{s0}) \,, \tag{7}$$

where

$$J(\psi'_{s0}) = \frac{1}{2}[M_{p,T}(\psi_{p0}, \psi'_{s0}) - \psi_{p,obs}]^{T} [M_{p,T}(\psi_{p0}, \psi'_{s0}) - \psi_{p,\ obs}] \,, \tag{8}$$

where T the matrix transpose, and $\psi_{s0}^{'*}$ is the optimal synoptic-scale initial field when the objective function attains the minimum.

In order to seek the minimum of the objective function, the gradient of the objective function with respect to the initial synoptic-scale field should be determined. The first variation of $J(\psi'_{s0})$ is

$$\delta J(\psi'_{s0}) = [M_{p,T}(\psi_{p0}, \psi'_{s0}) - \psi_{p,\ obs}, \mathrm{M}_{p,T}(\psi_{p0}, \psi'_{s0})\delta\psi'_{s0}] \,, \tag{9}$$

where $\mathrm{M}_{p,T}$ is the tangent linear approximation of propagator $M_{p,T}$.

Let $\mathrm{M}^{*}_{p,T}$ be the adjoint operator of $\mathrm{M}_{p,T}$, we have

$$\nabla J(\psi'_{s0}) = \mathrm{M}^{*}_{p,T}(\psi_{p0}, \psi'_{s0})[M_{p,T}(\psi_{p0}, \psi'_{s0}) - \psi_{p,\ obs}] \,. \tag{10}$$

The optimization algorithm is the limited memory BFGS method, which is an extension of the conjugate gradient method. It is widely used in predictability, sensitivity analysis, data assimilation and ensemble forecasting. Nonlinear optimization methods are already being used successfully in the study of atmospheric and oceanic science (Mu and Wang, 2001; Mu et al., 2003; Xu et al., 2004).

4. The optimal synoptic-scale initial field of dipole blocking patterns

4.1 *Results of the numerical experiment*

Considering that the mature blocking is usually in the upstream of the incipient blocking, the typical dipole blocking pattern defined is in the west of the initial blocking. In this paper, B_0=1.0, so that there are closed high and low pressure centers, as shown in Fig. 2. Certainly, the amplitude and the position can

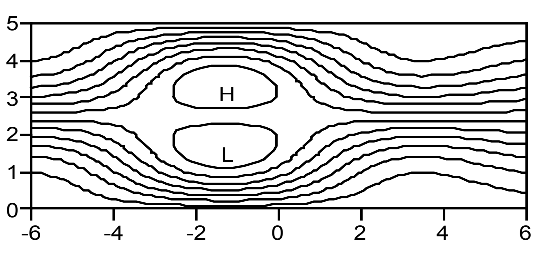

Fig. 2. Typical dipole blocking for B_0=1.0, CI is 0.28.

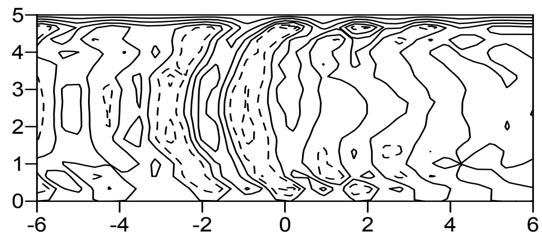

Fig. 3. Optimal synoptic-scale initial field for optimization time of $T = 3$ days (Day 0) (CI is 0.2), in which the solid lines represent the positive isolines and the dotted lines represent the negative isolines.

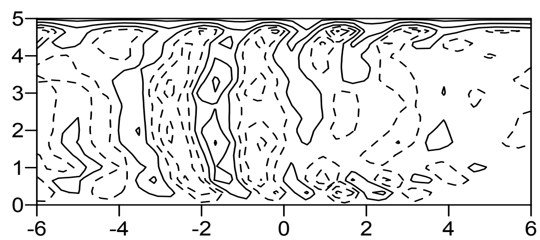

Fig. 4. Same as in Fig. 3 except with optimization time of $T = 5$ days (Day 0) (CI is 0.15).

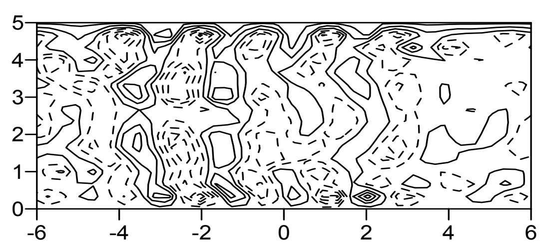

Fig. 5. Same as in Fig. 3 except with optimization time of $T = 7$ days (Day 0) (CI is 0.1).

exhibit small changes, but these do not influence the conclusions. We fix $T = 216, 360$, and 504 steps (corresponding to Days 3, 5 and 7) and the corresponding optimal synoptic-scale initial fields are shown in Figs. 3, 4 and 5 respectively. The results show that the common characteristics of these three pictures are the typical wave train structures in the upstream of the incipient blocking. Comparing these three pictures, it is also found that the longer the optimization time T is, the stronger the optimal synoptic-scale initial field is, which is easily understood from the point of view of energy. In addition, when the optimization time T is 7 days, there are two branches in the north-south direction in the optimal synoptic-scale initial field. Certainly, because of the solid boundary condition in the north-south direction, the values on the north/south boundary are not well disposed.

Now using Fig. 3 as the initial synoptic-scale field (Day 0), we simulate the evolution of the planetary-scale waves and synoptic-scale waves respectively. The results show that planetary-scale field develops into a typical dipole blocking and the synoptic-scale waves divide into two branches in the north-south direction at the given optimization time (Fig. 6), which is almost consistent with the forced envelope Rossby soliton theory (Luo, 1999, 2000, 2005a, b) and the numerical simulation (Jiang et al., 2005). For the optimization time interval being 5 and 7 days, respectively, we see similar results respectively. For simplicity, they are not shown here.

4.2 *The eddy feedback* $J_\mathrm{p} = -J(\psi_\mathrm{s}, \nabla^2\psi_\mathrm{s})_\mathrm{p}$

In order to investigate the contribution of the nonlinear self-interaction of synoptic- scale waves to the planetary-scale waves, the large-scale eddy feedback J_p field is calculated for the optimal initial synoptic-scale field and its evolution. The J_p field is shown in Fig. 7, when the optimization time T is 3 days. The results show that the J_p field possesses a low/high structure located in the upstream of the incipient blocking till the maturation of the blocking, in which the negative value region (dashed lines) represents the eddy-induced planetary-scale anticyclonic vorticity, and the positive value region (solid lines) represents the eddy-induced planetary-scale cyclonic vorticity in this whole process. During the onset stage of the vortex pair block, the eddy-induced planetary- scale anticyclonic vorticity is injected into the high pressure region of blocking, and then it promotes the development of the blocking anticyclone; but the eddy-induced planetary-scale cyclonic vorticity is injected into the low trough, and then it promotes the development of the low pressure of blocking. That is to say, the structure of the synoptic-scale waves in the upstream of the incipient blocking is very important for the onset of blocking. When the optimization time T is 7 days, the same results can be obtained (Fig. 8). This low/high J_p field of eddy-ignored planetary-scale vorticity transport is essential for the onset of blocking, which may be an indicator of blocking onset. The above results obtained by the optimization method are consistent with the results of the diagnosis by Luo et al. (2001).

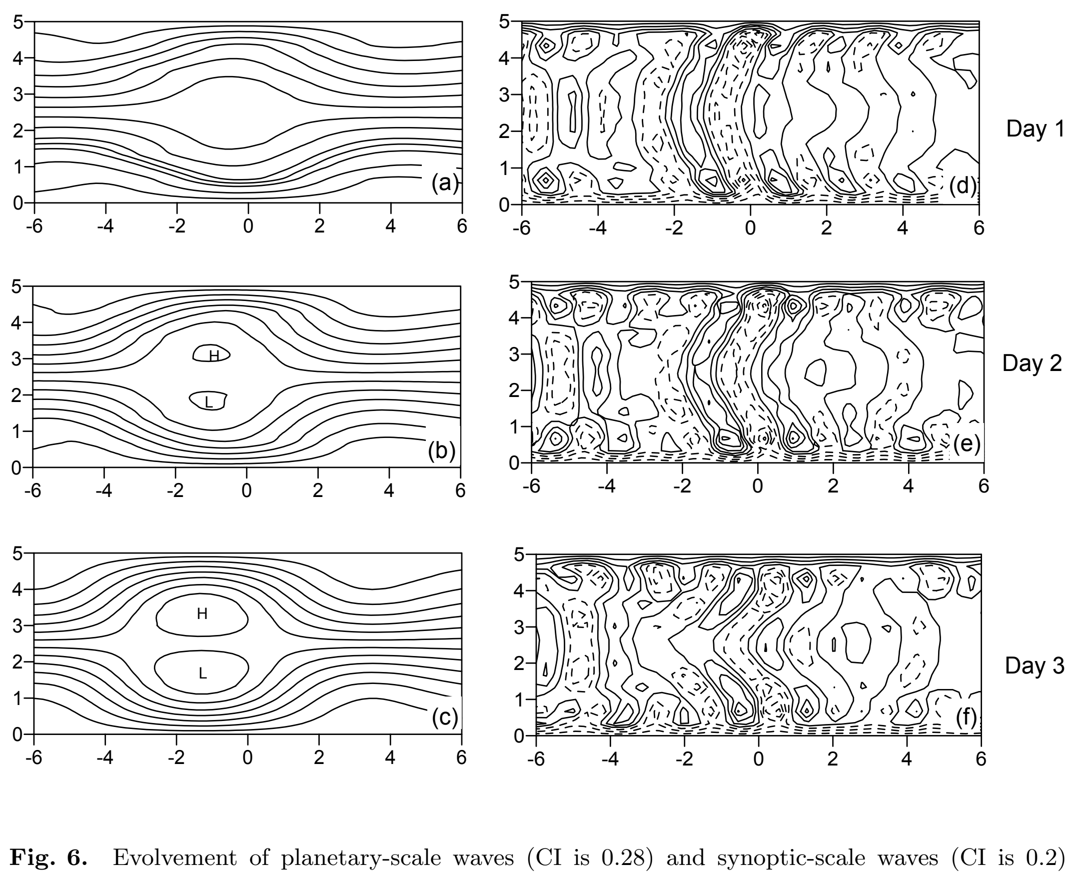

Fig. 6. Evolvement of planetary-scale waves (CI is 0.28) and synoptic-scale waves (CI is 0.2) respectively using Fig. 3 as the synoptic-scale initial field. (a) Day 1 of the planetary-scale waves, (b) Day 2 of the planetary-scale waves, (c) Day 3 of the planetary-scale waves; (d) Day 1 of the synoptic -scale waves, (e) Day 2 of the synoptic-scale waves, (f) Day 3 of the synoptic-scale waves.

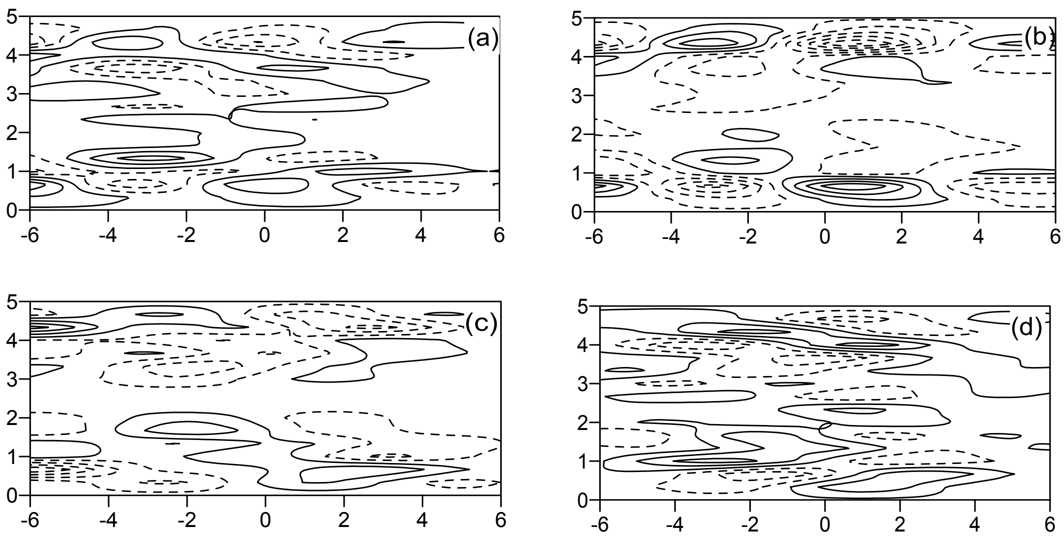

Fig. 7. $J_{\rm p}$ field $[J_{\rm p} = -J(\psi_{\rm s}, \nabla^2\psi_{\rm s})_{\rm p}]$ for the first 4 days (CI is 0.8) with an optimization time of T = 3 days. The solid lines represent the positive isolines and the dotted lines represent the negative isolines. (a) Day 0, (b) Day 1, (c) Day 2, (d) Day 3.

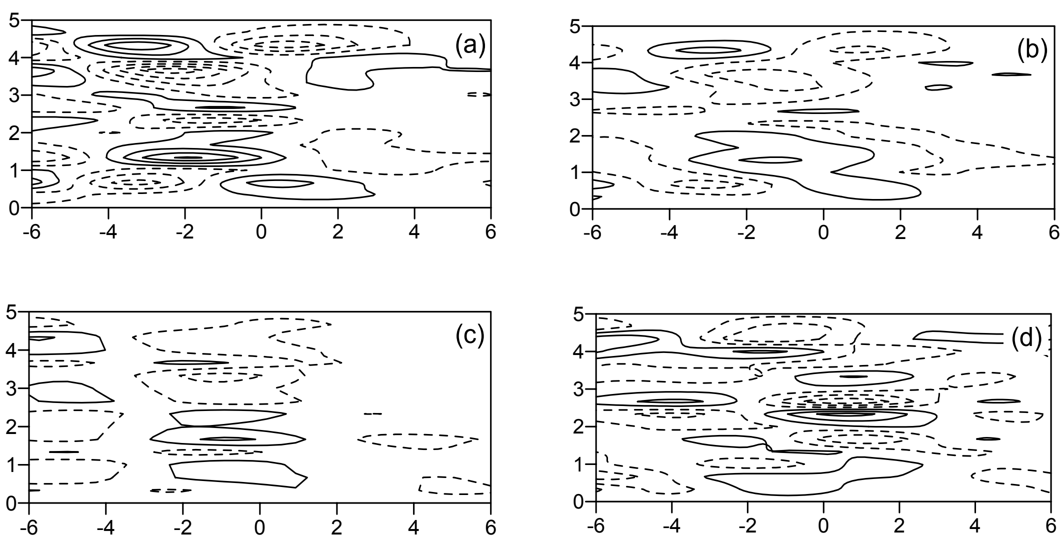

Fig. 8. Same as in Fig. 7 except with the optimization time of T = 7 days.

5. Discussion and conclusions

In this paper, the optimal synoptic-scale waves for the onset of blocking are obtained by solving the corresponding optimization problem numerically. Nonlinear optimization methods are used to study the block onset for the first time, which provides an advantageous tool for the study of the blocking precursors. This lays a foundation for the further study of initial perturbations for blocking, which overcomes the limit of traditional linear methods.

The results show that the prominent characteristics of the optimal synoptic-scale initial field for different optimization times are the wave train structures upstream of the incipient blocking. The longer the optimization time T is, the stronger the synoptic-scale waves are. From the analysis of the eddy-feedback $J_{\rm p}$ field, it is found that the large-scale low/high eddy-forcing pattern upstream of the incipient blocking is an essential precondition for the onset of dipole blocking, which is consistent with the observation analysis.

Besides this, other kinds of general objective functions based on the understanding of blocking can be adopted to investigate the effects of perturbations on the onset of blocking, because there are some limitations in this objective function, especially the definition of the typical blocking pattern. This work is just the beginning of a nonlinear optimization theory framework being applied to the study of the onset of blocking.

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