ADVANCES IN ATMOSPHERIC SCIENCES, VOL. 23, NO. 2, 2006, 224–234

Estimating the Retrievability of Temperature Profiles from Satellite Infrared Measurements

HUANG Jing*[1] (黄 静), QIU Chongjian[1] (邱崇践), MA Gang[2] (马 刚), and ZHANG Yanwu[3] (张艳武)

[1] *College of Atmospheric Sciences, Lanzhou University, Lanzhou* 730000

[2] *National Satellite Meteorological Center, China Meteorological Administration, Beijing* 100081

[3] *Cold and Arid Regions Environment and Engineering Research Institute, Chinese Academy of Sciences, Lanzhou* 730000

(Received 17 February 2005; revised 9 June 2005)

ABSTRACT

A method is developed to assess retrievability, namely the retrieval potential for atmospheric temperature profiles, from satellite infrared measurements in clear-sky conditions. This technique is based upon generalized linear inverse theory and empirical orthogonal function analysis. Utilizing the NCEP global temperature reanalysis data in January and July from 1999 to 2003, the retrievabilities obtained with the Atmospheric Infrared Sounder (AIRS) and the High Resolution Infrared Radiation Sounder/3 (HIRS/3) sounding channel data are derived respectively for each standard pressure level on a global scale. As an incidental result of this study, the optimum truncation number in the method of generalized linear inverse is deduced too. The results show that the retrievabilities of temperature obtained with the two datasets are similar in spatial distribution and seasonal change characteristics. As for the vertical distribution, the retrievabilities are low in the upper and lower atmosphere, and high between 400 hPa and 850 hPa. For the geographical distribution, the retrievabilities are low in the low-latitude oceanic regions and in some regions in Antarctica, and relatively high in mid-high latitudes and continental regions. Compared with the HIRS/3 data, the retrievability obtained with the AIRS data can be improved by an amount between 0.15 and 0.40.

Key words: meteorological satellites, generalized linear inverses, temperature profiles, EOF analysis, retrievability

doi: 10.1007/s00376-006-0224-x

1. Introduction

Radiance measurements of satellite infrared radiometers are widely used to retrieve atmospheric temperature and moisture profiles. Because the radiances arise from very thick layers of the atmosphere, the vertical resolution of the derived profiles from current operational satellite sounding instruments, for example, the TIROS-N Operational Vertical Sounder (TOVS), is poor. The poor vertical resolution is ascribed, in part, to the low spectral resolution. Therefore, high-performance vertical detectors with more observation channels have been increasingly incorporated into launched meteorological satellites during recent years. For example, the Atmospheric Infrared Sounder (AIRS) was launched into polar orbit on board the National Aeronautics and Space Administration's (NASA) Aqua platform on 4 May 2002. AIRS covers the spectrum range from 650 to 2700 cm^{-1} with 2378 spectral channels, the spectrum resolution is higher than 1200, and the absolute precision of the radiance measurements exceeds 0.2 K. This platform can measure the upper spectrum and can attain a high precision in infrared detection (Susskind et al., 1998; Wu et al., 2003). Other detectors with higher resolution are also being developed. However, the problem of atmospheric profile retrieval from satellite data is well known to be ill-posed (Zeng, 1974; Huang, 1996). The ill-posed nature of the problem may be more serious when more data channels are used in the retrieval. So the precision of retrieved atmospheric profiles from satellite data is limited in fact (Huang et al., 1992). We can use the limits of retrievability, which was first

*E-mail: huangjing03@st.lzu.edu.cn

proposed by Thompson et al. (1986), as a measure of ultimate precision or potential for retrieval. It is analogous with the limits of predictability in the numerical weather prediction problem. The estimation of retrievability for different satellite vertical detectors can provide very helpful information for the design of detectors and for the application of retrieval products from satellites and assimilation (Lorenc et al., 2000) of radiance data into numerical models. In theory, the retrieval precision was restricted by two major factors: (1) radiance observation error and (2) vertical resolution. Some theoretical analyses on the vertical resolution and precision of the retrieved temperature and humidity profiles have been performed since the 1970s (Huang et al., 1992; Backus and Gilbert, 1970; Conrath, 1972; Li, 1995; Rodgers, 1988; Smith et al., 1991; Thompson, 1982, 1991). Because of the ill-posed character of the retrieval problem, the retrieval solution is very sensitive to the observation error. So the vertical resolution that can be achieved in practice is less than the theoretical, analytical values. Huang et al. (1992) put forward the idea of effective resolution. Based on this concept, the accuracy of atmospheric temperature and humidity profiles retrieved from the High Resolution Infrared Radiation Sounder/2 (HIRS/2), the Geostationary Operational Environmental Satellite (GOES) I/M and the High-Resolution Interferometer Sounder (HIS) detector observations were compared with each other and the averaged retrieval errors were given at different heights. All of these were based on some observation station's sounding data and the observation error in brightness temperature was generally assumed to be 0.25 K. The averaged retrievability was represented in the above studies. However, the retrievability varies with region and season. So a more comprehensive and flexible method for global retrievability analyses is needed.

In this paper, a new method is developed to estimate the retrievability of atmospheric temperature profiles based on empirical orthogonal function (EOF) analysis and generalized linear inverse theory (GLIT). The EOF technique is used to extract the vertical structure of the atmosphere and GLIT is used to separate the atmospheric modes that can be retrieved from the satellite data. Utilizing the method, the change of the atmospheric temperature structure can be expressed with the first several EOFs, and then the distribution characteristics of the retrieval error can be directly estimated according to the radiative transfer equation (RTE). Compared with other methods, the computation requirements of this method are relatively small and can avoid the inversion of the covariance matrix. The method is described in section 2 and is compared with other methods in section 3. The method is applied to AIRS and HIRS/3 observations in section 4. Conclusions follow in section 5.

2. Method of estimating retrievability

When the linear approximation is applied and the surface skin temperature is known, the atmospheric temperature profile can be retrieved from satellite observations by solving the following integral equation:

$$\delta I_v = \int_0^{p_\mathrm{s}} K_v(p)\delta T(p)dp \, , \tag{1}$$

where I_v is the radiation amount reaching the satellite inductor at frequency $v, T(p)$ is the atmospheric temperature at pressure p, p_s is the surface pressure, $\delta I_v = I_v - I_{v,0}$, and $\delta T = T - T_0$, where the subscript 0 denotes the reference value. $K_v(p)$ is the variation kernel function and is given by the following formula when the atmosphere reference values are known

$$K_v(p) = \frac{dB_v[T_0(p)]}{dT}\frac{\partial \tau_v(p)}{\partial p} \, , \tag{2}$$

where $B(T)$ is the Planck function at temperature T, and τ_v is the atmospheric transmittance at frequency v. For practical applications, when the number of available observation channels is N and the number of layers of the profile to be retrieved is set to M, then N linear equations can be constructed by using discrete Eq. (1), and this is expressed by

$$\boldsymbol{\delta I} = \boldsymbol{K\delta T} \, , \tag{3}$$

where $\boldsymbol{\delta I}$ is the observation vector of dimension $N, \boldsymbol{\delta T}$ is the temperature vector of dimension M, and $\boldsymbol{K}$ is a matrix of dimension $N \times M$.

As mentioned in the introduction, the temperature retrieval problem is ill-posed. This means that the temperature profile solution is not unique and is sensitive to the observation errors. The properties of the solution can be examined by generalized inverse theory (Wiggins, 1972). This theory is based on singular value decomposition (SVD). A brief explain will be given as follows. In performing a singular value decomposition of matrix $\boldsymbol{K}, \boldsymbol{K}$ is decomposed into the form

$$\boldsymbol{K} = \boldsymbol{U\Lambda V}^\mathrm{T} \, , \tag{4}$$

where $\boldsymbol{U}$ is a matrix of order N and $\boldsymbol{V}$ is a matrix of order M. The superscript T denotes the matrix transpose. The column vectors of $\boldsymbol{U}$ and $\boldsymbol{V}^\mathrm{T}$ are the singular vectors and satisfy the orthogonal relation $\boldsymbol{u}_i, \boldsymbol{u}_j^\mathrm{T} = \delta_{ij}, \boldsymbol{v}_i v_j^\mathrm{T} = \delta_{ij}$, where δ_{ij} is the Dirac delta function. The eigenvectors $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ are also called the left and right vectors, respectively. The matrix $\boldsymbol{\Lambda}$ is an $N \times M$ matrix in which the nondiagonal elements are zero and the diagonal elements are the eigenvalues λ_i ordered from largest to smallest. Each pair of eigenvectors $(\boldsymbol{u}_i, \boldsymbol{v}_i)$ corresponds to an eigenvalue λ_i in $\boldsymbol{\Lambda}$. The ith eigenvalue λ_i, left eigenvector

$\boldsymbol{u}_i$, and right eigenvector $\boldsymbol{v}_i$, can be obtained by solving the following eigenvalue problems:

$$\boldsymbol{K}\boldsymbol{K}^{\mathrm{T}} = \lambda_i^2 \boldsymbol{u}_i \ , \ \ \boldsymbol{K}^{\mathrm{T}}\boldsymbol{K} = \lambda_i^2 \boldsymbol{v}_i \ , \tag{5}$$

Suppose the number of non-zero eigenvalues is h. It is easy to prove that the left eigenvectors and right eigenvectors correspond to these non-zero eigenvalues and that they are associated via the following equations:

$$\boldsymbol{K}\boldsymbol{v}_i = \lambda_i \boldsymbol{u}_i \ , \ \ \boldsymbol{K}^{\mathrm{T}}\boldsymbol{u}_i = \lambda_i \boldsymbol{v}_i \ , \quad i = 1, 2, \ldots h \ . \tag{6}$$

For zero-valued eigenvalues, i_i and $\boldsymbol{v}_i$ have the following relations:

$$\boldsymbol{K}^{\mathrm{T}}\boldsymbol{u}_i = 0 \ , \ \ i = h-1, \ h+2, \ldots, N \ , \tag{7}$$

$$\boldsymbol{K}\boldsymbol{v}_i = 0 \ , \ \ i = h+1, \ h+2, \ldots, M \ . \tag{8}$$

These eigenvectors are mutually orthonormal: $u_i^{\mathrm{T}}\boldsymbol{u}_j = \delta_{i,j}$, $\upsilon_i^{\mathrm{T}}\boldsymbol{v}_j = \delta_{i,j}$. Let $\boldsymbol{U}$ denote the N-dimension vector space spanned by the N left eigenvectors and call it a data space. The data space U can be partitioned into two subspaces U_h and U_0, where U_h is spanned by the h eigenvectors corresponding to non-zero eigenvalues and U_0 is spanned by the $N-h$ eigenvectors corresponding to the zero-valued eigenvalues. Similarly, the M-dimension vector space $\boldsymbol{V}$, called the parameter space, can be partitioned into $\boldsymbol{V}_h$ and $\boldsymbol{V}_0$. It can be seen from Eqs. (6)–(8) that only the projection of $\boldsymbol{\delta}T$ in subspace $\boldsymbol{V}_h$ is associated with that of $\boldsymbol{\delta I}$ in the data subspace $\boldsymbol{U}_h$, while the projection of $\boldsymbol{\delta T}$ in subspace $\boldsymbol{V}_0$ is independent of the observation. This implies that only the projection of $\boldsymbol{\delta T}$ in $\boldsymbol{V}_h$ has a relationship with the observation data and can be retrieved from observation. In other words, if the projection of $\boldsymbol{\delta I}$ in subspace $\boldsymbol{V}_0$, is not empty, retrieval errors of the temperature are always existent even if the observation and equation are perfect. This is the so-called resolution error.

If the observation errors are considered, the retrievability will decrease. From the above decomposition in Eqs. (4)–(6), $\boldsymbol{K}$ can be constructed by $\boldsymbol{U}_h$ and $\boldsymbol{V}_h$:

$$\boldsymbol{K} = \boldsymbol{U}_h \boldsymbol{\Lambda}_h \boldsymbol{V}_h^{\mathrm{T}} \ , \tag{9}$$

and then the solution of Eq. (3) (called the generalized linear inverse solution) can be written as

$$\boldsymbol{\delta T}_{\mathrm{r}} = (\boldsymbol{U}_h \boldsymbol{\Lambda}_h \boldsymbol{V}_h^{\mathrm{T}})^{-1} \boldsymbol{\delta I} = \boldsymbol{V}_h \boldsymbol{\Lambda}_h^{-1} \boldsymbol{U}_h^{\mathrm{T}} \boldsymbol{\delta I} \ , \tag{10}$$

where $\boldsymbol{\Lambda}_h$ is the $h \times h$ matrix containing h singular values along the diagonal, and $\boldsymbol{\delta T}_{\mathrm{r}}$ is the retrieved temperature vector. Assume that the observation errors are independent and have variance σ_{d}^2. The observation error vector is defined as ε_{d}. In substituting ε_{d} into Eq. (10), the retrieval error $\varepsilon_{\mathrm{rd}}$ caused by ε_{d} is obtained as

$$\boldsymbol{\varepsilon}_{\mathrm{rd}} = \boldsymbol{V}_h \boldsymbol{\Lambda}_h^{-1} \boldsymbol{U}_h^{\mathrm{T}} \boldsymbol{\varepsilon}_{\mathrm{d}} \ . \tag{11}$$

Calculating the vector product of $\varepsilon_{\mathrm{rd}}$ and its expectation value (denoted with angular brackets), we get the matrix:

$$\langle \varepsilon_{\mathrm{rd}} \varepsilon_{\mathrm{rd}}^{\mathrm{T}} \rangle = \sigma_{\mathrm{d}}^2 \boldsymbol{V}_h \boldsymbol{\Lambda}_h^{-2} \boldsymbol{V}_h^{\mathrm{T}} \ . \tag{12}$$

The variance of the retrieval error caused by the observation error is:

$$\sigma_{\mathrm{rd}}^2 = \frac{\langle \varepsilon_{\mathrm{rd}}^{\mathrm{T}} \varepsilon_{\mathrm{rd}} \rangle}{M} = \frac{\mathrm{tr}(\langle \varepsilon_{\mathrm{rd}} \varepsilon_{\mathrm{rd}}^{\mathrm{T}} \rangle)}{M} = \frac{\sigma_{\mathrm{d}}^2}{M} \mathrm{tr}(\boldsymbol{V}_h \boldsymbol{\Lambda}_h^{-2} \boldsymbol{V}_h^{\mathrm{T}}) = R\sigma_{\mathrm{d}}^2 \ , \tag{13}$$

where $\mathrm{tr}(\cdot)$ is the trace of a matrix,

$$R = \frac{\mathrm{tr}(\boldsymbol{V}_h \boldsymbol{\Lambda}_h^{-2} \boldsymbol{V}_h^{\mathrm{T}})}{M}$$

is the error amplification factor, and R_k is the one at pressure level p_k. So the retrieval error caused by the observation at p_k is $\sigma_{\mathrm{rd},k}^2 = R_k \sigma_{\mathrm{d}}^2$. It can be seen from Eq. (13) that R is mainly determined by the minimum of eigenvalue λ_h^2. If λ_h^2 is very small, the retrieval result will be very sensitive to observation errors. Thus, a truncation order $h' \leqslant h$ (Chou, 1986; Wunsch, 1978) should be given to assure the stability of the retrieval. In actual calculation λ_h^2 is set to zero if λ_h^2 is less than a specified value. h in all of the above formulas should be taken as h'. But in order to write expediently, we will continue to write h' as h hereinafter.

The error variance σ_{rd}^2 caused by observation is an averaged value over all the layers. In fact, the error and retrievability at each height can be estimated when the atmospheric temperature profiles are given. A useful method to represent the structure of the temperature profiles in terms of a minimal number of functions is the EOF technique, which is a common procedure in the temperature retrieval problem (Zeng, 1974; Li and Zeng, 1997). Consider a time series of temperature profiles represented by an $L \times M$ matrix $\boldsymbol{A}$, where L is the length of the time series and M is the number of vertical levels. Solving the eigenvalue problem

$$\boldsymbol{A}^{\mathrm{T}} \boldsymbol{A} q_i^{\mathrm{T}} = q_i^{\mathrm{T}} x_i \ , \tag{14}$$

we get M eigenvalues x_i and M eigenvectors q_i. Then the temperature anomaly at time l, namely δT_l can be expanded according to the orthogonal primary function:

$$\delta T_l = \sum_{i=1}^{M'} c_{il} q_i + \varepsilon_E \ , \tag{15}$$

where $M' \leqslant M$ is the truncation number, and ε_{E} is the truncation error. Real observation data analysis suggests that the convergence of the EOF is very fast so that we can obtain relatively accurate δT_l by a very low truncation order. Project the primary function q_i onto subspace $\boldsymbol{V}_h$:

$$q_i = \sum_{j=1}^{h} a_{ij} \boldsymbol{v}_j + e_i \ , \tag{16}$$

where $e_i = q_i - \sum_{j=1}^{h} a_{ij}\boldsymbol{v}_j$ is the projection of q_i onto subspace $\boldsymbol{V}_0$. The resolution error of δT at the ith mode is:

$$\varepsilon_{il} = c_{il}e_i = c_{il}\left(q_i - \sum_{j=1}^{h} a_{ij}\boldsymbol{v}_j\right) . \quad (17)$$

The error variance of resolution at level p_k is

$$\sigma^2_{\mathrm{s},i,k} = \langle \varepsilon^2_{i,l,k}\rangle = \langle c_i^2\rangle e^2_{i,k} , \quad (18)$$

where $e_{i,k}$ is the value of e_i at p_k. The total resolution error variance can be obtained by

$$\sigma^2_{\mathrm{s},k} = \sum_{i=1}^{M'} \langle c_i^2\rangle e^2_{i,k} , \quad (19)$$

where

$$\langle c_i^2\rangle = \frac{\lambda_i \langle \boldsymbol{\delta T}^{\mathrm{T}}\boldsymbol{\delta T}\rangle}{\sum_{i=1}^{n}\lambda_i} ,$$

which can be directly calculated from sample data. $\sigma^2_{\mathrm{e},k} = \langle \varepsilon^2_{\mathrm{E},k}\rangle$ is the truncation error of the EOF. When the model error is not considered, the total retrieval error is the square root of the variance sum containing the resolution error, the error caused by the observation, and the truncation error of the EOF:

$$\sigma_{\mathrm{r},k} = \sqrt{\sigma^2_{\mathrm{s},k} + \sigma^2_{\mathrm{rd},k} + \sigma^2_{\mathrm{e},k}} . \quad (20)$$

Then the retrievability at level p_k can be approximatively expressed based on the above analysis:

$$r_k = 1 - \sigma_{\mathrm{r},k}/\sigma_{T,k} , \quad (21)$$

where $\sigma_{T,K} = \sqrt{\langle \boldsymbol{\delta T}, \boldsymbol{\delta T}\rangle}$ is the mean square deviation of temperature at level p_k.

3. Test of the method

In order to test the credibility of the proposed method, a comparison between this method and other methods, the statistical-physical (SP) method and damped least-square (DLS) method, is given in the section. The detailed computing techniques and results are as follows.

3.1 *Brief introduction of the SP and DLS methods*

Following the SP method (Huang et al., 1992; Smith et al., 1991), a generalized statistical-physical solution of Eq. (3) can be written as:

$$\begin{aligned}\boldsymbol{\delta T}_{\mathrm{r}} &= (\boldsymbol{K}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{K} + \boldsymbol{B}^{-1})^{-1}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{\delta I} \\ &= \boldsymbol{C}\boldsymbol{\delta I} = \boldsymbol{C}(\boldsymbol{K}\boldsymbol{\delta T} + \boldsymbol{e}) , \end{aligned} \quad (22)$$

where $\boldsymbol{C}$ denotes the retrieval coefficient matrix; $\boldsymbol{B}$ is the sample statistical covariance matrix; $\boldsymbol{R}$ is the covariance of the brightness temperature error; $\boldsymbol{e}$ is assumed to be the brightness temperature measurement error; and superscripts T and -1 denote the transposition and inversion of a matrix, respectively. Then the covariance of the retrieval temperature error $\boldsymbol{G}, \boldsymbol{G} = \langle (\boldsymbol{\delta T} - \boldsymbol{\delta T}_{\mathrm{r}})(\boldsymbol{\delta T} - \boldsymbol{\delta T}_{\mathrm{r}})^{\mathrm{T}}\rangle$ can be deduced:

$$\begin{aligned}\boldsymbol{G} &= (\boldsymbol{I} - \boldsymbol{CK})\boldsymbol{\delta T}\boldsymbol{\delta T}^{\mathrm{T}}(\boldsymbol{I} - \boldsymbol{C}K)^{\mathrm{T}} + \boldsymbol{C}\boldsymbol{e}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}} \\ &= (\boldsymbol{I} - \boldsymbol{CK})\boldsymbol{B}(\boldsymbol{I} - \boldsymbol{CK})^{\mathrm{T}} + \boldsymbol{CRC}^{\mathrm{T}} = \boldsymbol{V} + \boldsymbol{M} , \end{aligned} \quad (23)$$

where $\boldsymbol{V} = (\boldsymbol{I} - \boldsymbol{CK})\boldsymbol{B}(\boldsymbol{I} - \boldsymbol{CK})^{\mathrm{T}}; \boldsymbol{M} = \boldsymbol{CRC}^{\mathrm{T}}; \boldsymbol{I}$ is the identity matrix. Terms $\boldsymbol{V}$ and $\boldsymbol{M}$ can be defined as the vertical resolution component error and the measurement noise component error, respectively. The total root-mean-square (rms) retrieval error $\sigma_{\mathrm{r},k}$ is the square root of the diagonal elements of matrix $\boldsymbol{G}$, if interlevel correlations are ignored. If we substitute $\sigma_{\mathrm{r},k}$ into Eq. (21), then the retrievability can be solved.

Another method (DLS) proposed by Smith et al. (1972) solves Eq. (3) by minimizing the quadratic form

$$J(\boldsymbol{\delta T}) = (\boldsymbol{K}\boldsymbol{\delta T} - \boldsymbol{\delta I})^{\mathrm{T}}(\boldsymbol{K}\boldsymbol{\delta T} - \boldsymbol{\delta I}) + \gamma\boldsymbol{\delta T}^{\mathrm{T}}\boldsymbol{\delta T} , \quad (24)$$

where γ is the damping coefficient. Differentiation with respect to the elements of $\boldsymbol{\delta T}$ yields the normal equation

$$(\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K} + \gamma\boldsymbol{I})\boldsymbol{\delta T} = \boldsymbol{K}^{\mathrm{T}}\boldsymbol{\delta I} , \quad (25)$$

which has the unique solution

$$\boldsymbol{\delta T}_{\mathrm{r}} = (\boldsymbol{K}^{\mathrm{T}}\boldsymbol{K} + \gamma\boldsymbol{I})^{-1}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{\delta I} = \boldsymbol{C}'(\boldsymbol{K}\boldsymbol{\delta T} + \boldsymbol{e}) . \quad (26)$$

A comparison of Eqs. (22) and Eq. (26) indicates that Eq. (22) is a more general solution than Eq. (26). However, Eq. (22) is impractical since the actual covariance of the errors of particular guess profiles (the elements of $\langle \boldsymbol{\delta T}\boldsymbol{\delta T}^{\mathrm{T}}\rangle$) is difficult to estimate. So Smith et al. (1972) estimated the γ as:

$$\gamma = \frac{\sigma^2_{\mathrm{d}}}{\sigma^2_{\mathrm{b}}} , \quad (27)$$

where scalars σ^2_{d} and σ^2_{b} are the error variances of the observation and background, respectively. Once the coefficient γ is determined, the retrieved temperature profiles can be given by Eq. (26). In fact, the DLS method can be considered as a simplification of the SP method on the assumption that the background errors are random. But actually they are not random, so the retrieval error cannot be estimated following Eq. (23). We can only calculate the retrieval error for each atmosphere profile by comparing the retrieval with the "true" profile directly. This can be implemented by the following scheme: (1) take each sample profile as the "true" profile; (2) the observations are calculated by adding random noise to the simulated brightness

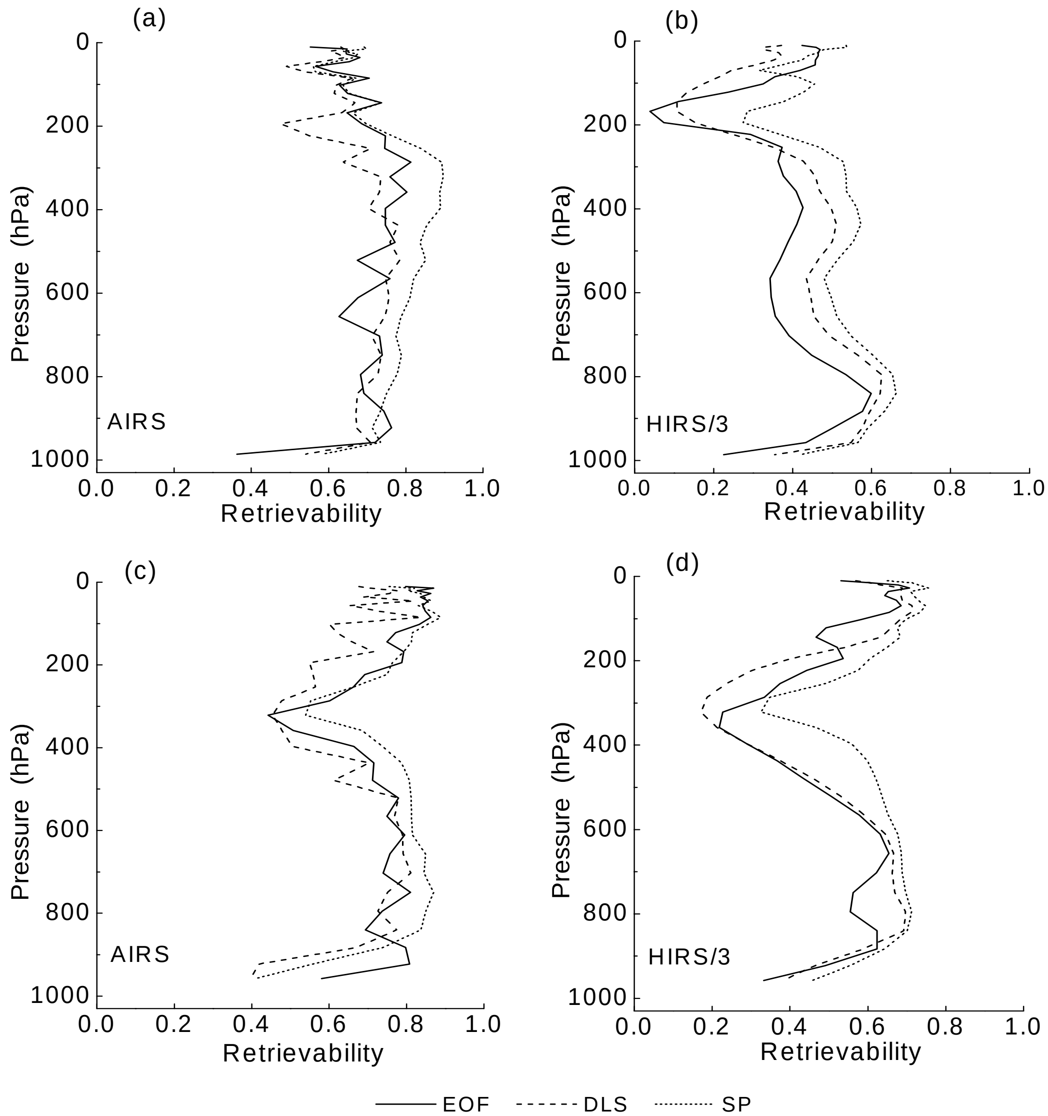

Fig. 1. The retrievabilitis obtained with AIRS and HIRS/3 data by three methods at point A (a and b) and B (c and d) (EOF: the method combining EOF and SVD; DLS: damped least-square method; SP: statistical-physical method).

temperature which were obtained from the "true" profiles via RTE; (3) a set of retrieval temperature profiles from these "observations" are used to calculate the mean error variances. In our test, a total five years' of profiles from the NCEP reanalysis dataset for each point and a set of "observations" with 100 members are used for each profile. This means that the retrieval operations with 100 times the profile numbers are performed for the calculation of the mean error variances at each point. Therefore, this requires much calculation time.

The SP method is widely used in retrieval problems. A drawback is the required full knowledge of the background error covariance and the inversion of the error covariance. Assuming the background errors are random, DLS simplifies SP to make it easier to implement. But the estimation of the damping coefficient γ in DLS is factitious to a certain extent and the covariance of the retrieval error cannot be estimated by Eq. (23) directly. So the calculation requirements are relatively large through computing the mean retrievability. While using the method proposed in this paper, the retrieval error can be estimated from RTE directly. This avoids the randomness of estimating γ and the large calculation requirements in the DLS method; moreover, it avoids the calculation of inverting the covariance matrix in the SP method.

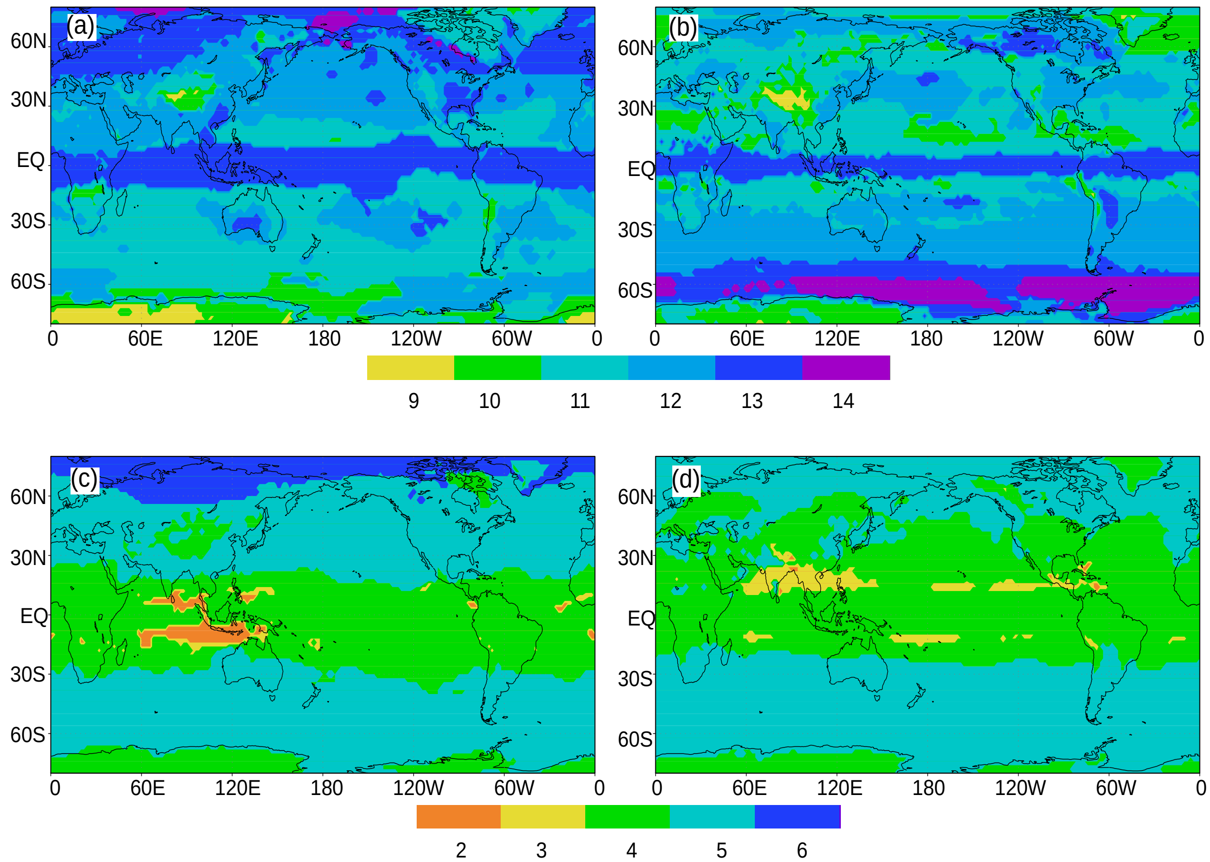

Fig. 2. The optimum truncation order h. (a) h obtained with AIRS data in January; (b) h obtained with AIRS data in July; (c) h obtained with HIRS/3 data in January; (b) h obtained with HIRS/3 data in July.

3.2 *Comparison of the three methods*

We utilize the three methods mentioned above to compute the retrievability. Following Smith et al. (1972) and Huang et al. (1992), an rms radiance error equivalent to a brightness temperature measurement error of 0.25 K is assumed. The AIRS observations are from 324 channels accepted by the U. K. Met Office and the HIRS/3 observations are from 12 channels. The atmosphere radiative transfer model is RTTOV7 employed by the European Center for Medium range Weather Forecasting (ECMWF). The atmospheric relative humidity and temperature profiles are taken from the NCEP reanalysis dataset in January from 1999 to 2003. The reference field is the mean temperature profile.

Two points A (30°N, 120°E) and B (60°N, 120°E) were chosen to perform the comparison because of the large calculation requirements in DLS. In the method proposed in this paper, the truncation order of the EOF is 11 and the truncation order of the SVD is the value corresponding to the minimum retrieval error (see section 2). The vertical distribution of the retrievability calculated by the three methods at the two points in January is shown in Fig. 1.

It is shown from Fig. 1 that the retrievabilities obtained by the proposed method are similar to the results obtained by the DLS method, but they were a bit smaller than those of SP. The reason is that the full information about background errors, an additional piece of information, was used in the SP method. The comparison also shows that the difference among the three methods is less with the AIRS data than with the HIRS/3 data. This illuminates that the influence of the background error covariance is less with the AIRS data than with the HIRS/3 data, which occurs because the spectral resolution is higher and more information is contained in AIRS.

In the DLS method, the damping coefficient γ is estimated based on the "true" temperature profiles, but it may not be so accurate in actual calculations, while the compared results between DLS and the proposed method are very similar and only a bit smaller than those of the SP method. All of this indicates that the method combining EOF and SVD is feasible and can compute the global retrievability expediently. So it can be used to compute the global retrievability and be used as a practical retrieval method. In the next section, a globe retrievability image will be given by the method that combines the SVD and EOF techniques.

4. Result analyses

Utilizing the error estimate method in section 2, the global retrievability distribution of atmospheric temperature profiles can be calculated with AIRS and HIRS/3 data. NCEP reanalysis data with $1^\circ \times 1^\circ$ resolution in January and July from 1999 to 2003 are used as the atmosphere parameter samples. The reference fields are produced by averaging the temperature profiles over the full month (January or July) at each spot. The temperature anomaly can be calculated, which can then be decomposed with the EOF analysis. The vertical structure of the temperature anomaly in most regions can be represented by the first 11 truncated EOF vectors with the average variance larger than ninety-eight percent. So the truncation order of 11 is used in the following calculation.

4.1 *Optimum truncation order in SVD*

It was seen from the last section that resolution errors decrease when the truncation order in SVD increases, while the errors caused by observation increase. A good truncation order should be selected by considering both stability and resolution. There have been many studies on this issue because the choice of truncation order will greatly influence the precision of the retrieval solution (Thompson, 1991; Li et al., 2001). When the observation error σ_d^2 and atmosphere parameter samples are given, the mean retrieval error over all levels can be calculated with different truncation orders by the method in the above section. The optimum truncation order is the value that corresponds to the smallest error. Figure 2 is the distribution of optimum truncation order obtained with AIRS and HIRS/3 data in January and July. Here the assumption that the rms error of the brightness temperature is 0.25 K is applied. It is shown in Figs. 2a and 2b that the truncation order with the AIRS data in most regions from 60°N to 60°S in January is between 11 and 13. But the truncation order near the Tibetan Plateau is lower than in other regions, thus h can be set to 9 (in July) or 10 (in January). The truncation order reaches 13 in the equatorial regions. Also, h at high latitudes in the Northern Hemisphere in January and the Southern Hemisphere in July is relatively large ($\geqslant$13). Similar to the AIRS analysis result, in Fig. 2c and 2d, h is also low near the Tibetan Plateau when the HIRS/3 data are used. And h in low latitude regions and some regions of Antarctica has a smaller value than in middle-high latitude regions. In January, h is 2 at some equatorial regions of the Indian Ocean. In July, the regions corresponding to small h are extended. Generally, the regions corresponding to small h spread northwards. In July, in most regions of middle-low latitudes of the Northern Hemisphere, h is smaller than or equal to 4, while h in the Southern Hemisphere is similar to that in January. Different from the results using AIRS data, the values of h with the HIRS/3 data have a more distinct banding distribution characteristic, the values in the equatorial area are the smallest, and all values from the HIRS/3 data are far less than those from the AIRS data. The magnitude of h represents the information of the atmospheric temperature anomaly retained in the satellite data. The larger the h, the more information can be retrieved from the satellite data. Of course, the AIRS data contains more information on the atmospheric temperature. However, if the effective information ratio is denoted by h/N, the effective information ratio of AIRS data is less than that of the HIRS/3 data. This indicates that the independence of each channel of the AIRS data is lower than that of the HIRS/3 data.

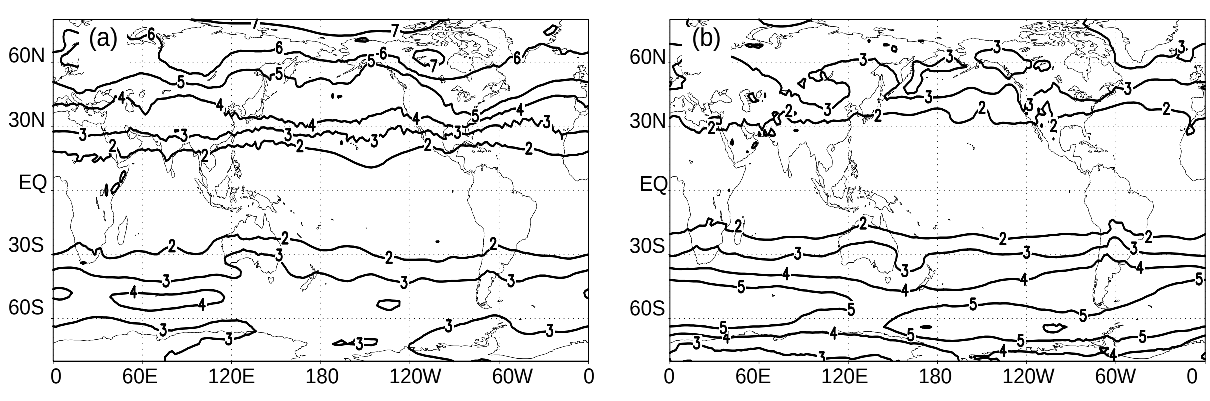

Fig. 3. The mean square deviation (K) of temperature over all levels in January (a) and July (b).

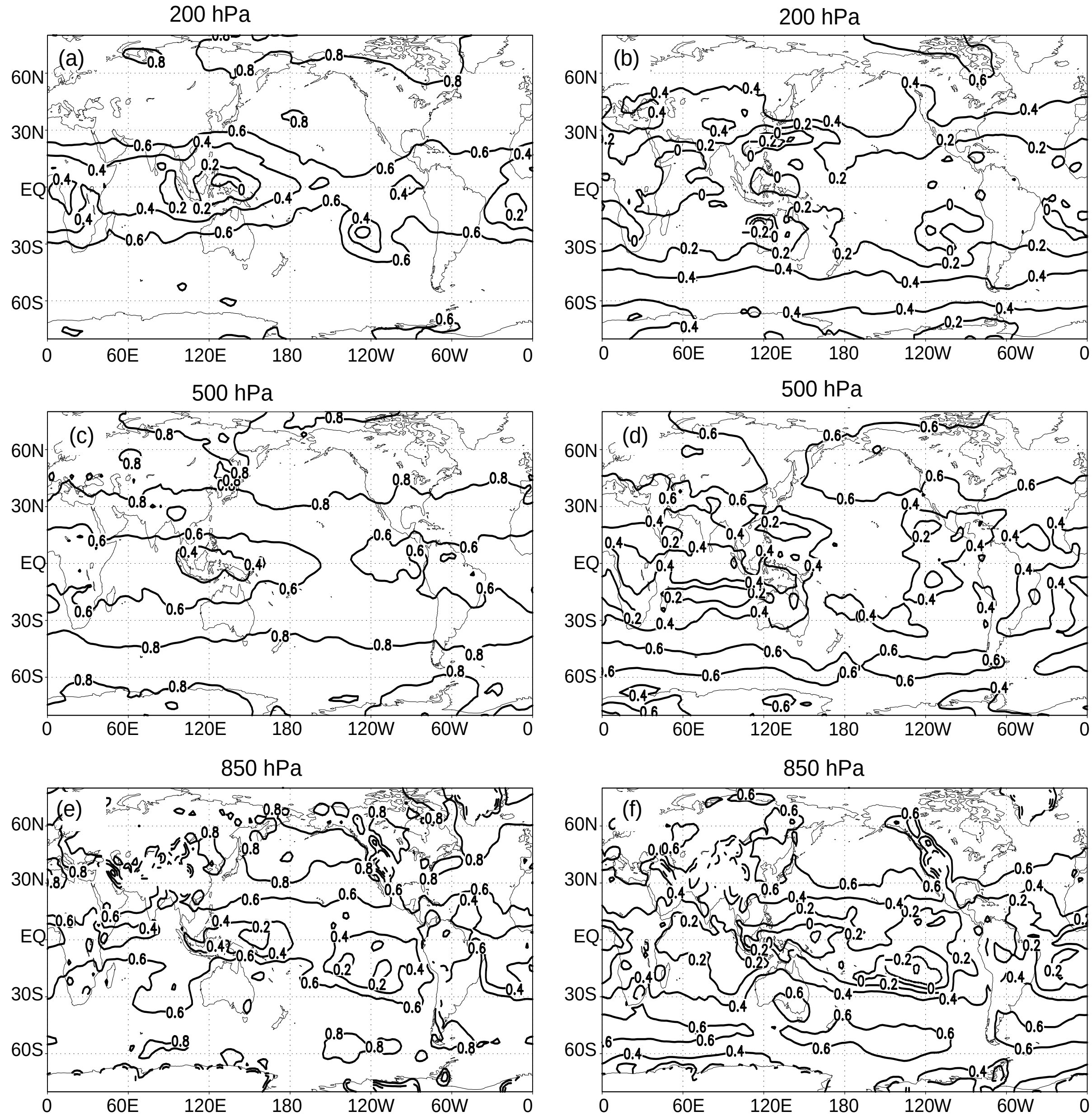

Fig. 4. The retrievability of temperature obtained with AIRS (a, c, e) and HIRS/3 (b, d, f) data in January.

In fact, the distribution of optimum truncation order is dependent on the distribution of the mean square deviation of temperature when we calculated it at each grid point. This feature is obvious in the HIRS/3 data. Figure 3 is the distribution of averaged mean square deviation of temperature over all levels. It shows that the values in low latitude regions are the minimum and that they are larger in winter (NH: January, SH: July) than in summer in mid-high latitude regions. This explains that a high (low) mean square deviation of temperature corresponds to high (low) optimum truncation order.

4.2 *Retrievability analyses*

Using the proposed method, the retrievability can be calculated according to the optimum truncation order at each isobaric surface. Figure 4 is the global retrievability at 200 hPa, 500 hPa, and 850 hPa in January. For the three pressure levels, the retrievabilities are all relatively low in the low latitude areas near the equator. At 200 hPa, for the AIRS data, there are two distinct small-value (<0.2) regions, which lie over the West Pacific Ocean at the equator and the

Atlantic Ocean east of South America, while a high retrievability of 0.6–0.8 is located in the mid-high latitudes. For the HIRS/3 data, the retrievabilities in the 30°N–30°S regions are smaller than 0.2 in most regions. The value in other regions is about 0.4. At 500 hPa, the distribution of retrievability is similar to that at 200 hPa in many regions. But its magnitude is about 0.2 higher than that at 200 hPa and the difference of retrievability between low latitudes and mid-high latitudes becomes small. The retrievability at this level is higher than that at 200 hPa and 850 hPa in some regions, especially in low latitude areas. At 850 hPa, the retrievabilities are obviously tied to landform. And the values are smaller than those at 500 hPa in most low-latitude regions. But the difference of retrievability between 850 hPa and 500 hPa in mid-high latitude regions is not large. In general, the distribution characteristics of retrievabilities obtained with the two datasets are similar, but the values from AIRS data are about 0.15–0.4 higher than those from HIRS/3 data in the same latitude regions. This indicates that AIRS data contains more information of the atmospheric temperature profile. The distribution of retrievability in July is similar to January. There exists little difference in most regions and so the figures are omitted here.

In order to reflect the character of the vertical distribution, the global retrievability at each height is averaged in January and July, respectively. The globe mean retrievability profiles are calculated and shown in Fig. 5. It can be seen that the vertical variation of the retrievabilities obtained with AIRS and HIRS/3 data are similar to each other regardless of whether in January or in July. The maximums of the profiles appear near 500 hPa and the magnitudes in the upper air and near the ground are relatively low. The retrievabilities from AIRS data are over 0.5 at most heights and much higher than those from HIRS/3 data. The differences in the retrievabilities obtained with the two datasets are relatively large above 400 hPa. This indicates that large improvements in the retrieval can be gained in the mid-upper atmosphere. Figure 6 is the averaged retrievability by latitude in January and July with the two kinds of data. It is shown that the improvements in retrievability from the AIRS data are the highest in the low latitude areas. In January in particular, the improvement in the retrievability exceeds 0.3. For the two kinds of data, the retrievabilities in the Northern Hemisphere in January are better than in July, while the situation in the Southern Hemisphere is the reverse. Another characteristic is that the retrievabilities of temperature exhibit a minimum in the low latitude regions and they increase with increasing latitude. But in the regions south of 50°S, the retrievability decreases with increasing (absolute) latitude.

In conclusion, the distribution characteristics of retrievability by latitude are related to the temporal variability of temperature. The lesser temporal variability in the low latitude and oceanic regions indicates that the deviation between actual atmospheric profiles and the background are relatively small, and that less effective information can be obtained from observations. So the significance of the retrieval is not correspondingly large and the retrievabilities are relatively small. The distribution characteristic of retrievability by height is that the values in the middle atmosphere are somewhat large. The reason is that the weight function in many channels covers the middle atmosphere and the information of satellite observations is reflected more in the middle atmosphere.

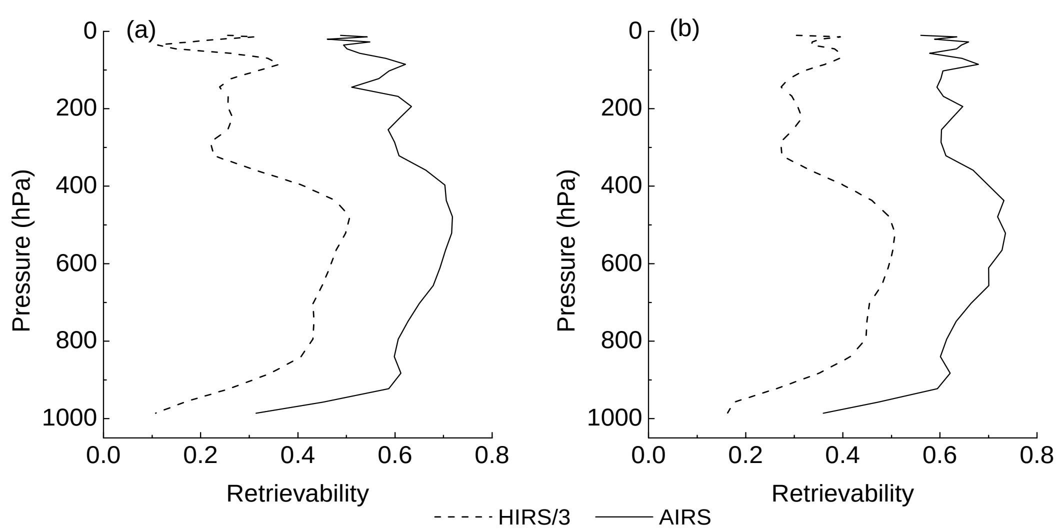

Fig. 5. Average retrievability at each height in (a) January and (b) July.

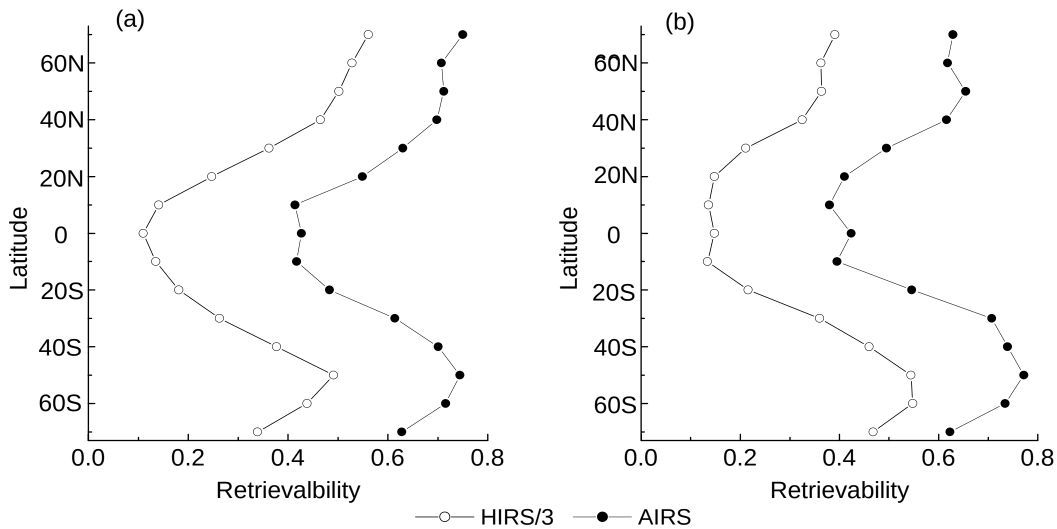

Fig. 6. Average retrievability by latitude in (a) January and (b) July.

5. Summary and discussion

The retrievability of temperature from satellite remote sensing data can be comprehended as the ultimate precision of retrieval. A method for estimating the retrievability of the clear-sky temperature profiles is developed based on GLIT and EOF analysis in this paper. Utilizing the proposed method, the retrievability at each height can be calculated easily when the observation error is given. The main modes of the atmospheric vertical structure can be obtained by EOF analysis. Using five years' of NCEP temperature reanalysis data as the samples, the global distribution of the retrievabilities of atmospheric temperature are calculated with the infrared detector data of AIRS and HIRS/3. The optimum truncation order in GLIT is estimated too. The basic characteristics are as follows:

(1) The optimum truncation order decides the effective information offered by the satellite measurements. The results show that the optimum truncation order is small in low latitude regions and high in mid-high latitude regions. This characteristic is related to the temporal variability of the atmospheric temperature. Using the AIRS data, the optimum truncation order in most areas is between 10 and 14. It is large, relatively, in high latitude areas of the Northern Hemisphere in January and in the Southern Hemisphere in July. The optimum truncation order is between 3 and 7 when using the HIRS/3 data. Thus, the AIRS data can provide more information for the temperature retrieval, but it also contains more invalid information.

(2) The retrievability of temperature is low in the upper and lower atmosphere, and high between 400 hPa and 850 hPa. In geographical distribution, the retrievabilities are low in the low latitude marine regions and in some regions in Antarctica, and relatively high in mid-high latitude regions and continental regions. This is consistent with the distribution of the optimum truncation order and partly represents the relationship between retrievability and the variability of temperature.

(3) In comparing the retrievabilities obtained with the AIRS and HIRS/3 data, the former are 0.15–0.4 higher than the latter and the retrievabilities obtained with the AIRS data are improved more evidently in the low latitude regions. This implies that using the AIRS data can improve the temperature retrieval in theory. In particular in using the AIRS data, the retrievability at 500 hPa in some mid-high latitude regions can reach 0.8. But the actual retrieval precision is still lower than this value. This indicates that existing retrieval methods (including radiance models) and the data procedures can be further improved.

It should be pointed out that the calculations in this paper were completed under clear-sky conditions and that a hypothetical observation error was used. Thus, the results may not fully represent the actual precision of a real temperature retrieval. But the distribution characteristics of retrievability in three dimensions can still provide useful information in the application of satellite data. In addition, the optimum truncation order obtained in this paper has some actual application merit in the retrieval process.

Acknowledgments. The authors would like to thank Dr. Liu Shun for his valuable suggestions. This study was supported by the National Natural Science Foundation of China (Grant Nos. 40375009 and 40305003).

REFERENCES

Backus, G. E., and F. Gilbert, 1970: Uniqueness in the inversion of inaccurate gross earth data. *Philos. Trans. Roy. Soc. London*, **266**A, 123–192.

Chou Jifan, 1986: *Long-Term Numerical Weather Prediction*. China Meteorological Press, Beijing, 216–231. (in Chinese)

Conrath, B. J., 1972: Vertical resolution of temperature profiles obtained from remote radiation measurement. *J. Atmos. Sci.*, **29**, 1262–1272.

Huang, H. L., W. L. Smith, and H. M. Woolf, 1992: Vertical resolution and accuracy of atmospheric infrared sounding spectrometers. *J. Appl. Meteor.*, **31**, 265–274.

Huang Sixun, 1996: Inversion and ill-posed problem solutions in atmospheric remote sensing. *Adv. Atmos. Sci.*, **13**(4), 489–504.

Li Jun, 1995: The capability of atmospheric profile retrieval from satellite high resolution infrared sounder radiance. *Adv. Atmos. Sci.*, **12**(2), 254–258.

Li Jun, and Zeng Qingcun, 1997: Infrared remote sensing of clear atmosphere and related inversion problem. Part II: Experimental study. *Scientia Atmospherica Sinica*, **21**(2), 214–222. (in Chinese)

Li Ping, Wang Chunyong, Xu Houze, and Xiong Xiong, 2001: The discussion of singular value decomposition's application in geophysical inversion. *Progress in Natural Science*, **11**(8), 891–896. (in Chinese)

Lorenc, A. C., and Coauthors, 2000: The Met. Office global three-dimensional variational data assimilation scheme. *Quart. J. Roy. Meteor. Soc.*, **126**, 2991–3012.

Rodgers, C. D., 1988: A general error analysis for profile retrieval. *Advances in Remote Sensing Retrieval Methods*, A. Deepak, Ed., A. Deepak Publishing, 285–298.

Smith, W. L., H. M. Woolf, and H. E. Fleming, 1972: Retrieval of atmospheric temperature profiles from satellite measurements for dynamical forecasting. *J. Appl. Meteor.*, **11**, 113–122.

Smith, W. L., H. M. Woolf, and H. E. Revercomb, 1991: Linear simultaneous solution for temperature and absorbing constituent profiles from radiance spectra. *Appl. Opt.*, **30**, 1117–1123.

Susskind, J., C. Barnet, and J. Blaisdell, 1998: Determination of atmospheric and surface parameters from simulated AIRS/AMSU/HSB sounding data: Retrieval and cloud clearing methodology. *Adv. Space Res.*, **21**, 369–384.

Thompson, O. E., 1982: HIRS-AMTS satellite sounding system test—Theoretical and empirical vertical resolving power. *J. Appl. Meteor.*, **21**, 1550–1562.

Thompson, O. E., 1991: Regularizing the satellite temperature-retrieval problem through singular-value decomposition of the radiative transfer physics. *Mon. Wea. Rev.*, **120**, 2314–2328.

Thompson, O. E., D. D. Dazlich, and Yu-Tai Hou, 1986: The ill-posed nature of the satellite temperature retrieval problem and the limits of retrievability. *J. Atmos. Oceanic Technol.*, **3**, 643–649.

Wiggins, R. A., 1972: The general linear inverse problem: Implication of surface waves and free oscillations for Earth structure. *Rev. Geophys. Space Phys.*, **10**, 251–285.

Wu Xuebao, Zhang Fengying, and Zhu Yuanjing, 2003: Retrieval of thermodynamic parameters using high spectral resolution infrared measurements from satellites and aircraft. *Meteorology Science and Technology,* **31**(4), 201–205. (in Chinese)

Wunsch, C., 1978: The North Atlantic general circulation west of 50°W determined by inverse methods. *Rev. Geophys. Space Phys.*, **16**(4), 583–620.

Zeng Qingcun, 1974: *Principles of Infrared Remote Sounding of the Atmosphere*. Science Press, Beijing, 1–76. (in Chinese)