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A Simplified Scheme of the Generalized Layered Radiative Transfer Model

DAI Qiudan*(戴秋丹) and SUN Shufen (孙菽芬)

State Key Laboratory of Numerical Modeling for Atmospheric Sciences and Geophysical Fluid Dynamics, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029

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ABSTRACT

In this paper, firstly, a simplified version (SGRTM) of the generalized layered radiative transfer model (GRTM) within the canopy, developed by us, is presented. It reduces the information requirement of inputted sky diffuse radiation, as well as of canopy morphology, and in turn saves computer resources. Results from the SGRTM agree perfectly with those of the GRTM. Secondly, by applying the linear superposition principle of the optics and by using the basic solutions of the GRTM for radiative transfer within the canopy under the condition of assumed zero soil reflectance, two sets of explicit analytical solutions of radiative transfer within the canopy with any soil reflectance magnitude are derived: one for incident diffuse, and the other for direct beam radiation. The explicit analytical solutions need two sets of basic solutions of canopy reflectance and transmittance under zero soil reflectance, run by the model for both diffuse and direct beam radiation. One set of basic solutions is the canopy reflectance $\alpha_{\rm f}$ (written as α_1 for direct beam radiation) and transmittance $\beta_{\rm f}$ (written as β_1 for direction beam radiation) with zero soil reflectance for the downward radiation from above the canopy (i.e. sky), and the other set is the canopy reflectance ($\alpha_{\rm b}$) and transmittance $\beta_{\rm b}$ for the upward radiation from below the canopy (i.e., ground). Under the condition of the same plant architecture in the vertical layers, and the same leaf adaxial and abaxial optical properties in the canopies for the uniform diffuse radiation, the explicit solutions need only one set of basic solutions, because under this condition the two basic solutions are equal, i.e., $\alpha_{\rm f} = \alpha_{\rm b}$ and $\beta_{\rm f} = \beta_{\rm b}$. Using the explicit analytical solutions, the fractions of any kind of incident solar radiation reflected from (defined as surface albedo, or canopy reflectance), transmitted through (defined as canopy transmittance), and absorbed by (defined as canopy absorptance) the canopy and other properties pertinent to the radiative transfer within the canopy can be estimated easily on the ground surface below the canopy (soil or snow surface) with any reflectance magnitudes. The simplified transfer model is proven to have a similar accuracy compared to the detailed model, as well as very efficient computing.

Key words: generalized layered canopy radiative transfer model, simplified model, analytical solutions, basic solutions, adaxial, abaxial, leaf optical properties

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1. Introduction

The generalized layered radiative transfer model (GRTM), dealing with incident direct beam or diffuse radiation transfer in the vegetation canopy with either even or uneven leaf optical properties, has been developed. In this model, full upward or downward hemispherical diffuse radiation is represented by the sum of its nine sub-components distributed in nine classes of 10 degrees each (Dai and Sun, 2006). In addition, the leaf inclination angle distribution in each layer is also described by the sum of nine sub-parts distributed in the same nine inclination sectors. By considering the light-leaf geometric optics relationship in each layer in detail, the model can provide the key components of radiation transfer more accurately within a canopy, such as the reflection from, transmission through and absorption by a canopy, and the portion of ground absorption, which are all very essential issues for the land surface processes models. Compared with other radiative transfer models, e.g., the widely used two-stream model in land surface models, this model can

*E-mail: dqd@mail.iap.ac.cn

not only provide more accurate results but can also be used in more general cases: in the anisotropic distribution of both incident sky radiation and diffuse radiation within the canopy; uneven optical properties of adaxial and abaxial leaf surfaces; differing leaf angle distributions in each layer; and different plants in the layers. It greatly expands the application of the research on radiative transfer within the canopy (Dai and Sun, 2006, 2007).

There are thousands and thousands of grid cells on the global land surface with a size of around 100 km×100 km in the current GCM model, which demands a more advanced computer system. So, the land surface model dealing with physical processes of the surface in the GCM models requires its subcomponents to be as simple as possible, but without losing its precision. For example, the SSiB model is a simplified version of SiB (Sellers et al., 1986), with the simplifications made on three main aspects, among which the most important one is for the estimation of radiation transfer fluxes in the canopy (Xue et al., 1991). For the same reason, the GRTM also needs to be as simple as possible, but what is the criterion for the GRTM simplification without losing its precision? It was suggested that an absolute accuracy of 0.02–0.05 of the albedo estimation is demanded for climate studies because it is found that an error greater than this range in albedo estimation on the ground surface will reduce the accuracy of GCM simulation (Henderson-Sellers and Wilson, 1983; Sellers, 1993; Lean and Rowntree, 1997). So, the criterion that the absolute error of the reflectance calculation from the canopy is less than 0.02 is also used for simplification of the GRTM below.

As mentioned above, the GRTM (Dai and Sun, 2006) divided the full hemispherical diffuse radiation into nine sub-streams in nine radiation inclination angle sectors, and also divided the leaf inclination angle distribution of the canopy into nine classes (0°–10°, 10°–20°, 20°–30°, 30°–40°, 40°–50°, 50°–60°, 60°–70°, 70°–80°, and 80°–90°). The results from the GRTM were compared with those of the model with a finer division of the full hemisphere diffuse radiation, such as 18 sub-streams in 18 inclination angle sectors. This comparison demonstrated that the differences from the two models were very small, meaning the accuracy of the GRTM is believable and could serve as the basic model for simplification.

Many real leaf angle distributions for canopies have been grouped into classes typified by names such as spherical, planophile and erectophile etc. (De Wit, 1965). The measurement data for leaf inclination angle distribution of vegetation with a higher resolution of nine inclination angle classes has been difficult to obtain, with few successes in the literature. However, the empirical data for three classes can be provided with less difficulty than for the nine classes employed in most other studies (Goudriaan, 1988). Ross's book provided a lot of data for three classes of leaf angle distributions in real plant stands (Ross, 1981). In this paper, the possibility of simplifying the GRTM with three leaf angle classes (0°–30°, 30°–60° and 60°–90°) is also examined.

In section 3, below, different simplification schemes of the GRTM are tested and the results from the schemes are compared with the results from the GRTM. The criterion of the reflectance difference being less than 0.02 between the GRTM and each simplified version is employed to decide whether the tested simplified version could be accepted or not. Detailed test design and comparison between the GRTM and each simplified version will also be presented in the same section.

In the GRTM, or other radiative transfer models in the canopy (Wang, 2005; Li et al., 1995; Ni and Woodcock, 2000), the radiative transfer process is dependent not only on the leaf inclination angle distribution and leaf optical properties, but also on the optical properties of the ground surface (soil or snow surface) below the canopy, especially on soil (or snow) surface reflectance ρ_s because reflectance is the bottom boundary condition for the solution of solar radiative transfer within the entire canopy system. The conditions of an underlying ground surface varies, which makes the optical properties highly changeable. For example, ρ_s changes greatly with the color, wetness, soil texture and the existence of snow cover. Previously, the radiative transfer models in the canopy, e.g., the GRTM or the two-stream model, should be run repeatedly when there is any change in ρ_s, which of course consumes computer resources. However, both the GRTM and the two-stream model are linear systems and thus their solutions should obey the superposition principle. Applying the superposition principle in optics to the basic radiative transfer solutions within the canopy under the condition of zero soil surface reflectance ($\rho_s = 0$), we trace and analyze the successive scattering phenomenon between the underlying surface and the plant system, and have derived two sets of general and explicit analytical solutions. The derivation method employed here was also used in an atmospheric radiation transfer study (Liou, 2002). The two sets of solutions can be easily used to quantify the radiative transfer of either incident direct beam, or diffuse radiation, with any kind of distribution for the canopy, with either the same or different adaxial and abaxial leaf optical properties; and by using these derived solutions, it will greatly save on computer resources. It is obvious that the methodology to derive the explicit analytical solutions used here can be applied to other radiative transfer models within the canopy, such as the two-stream model. The derivations and the verifications of the analytical solutions for any soil surface reflectance are provided in section 4.

At the same time, changes in the leaf area index (LAI) with the seasons will affect the radiative transfer process. In order to further simplify the radiative transfer in the canopy for various LAI values, we fit the basic solutions versus different LAI values, and acquire the corresponding fitting formulas under diffuse and a specific incident direct beam radiation. The results from fitting the basic solutions combined with the explicit analytical solutions are compared with those of the model, also presented in section 4.

2. GRTM physics

The full GRTM model is described in detail by Dai and Sun (2006), with only basic physics equations presented as:

$$\begin{aligned}\phi_{\rm d}(\beta_k', j+1) = {} & \phi_{\rm d}(\beta_k', j) \sum_{\lambda_n=1}^{N} g(\lambda_n, L_j) M_{\rm t}(\beta_k', \lambda_n) + \\ & \sum_{\lambda_n=1}^{N} g(\lambda_n, L_j) B_l(\beta_k', \lambda_n) \sum_{\beta_k=1}^{K} M_i(\beta_k, \lambda_n) \times \\ & [\phi_{\rm d}(\beta_k, j)(\rho_j \zeta_{\rm f} + \rho_j' \zeta_{\rm b} + \tau_j \xi_{\rm f} + \tau_j' \xi_{\rm b}) + \\ & \phi_{\rm u}(\beta_k, j+1)(\rho_j \xi_{\rm b} + \rho_j' \xi_{\rm f} + \tau_j \zeta_{\rm b} + \tau_j' \zeta_{\rm f})] + \\ & \sum_{\lambda_n=1}^{N} g(\lambda_n) B_l(\beta_k', \lambda_n) I_{\rm B}(z=L_j) dL_j \frac{G(\beta_0, \lambda_n)}{\sin\beta_0} \times \\ & (\rho_j \zeta_{\rm f,B} + \rho_j' \zeta_{\rm b,B} + \tau_j \xi_{\rm f,B} + \tau_j' \xi_{\rm b,B})\ , \end{aligned} \tag{1a}$$

$$\begin{aligned}\phi_{\rm u}(\beta_k', j) = {} & \phi_{\rm u}(\beta_k', j+1) \sum_{\lambda_n=1}^{N} g(\lambda_n, L_j) M_{\rm t}(\beta_k', \lambda_n) + \\ & \sum_{\lambda_n=1}^{N} g(\lambda_n, L_j) B_l(\beta_k', \lambda_n) \sum_{\beta_k=1}^{K} M_i(\beta_k, \lambda_n) \times \\ & [\phi_{\rm d}(\beta_k, j)(\rho_j \xi_{\rm f} + \rho_j' \xi_{\rm b} + \tau_j \zeta_{\rm f} + \tau_j' \zeta_{\rm b}) + \\ & \phi_{\rm u}(\beta_k, j+1)(\rho_j \zeta_{\rm b} + \rho_j' \zeta_{\rm f} + \tau_j \xi_{\rm b} + \tau_j' \xi_{\rm f})] + \\ & \sum_{\lambda_n=1}^{N} g(\lambda_n, L_j) B_l(\beta_k', \lambda_n) I_{\rm B}(z=L_j) dL_j \frac{G(\beta_0, \lambda_n)}{\sin\beta_0} \times \\ & (\rho_j \xi_{\rm f,B} + \rho_j' \xi_{\rm b,B} + \tau_j \zeta_{\rm f,B} + \tau_j' \zeta_{\rm b,B})\ , \end{aligned} \tag{1b}$$

where β_k ($k = 1, \ldots, 9$) is the inclination of the incident diffuse light; β_0 is the inclination angle of the sun; β_k' is the inclination of the scattered radiation; $\phi_{\rm u}(\beta_k', j)$ and $\phi_{\rm d}(\beta_k', j)$ are the upward and downward scattered radiation fluxes at inclination angle β_k' between layer j and $j-1, \rho_j$ and τ_j are the adaxial leaf reflectance and transmittance; and ρ_j' and τ_j' the abaxial leaf reflectance and transmittance for layer j; $g(\lambda_n, L_j)$ is the leaf angle distribution function for a leaf with an inclination angle of λ_n at layer j in the canopy; $G(\beta_0, \lambda_n)$ is the G function, which is the projection of leaves inclined at λ_n to the solar beam direction with inclination β_0; and $\overline{G(\beta_0)}$ is the average G function of leaves with angle distribution $g(\lambda_n, L_j)$ to the solar beam direction β_0. L_j is the cumulative LAI from the canopy top to layer j; $M_i(\beta_k, \lambda_n)$ is the intercepted coefficient of layer j; and $M_i(\beta_k, \lambda_n) = dL_j G(\beta_k, \lambda_n)/\sin\beta_k$, where dL_j is the LAI of layer $j \cdot M_t(\beta_k, \lambda_n)$ is the penetration coefficient of layer $j; \xi_{\rm f}\ \xi_{\rm b}, \zeta_{\rm f}$ and $\zeta_{\rm b}$ in Eqs. (1a) and (1b) are the adaxial and abaxial leaf reflectance and transmittance distribution functions for the upscattering of the downward diffuse radiation; $\xi_{\rm f,\ B}, \xi_{\rm b,\ B}, \zeta_{\rm f,\ B}$ and $\zeta_{\rm b,\ B}$ are the same distribution functions for the solar beam, indicated by the subscript B. All these functions depend on the incident light inclination angle β_k (β_0 for solar beam), the scattered light inclination angle β_k' and the leaf inclination angle λ_n. For the detailed derivations and expressions of the above parameters, please refer to Dai and Sun (2006). $B_l(\beta_k', \lambda_n)$ is the anisotropic scattering distribution function, defined as:

$$B_l(\beta_k', \lambda_n) = \frac{B_{\rm u}(\beta_k') M_i(\beta_k', \lambda_n)}{\sum_{\beta_k=1}^{9} B_{\rm u}(\beta_k) M_i(\beta_k, \lambda_n)}\ ,$$

where $B_{\rm u}(\beta_k')$ is the distribution function for isotropic diffuse radiation (Goudriaan, 1977).

3. Simplification of the GRTM

As mentioned in the introduction, three tests are designed to simplify the GRTM. Test 1 is the simplification of the leaf inclination angle classes of the GRTM: keeping the incident radiation sectors as that in the GRTM, the nine leaf angle classes are reduced to three. Test 2 is the simplification of the radiation angle sectors of the GRTM: keeping the leaf angle distribution as nine angle classes, the radiation angle distribution division is reduced to three angle sectors. Test 3 is the simplification of both the radiation angle sectors and the leaf angle classes of the GRTM: the nine radiation angle sectors are reduced to three, and at the same time, so are the nine leaf angle classes. The results of the three tests were compared with those of the original GRTM for direct beam and diffuse radiation, respectively, under various conditions: these include different n1−n12 combinations of the LAI; leaf reflectance and transmittance, and soil reflectance (see Appendix); and different types of canopies, with spher-

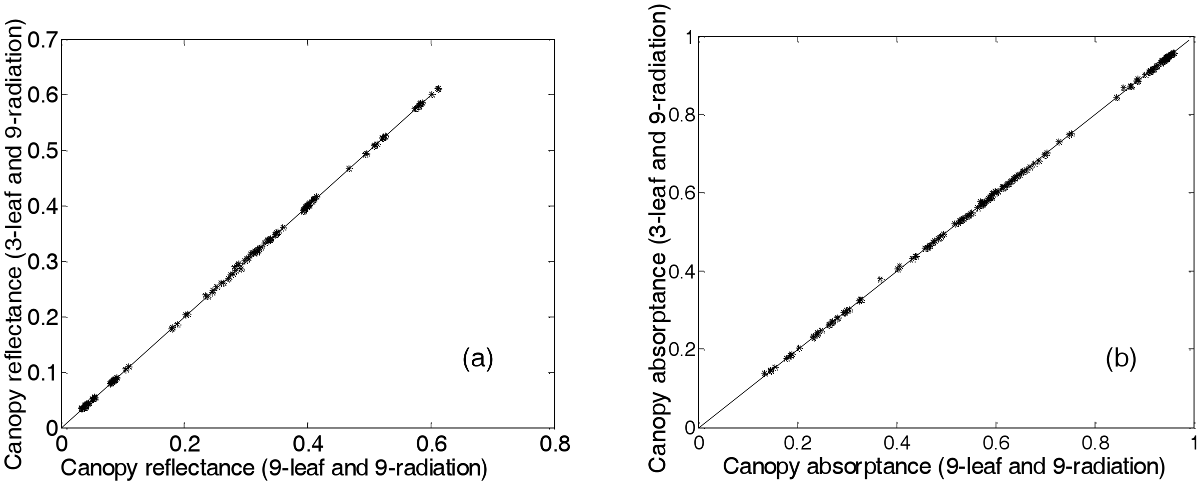

Fig. 1. Comparison of (a) the canopy reflectance and (b) absorptance by the GRTM (with nine radiation angle sectors and nine leaf angle classes) with those by test 1 (with nine radiation angle sectors and three leaf angle classes) under direct beam radiation.

ical, planophile and erectophile leaves.

3.1 *Simplification of the leaf angle classes (test 1)*

It is very clear that the results from the simplified model in test1 are in good agreement with those from the GRTM under incident direct radiation. Figure 1 shows a comparison of the canopy reflectance and absorptance by the GRTM with those in test 1. It can be seen that the canopy reflectance and absorptance from the two models are almost the same. The maximum differences between them of the canopy reflectance and absorptance are 0.008 and 0.005 respectively, and the former is much less than an acceptable error of 0.02.

It is also shown that the results for diffuse radiation from the simplified model of test 1 are completely the same as those from the GRTM (figure omitted). The maximum differences of the canopy reflectance and absorptance are both 0.002, and the former is also much less than 0.02.

Therefore, the GRTM can at least be simplified reasonably to the model with three leaf angle classes for the leaf angle distribution.

3.2 *Simplification of the radiation angle sectors (test 2)*

By comparison of the canopy reflectance and absorptance results of the GRTM and those of test 2 (with three radiation sectors and nine leaf angle classes), we can see (figure omitted) that the two results are completely the same under both direct and diffuse radiation. Under direct radiation, the maximum canopy reflectance difference is 0.01; and the maximum absolute and relative canopy absorptance differences are 0.011 and 3.8% respectively. Under diffuse radiation, the maximum canopy reflectance difference is 0.009; and the maximum absolute and relative canopy absorptance differences are 0.011 and 2.1% respectively.

We also compared the results of the model with several other radiation angle sectors with those of the GRTM under diffuse radiation and found that the canopy reflectance results of the model with less than three radiation angle sectors are coarse and can bring differences of over 0.02. Table 1 is the comparison of the results by the GRTM and by the model with one, two, three, nine and eighteen radiation angle sectors (r1, r2, r3, r9, r18), with a 45° leaf inclination angle. We can see that the radiative transfer results with three, nine and eighteen radiation angle sectors are the same, but large differences appeared in the comparison results of the model with only one radiation angle zone (one stream upward and one downward) with the GRTM. The canopy reflectance difference is 0.02 for the model with two radiation angle sectors (two streams upward and two downward) compared with the GRTM with two radiation angle sectors.

Therefore, the GRTM could also be simplified reasonably into the model with three radiation angle sectors, which corresponds to a six-stream model.

3.3 *Simplification of both the radiation angle sectors and the leaf angle classes (test 3)*

From the comparison of the canopy reflectance and absorptance results by the GRTM with those by the simplified model (with three radiation angle sectors and three leaf angle classes), we can see (figure omitted) that the results of the two models are the same

Table 1. Comparisons of the results by the GRTM with one, two, three, nine and eighteen radiation angle sectors.

		Radiation angle sectors				
		r1	r2	r3	r9	r18
Components of the radiation fluxes						
Canopy reflectance		0.415	0.396	0.385	0.377	0.376
Canopy transmittance		0.159	0.107	0.115	0.113	0.111
Canopy absorptance		0.459	0.518	0.523	0.532	0.535
Ground absorption		0.127	0.085	0.092	0.09	0.089
Difference						
	r1−r18	r2−r18	r2−r9	r3−r18	r3−r9	r9−r18
Canopy reflectance	0.039	0.020	0.019	0.009	0.008	0.001
Canopy transmittance	0.048	−0.004	−0.006	0.004	0.002	0.002
Canopy absorptance	−0.076	−0.017	−0.014	−0.012	−0.009	0
Ground absorption	0.038	−0.004	−0.005	0.003	0.002	0.001

Note: $\bar{\lambda} = 45°$, n10: LAI=5, ρ_l=0.5, τ_l=0.3, ρ_s=0.2. r1, r2, r3, r9 and r18 stand for 1, 2, 3, 9, and 18 radiation angle sectors respectively.

under both direct and diffuse radiation. Under direct radiation, the maximum canopy reflectance difference is 0.013; and the maximum absolute and relative canopy absorptance differences are 0.013 and 3.9% respectively. Under diffuse radiation, the maximum canopy reflectance difference is 0.011; and the maximum absolute and relative canopy absorptance differences are 0.011 and 2.2% respectively.

Therefore, the GRTM can be further simplified reasonably into the model with three radiation angle sectors (corresponding to a six-stream model) and three leaf angle classes model (simplified model), and this is the final simplified version of GRTM (SGRTM) used later. Using the SGRTM, computer resources are greatly saved, the requirement of information on vegetation morphology is reduced, and the restriction of inputting diffuse radiation (such as isotropic diffuse radiation) is alleviated.

4. Explicit analytical solutions for the radiative transfer in the canopy

4.1 *Analytical solutions for the radiative transfer under diffuse radiation*

Without losing generality, the incident downward diffuse radiation from the atmosphere into the plant canopy can be set to unity. In order to derive the explicit analytical solutions of diffuse radiation transfer with various soil reflectance values, a basic solution of diffuse radiation transfer within the canopy but with zero soil reflectance needs to be solved first. Under zero soil reflectance, the canopy reflectance and transmittance for a unit diffuse radiation flux incident downward from the top of the canopy are noted as α_f and β_f, then canopy absorptance is $(1-\alpha_f-\beta_f)$; and the canopy reflectance, transmittance, and absorptance are α_b, β_b, and $(1-\alpha_b-\beta_b)$ respectively, for a unit diffuse radiation flux emitted upward from the bottom of the canopy. Subscripts f and b mean downward and upward direction respectively.

In a real situation, soil reflectance ρ_s is not equal to zero, so the soil will reflect and absorb part of the transmitted radiation β_f again. Referring to Fig. 2, the absorbed radiation from the transmitted radiation β_f by the soil is $(1-\rho_s)\beta_f$, and the reflected radiation is $\rho_s\beta_f$. Then $\rho_s\beta_f$ will be scattered by the canopy again: reflected, absorbed and transmitted by the canopy. The downward reflected radiation from the reflected radiation $\rho_s\beta_f$ by the canopy is $\rho_s\alpha_b\beta_f$, the upward transmitted radiation is $\rho_s\beta_f\beta_b$, and the absorbed radiation is $\rho_s\beta_f(1-\alpha_b-\beta_b)$. The reflected radiation $\rho_s\alpha_b\beta_f$ moving downward to the soil surface will be reflected and absorbed again by the soil. This successive radiative scattering transfer process between the underlying soil and the vegetation system continues infinitely. The total canopy reflectance (or albedo), canopy transmittance, canopy absorptance, and ground absorption, below the canopy can be obtained through summing up all the upward radiations leaving the canopy top to the atmosphere, all the downward radiations leaving the canopy bottom to the soil, all the radiations absorbed by the canopy, and all the ground absorbed radiations. It is found that forms of the summation equations are infinite series similar to Taylor series expansions. After analysis, it is found the series is convergent to the very simple analytical solution indicated by Eqs. (2)–(5). Thus the canopy reflectance α_c, canopy transmittance β_c, canopy absor-

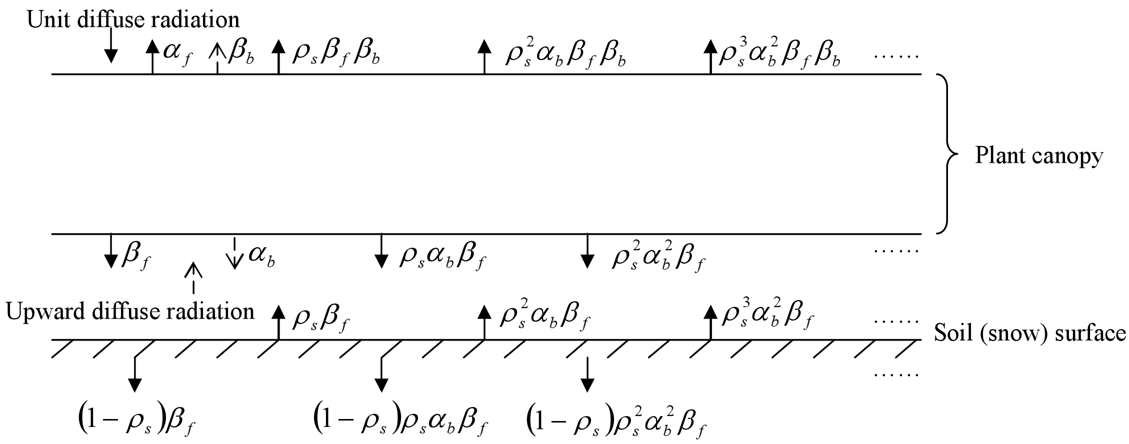

Fig. 2. Schematic figure of radiative transfer in the canopy under diffuse radiation.

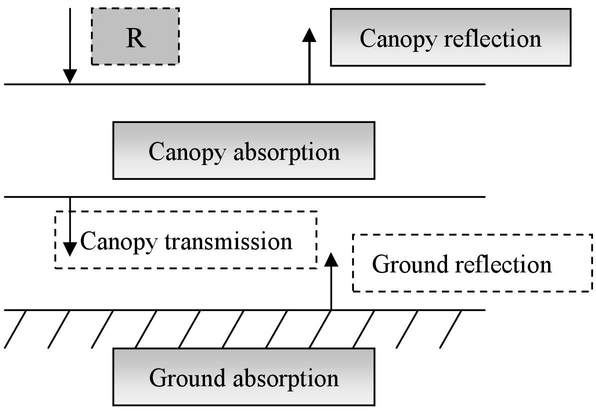

Fig. 3. Sketch map of radiative transfer in the canopy.

ptance γ_c, and ground absorption γ_g can be derived as:

$$\begin{aligned}\text{Canopy reflectance } \alpha_\mathrm{c} &= \alpha_\mathrm{f} + \rho_\mathrm{s}\beta_\mathrm{f}\beta_\mathrm{b} + \rho_\mathrm{s}^2\alpha_\mathrm{b}\beta_\mathrm{f}\beta_\mathrm{b} + \\ &\quad \rho_\mathrm{s}^3\alpha_\mathrm{b}^2\beta_\mathrm{f}\beta_\mathrm{b} + \cdots + \rho_\mathrm{s}^n\alpha_\mathrm{b}^{n-1}\beta_\mathrm{f}\beta_\mathrm{b} + \cdots \\ &= \alpha_\mathrm{f} + \rho_\mathrm{s}\beta_\mathrm{f}\beta_\mathrm{b}(1 + \rho_\mathrm{s}\alpha_\mathrm{b} + \rho_\mathrm{s}^2\alpha_\mathrm{b}^2 + \\ &\quad \rho_\mathrm{s}^3\alpha_\mathrm{b}^3 + \cdots + \rho_\mathrm{s}^{n-1}\alpha_\mathrm{b}^{n-1} + \cdots) \\ &= \alpha_\mathrm{f} + \rho_\mathrm{s}\beta_\mathrm{f}\beta_\mathrm{b}\left(\frac{1}{1-\rho_\mathrm{s}\alpha_\mathrm{b}}\right) ,\end{aligned} \tag{2}$$

$$\begin{aligned}\text{Canopy transmittance } \beta_\mathrm{c} &= \beta_\mathrm{f} + \rho_\mathrm{s}\alpha_\mathrm{b}\beta_\mathrm{f} + \rho_\mathrm{s}^2\alpha_\mathrm{b}^2\beta_\mathrm{f} + \\ &\quad \rho_\mathrm{s}^3\alpha_\mathrm{b}^3\beta_\mathrm{f} + \cdots + \rho_\mathrm{s}^n\alpha_\mathrm{b}^n\beta_\mathrm{f} + \cdots \\ &= \beta_\mathrm{f}(1 + \rho_\mathrm{s}\alpha_\mathrm{b} + \rho_\mathrm{s}^2\alpha_\mathrm{b}^2 + \rho_\mathrm{s}^3\alpha_\mathrm{b}^3 + \cdots + \rho_\mathrm{s}^n\alpha_\mathrm{f}^n + \cdots) \\ &= \beta_\mathrm{f}\left(\frac{1}{1-\rho_\mathrm{s}\alpha_\mathrm{b}}\right) ,\end{aligned} \tag{3}$$

$$\begin{aligned}\text{Canopy absorptance } \gamma_\mathrm{c} &= (1-\alpha_\mathrm{f}-\beta_\mathrm{f}) + \\ &\quad \rho_\mathrm{s}\beta_\mathrm{f}(1-\alpha_\mathrm{b}-\beta_\mathrm{b}) + \rho_\mathrm{s}^2\alpha_\mathrm{b}\beta_\mathrm{f}(1-\alpha_\mathrm{b}-\beta_\mathrm{b}) + \cdots + \\ &\quad \rho_\mathrm{s}^n\alpha_\mathrm{b}^{n-1}\beta_\mathrm{f}(1-\alpha_\mathrm{b}-\beta_\mathrm{b}) + \cdots \\ &= (1-\alpha_\mathrm{f}-\beta_\mathrm{f}) + (1-\alpha_\mathrm{b}-\beta_\mathrm{b})\times \\ &\quad [\rho_\mathrm{s}\beta_\mathrm{f}(1 + \rho_\mathrm{s}\alpha_\mathrm{b} + \rho_\mathrm{s}^2\alpha_\mathrm{b}^2 + \cdots + \rho_\mathrm{s}^{n-1}\alpha_\mathrm{b}^{n-1} + \cdots)] \\ &= (1-\alpha_\mathrm{f}-\beta_\mathrm{f}) + (1-\alpha_\mathrm{b}-\beta_\mathrm{b})\left(\rho_\mathrm{s}\beta_\mathrm{f}\frac{1}{1-\rho_\mathrm{s}\alpha_\mathrm{b}}\right) ,\end{aligned} \tag{4}$$

$$\begin{aligned}\text{Ground absorption } \gamma_\mathrm{g} &= (1-\rho_\mathrm{s})\beta_\mathrm{f} + \\ &\quad (1-\rho_\mathrm{s})\rho_\mathrm{s}\alpha_\mathrm{b}\beta_\mathrm{f} + (1-\rho_\mathrm{s})\rho_\mathrm{s}^2\alpha_\mathrm{b}^2\beta_\mathrm{f} + \\ &\quad (1-\rho_\mathrm{s})\rho_\mathrm{s}^3\alpha_\mathrm{b}^3\beta_\mathrm{f} + \cdots + (1-\rho_\mathrm{s})\rho_\mathrm{s}^n\alpha_\mathrm{b}^n\beta_\mathrm{f} + \cdots \\ &= (1-\rho_\mathrm{s})\beta_\mathrm{f}(1 + \rho_\mathrm{s}\alpha_\mathrm{b} + \rho_\mathrm{s}^2\alpha_\mathrm{b}^2 + \\ &\quad \rho_\mathrm{s}^3\alpha_\mathrm{b}^3 + \cdots + \rho_\mathrm{s}^n\alpha_\mathrm{b}^n + \cdots) \\ &= (1-\rho_\mathrm{s})\beta_\mathrm{f}\frac{1}{1-\rho_\mathrm{s}\alpha_\mathrm{b}} .\end{aligned} \tag{5}$$

Using the above derivation, the real canopy reflectance, canopy absorptance, canopy transmittance, and ground absorption under different soil reflectance ρ_s can easily be calculated, as long as two sets of basic solutions of canopy reflectance and canopy transmittance (one set of solution is α_f and β_f; the other set is α_b and β_b) under the condition of zero soil reflectance ($\rho_\mathrm{s} = 0$) is obtained from either the GRTM or its simplified model.

Figure 3 is the schematic map of radiative transfer in the canopy, in which R is the downward radiation, and R is separated into canopy absorption (R_c), ground absorption (R_g) and canopy reflection (R_r), canopy transmission (R_t) minus ground reflection (R_p) is ground absorption (R_g). For

$$R_\mathrm{p} = R_\mathrm{t} \times \rho_\mathrm{s} ,$$

ground absorption can also be expressed as:

$$R_\mathrm{g} = R_\mathrm{t} \times (1-\rho_\mathrm{s}) .$$

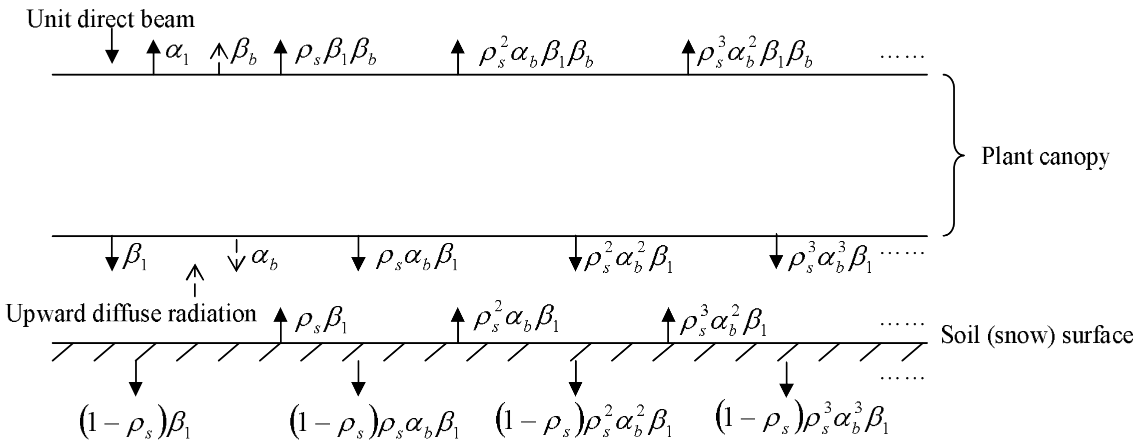

Fig. 4. Schematic figure of radiative transfer in the canopy under direct beam radiation.

If A is a unit radiation flux, the radiation absorption, reflection and transmission by the canopy is equal to the canopy reflectance, absorptance, and transmittance, respectively. We can see the radiative transfer components can be calculated by the analytical solutions once the canopy reflectance and canopy transmittance with zero soil reflectance (basic solutions) are given. The analytical solutions [Eqs. (2)–(5)] verify this, and also the rationality of the derived results. So, for the following derivations, we will give only the expressions of canopy reflectance and transmittance, respectively, and omit the expressions of other components.

For canopies with plants of the same architecture (e.g., the same vertical shapes, and the same adaxial and abaxial leaf optical properties) under the uniform radiation distribution, $\alpha_{\rm f}=\alpha_{\rm b}, \beta_{\rm f}=\beta_{\rm b}$, the analytical formulas can be more simple and they only need one set of the basic solutions, e.g., $\alpha_{\rm f}$ and $\beta_{\rm f}$. The canopy reflectance and transmittance can be written as:

$$\begin{aligned}\alpha_{\rm c} &= \alpha_{\rm f}+\rho_{\rm s}\beta_{\rm f}^2+\rho_{\rm s}^2\alpha_{\rm f}\beta_{\rm f}^2+\rho_{\rm s}^3\alpha_{\rm f}^2\beta_{\rm f}^2+\cdots+ \\ &\quad \rho_{\rm s}^n\alpha_{\rm f}^{n-1}\beta_{\rm f}^2+\cdots \\ &= \alpha_{\rm f}+\rho_{\rm s}\beta_{\rm f}^2(1+\rho_{\rm s}\alpha_{\rm f}+\rho_{\rm s}^2\alpha_{\rm f}^2+ \\ &\quad \rho_{\rm s}^3\alpha_{\rm f}^3+\cdots+\rho_{\rm s}^{n-1}\alpha_{\rm f}^{n-1})+\cdots \\ &= \alpha_{\rm f}+\rho_{\rm s}\beta_{\rm f}^2\left(\frac{1}{1-\rho_{\rm s}\alpha_{\rm f}}\right) ,\end{aligned} \tag{6}$$

$$\begin{aligned}\beta_{\rm c} &= \beta_{\rm f}+\rho_{\rm s}\alpha_{\rm f}\beta_{\rm f}+\rho_{\rm s}^2\alpha_{\rm f}^2\beta_{\rm f}+\rho_{\rm s}^3\alpha_{\rm f}^3\beta_{\rm f}+\cdots+ \\ &\quad \rho_{\rm s}^n\alpha_{\rm f}^n\beta_{\rm f}+\cdots \\ &= \beta_{\rm f}(1+\rho_{\rm s}\alpha_{\rm f}+\rho_{\rm s}^2\alpha_{\rm f}^2+\rho_{\rm s}^3\alpha_{\rm f}^3+\cdots+ \\ &\quad \rho_{\rm s}^n\alpha_{\rm f}^n+\cdots) \\ &= \beta_{\rm f}\left(\frac{1}{1-\rho_{\rm s}\alpha_{\rm f}}\right) .\end{aligned} \tag{7}$$

4.2 *Analytical solutions for the radiative transfer under direct radiation*

The situation is similar for the direct radiation (see Fig. 4). The canopy reflectance and transmittance are α_1 and β_1, respectively, with zero soil reflectance under a beam of direct radiation. α_1 and β_1 are the functions of the solar inclination angle. The direct beam radiation is scattered into diffuse radiation, once it strikes the leaves. So, the canopy reflectance $\alpha_{\rm b}$ and transmittance $\beta_{\rm b}$ with zero soil reflectance under diffuse radiation incident upward from the bottom of the canopy are to be known. Denote $\rho_{\rm s}$ as the soil (snow) reflectance, similar to the derivations in section 4.1, and the canopy reflectance and transmittance of the canopy and soil system under direct beam radiation are written as:

$$\begin{aligned}\alpha_{\rm c} &= \alpha_1+\rho_{\rm s}\beta_1\beta_{\rm b}+\rho_{\rm s}^2\alpha_{\rm b}\beta_1\beta_{\rm b}+\rho_{\rm s}^3\alpha_{\rm b}^2\beta_1\beta_{\rm b}+\cdots+ \\ &\quad \rho_{\rm s}^n\alpha_{\rm b}^{n-1}\beta_1\beta_{\rm b}+\cdots \\ &= \alpha_1+\rho_{\rm s}\beta_1\beta_{\rm b}(1+\rho_{\rm s}\alpha_{\rm b}+\rho_{\rm s}^2\alpha_{\rm b}^2+ \\ &\quad \rho_{\rm s}^3\alpha_{\rm b}^3+\cdots+\rho_{\rm s}^{n-1}\alpha_{\rm b}^{n-1}+\cdots) \\ &= \alpha_1+\rho_{\rm s}\beta_1\beta_{\rm b}\left(\frac{1}{1-\rho_{\rm s}\alpha_{\rm b}}\right) ,\end{aligned} \tag{8}$$

$$\begin{aligned}\beta_{\rm c} &= \beta_1+\rho_{\rm s}\alpha_{\rm b}\beta_1+\rho_{\rm s}^2\alpha_{\rm b}^2\beta_1+\rho_{\rm s}^3\alpha_{\rm b}^3\beta_1+\cdots+ \\ &\quad +\rho_{\rm s}^n\alpha_{\rm b}^n\beta_1+\cdots \\ &= \beta_1(1+\rho_{\rm s}\alpha_{\rm b}+\rho_{\rm s}^2\alpha_{\rm b}^2+\rho_{\rm s}^3\alpha_{\rm b}^3+\cdots+ \\ &\quad \rho_{\rm s}^n\alpha_{\rm b}^n+\cdots) \\ &= \beta_1\left(\frac{1}{1-\rho_{\rm s}\alpha_{\rm b}}\right) .\end{aligned} \tag{9}$$

Note that $\alpha_{\rm c}$ and $\beta_{\rm c}$ are also the functions of the solar inclination angle.

Taking α_1 and β_1 as the same forms of $\alpha_{\rm f}$ and $\beta_{\rm f}$ respectively, the explicit analytical solutions can be

Table 2. Comparison of the radiative transfer results by the model and the analytical solutions for different adaxial and abaxial leaf optical properties under diffuse radiation.

$\rho_l = 0.5, \rho_l' = 0.3, \tau_l = 0.1, \tau_l' = 0.2$								
	Canopy reflectance α_c		Canopy transmittance β_c		Canopy absorptance γ_c		Ground absorption γ_g	
ρ_s	Model result	Formula result	Model result	Formula result	Model result	Formula result	Model result	Formula result
0.0	0.168		0.515		0.317		0.515	
0.1	0.196	0.196	0.521	0.521	0.335	0.335	0.469	0.469
0.2	0.224	0.225	0.528	0.528	0.353	0.353	0.422	0.422
0.3	0.254	0.254	0.534	0.534	0.372	0.372	0.374	0.374
0.4	0.284	0.284	0.541	0.541	0.392	0.391	0.324	0.324
0.5	0.315	0.315	0.548	0.548	0.411	0.411	0.274	0.274
0.6	0.346	0.347	0.554	0.555	0.432	0.431	0.222	0.222
0.7	0.379	0.380	0.562	0.562	0.453	0.452	0.168	0.169
0.8	0.412	0.413	0.569	0.569	0.474	0.473	0.114	0.114

Note: α_b=0.119, β_b=0.538.

Table 3. Comparison of the radiative transfer results by the model and the analytical solutions for different adaxial and abaxial leaf optical properties under direct beam radiation.

$\rho_l = 0.5, \rho_l' = 0.3, \tau_l = 0.1, \tau_l' = 0.2$								
	Canopy reflectance α_c		Canopy transmittance β_c		Canopy absorptance γ_c		Ground absorption γ_g	
ρ_s	Model result	Formula result	Model result	Formula result	Model result	Formula result	Model result	Formula result
0.0	0.155		0.559		0.287		0.559	
0.1	0.185	0.185	0.565	0.566	0.306	0.305	0.509	0.509
0.2	0.216	0.217	0.572	0.573	0.326	0.325	0.458	0.458
0.3	0.248	0.249	0.579	0.580	0.347	0.346	0.405	0.406
0.4	0.281	0.281	0.586	0.587	0.368	0.367	0.352	0.352
0.5	0.314	0.315	0.594	0.594	0.389	0.388	0.297	0.297
0.6	0.349	0.349	0.601	0.602	0.411	0.410	0.240	0.241
0.7	0.384	0.385	0.609	0.610	0.434	0.432	0.183	0.183
0.8	0.420	0.421	0.617	0.618	0.457	0.456	0.123	0.124

Note: α_b=0.119, β_b=0.538.

written in general formula forms as in Eqs. (2)–(5). However, note that under direct beam radiation, they are the functions of the solar inclination angle.

4.3 *Verification of the analytical solutions*

The analytical solutions derived above were verified in canopies with various representative combinations of different LAIs, leaf angle distributions, and leaf optical properties, under separate direct beam and diffuse radiation. The results of radiative transfer components, such as canopy reflectance, transmittance, absorptance, and ground absorption etc. by the analytical solutions are all considerably accurate compared to the results by the GRTM model. Table 2 and Table 3 show the comparison of the results by formulas and those by the GRTM model under diffuse and direct beam radiation respectively, for the same canopy: leaf angle distribution is spherical; LAI=1; leaf adaxial reflectance $\rho_l = 0.5$, abaxial reflectance $\rho_l' = 0.3$, leaf adaxial transmittance $\tau_l = 0.1$, abaxial transmittance $\tau_l' = 0.2$. We can see that the analytical solution formulas results are accurate. Table 4 gives the comparison of the analytical solution formulas results with the model results under uniform diffuse radiation for the canopy of LAI=1, leaf reflectance $\rho_l = 0.5$, leaf transmittance $\tau_l = 0.3$, and a leaf angle of 50°. It needs only one set of basic solutions of canopy reflectance and transmittance, e.g., α_f and β_f are 0.230 and 0.597 respectively, in Table 4. We can see that the two model results are coincident. Other verification results are similar to those in Table 5, and the results by the formulas are accurate and there is no

Table 4. Comparison of the radiative transfer results by the model and the analytical solutions under diffuse radiation.

ρ_s	Canopy reflectance α_c		Canopy transmittance β_c		Canopy absorptance γ_c		Ground absorption γ_g	
	Model result	Formula result	Model result	Formula result	Model result	Formula result	Model result	Formula result
0	0.230		0.597		0.173	0.173	0.597	0.597
0.1	0.267	0.267	0.610	0.611	0.184	0.184	0.549	0.550
0.2	0.305	0.305	0.625	0.626	0.195	0.195	0.500	0.501
0.3	0.345	0.345	0.641	0.641	0.206	0.206	0.449	0.449
0.4	0.387	0.387	0.657	0.658	0.218	0.219	0.394	0.395
0.5	0.432	0.431	0.674	0.675	0.231	0.231	0.337	0.337
0.6	0.478	0.478	0.692	0.693	0.245	0.245	0.277	0.277
0.7	0.527	0.527	0.711	0.712	0.260	0.259	0.213	0.214
0.8	0.579	0.579	0.730	0.732	0.275	0.274	0.146	0.146

Note: LAI=1, ρ_l=0.5, τ_l=0.3, $\bar{\lambda} = 50°$.

Table 5. Comparison of the canopy reflectance and transmittance results by the model (two-stream model) and those by the analytical solutions.

β_s	Canopy reflectance α_c		Canopy transmittance β_c	
	Model result	Formula result	Model result	Formula result
0	0.363		0.207	
0.1	0.367	0.367	0.215	0.215
0.2	0.372	0.372	0.223	0.224
0.3	0.377	0.377	0.232	0.233
0.4	0.383	0.383	0.242	0.243
0.5	0.389	0.389	0.253	0.253
0.6	0.396	0.396	0.265	0.265
0.7	0.403	0.403	0.278	0.278
0.8	0.411	0.411	0.292	0.292

need to discuss it any further.

The analytical solution formulas results are in good agreement with the model results for the radiative transfer in the canopy with different adaxial and abaxial leaf optical properties.

The analytical solutions are also suitable for the two-stream model. Table 5 is the comparison of the results by the two-stream model and the analytical solution formulas for the canopy of LAI=5, leaf reflectance $\rho_l = 0.5$, and leaf transmittance $\tau_l = 0.3$ and vertical leaf angle distribution under diffuse radiation. We can see that the canopy reflectance and transmittance from the two-stream model and the formulas are the same.

4.4 *Expansion of the analytical solutions for the radiative transfer in the canopy with the same adaxial and abaxial leaf optical properties*

There are various vegetation types covered on the land surface, which have been divided into 18 and 13 types in BATS (Dickinson et al., 1993) and SiB2 (Sellers et al., 1996), respectively. Due to varying soil moisture across the seasons, reflectance of the soil under the vegetation can change a great deal, and the vegetation itself can also vary considerably, such as, for example, its LAI. The impact and feedback of changes in vegetation during different growth periods are also important to investigate in climate studies. At present, some land surface process models have coupled physical processes and plant physiological and ecological processes in the atmosphere over the land surface, vegetation, and soil, to construct vegetation–atmosphere bidirectional interacting models, e.g., the AVIM model (Atmosphere-Vegetation Interaction Model) (Ji, 1995). Due to computer resource limitations, simple and accurate predictions by the land surface model are required in the climate model. For a vegetation species, we fit the basic solutions for its different growth seasons, and couple with the derived analytical solution formulas which no doubt can decrease the amount of computing required and meet the needs of the climate model.

For vegetation with a spherical leaf angle distribution, its leaf reflectance ρ_l and transmittance τ_l are

Table 6. Comparison of the radiative transfer results by the model and those by fitting coupling analytical solutions (VIS waveband, diffuse radiation).

ρ_s	Canopy reflectance α_c		Canopy transmittance β_c		Canopy absorptance γ_c		Ground absorption γ_g	
	Model result	Fitting result	Model result	Fitting result	Model result	Fitting result	Model result	Fitting result
0	0.047	0.047	0.054	0.055	0.899	0.898	0.054	0.055
0.1	0.047	0.047	0.054	0.055	0.904	0.903	0.049	0.050
0.2	0.048	0.048	0.055	0.055	0.909	0.908	0.044	0.044
0.3	0.048	0.048	0.055	0.056	0.914	0.913	0.038	0.039
0.4	0.048	0.048	0.055	0.056	0.919	0.918	0.033	0.034
0.5	0.049	0.049	0.055	0.056	0.924	0.923	0.028	0.028
0.6	0.049	0.049	0.056	0.056	0.929	0.929	0.022	0.023
0.7	0.049	0.049	0.056	0.057	0.934	0.934	0.017	0.017
0.8	0.050	0.050	0.056	0.057	0.939	0.939	0.011	0.011

Table 7. Comparison of the radiative transfer results by the model and those by fitting coupling analytical solutions (NIR waveband, diffuse radiation).

ρ_s	Canopy reflectance α_c		Canopy transmittance β_c		Canopy absorptance γ_c		Ground absorption γ_g	
	Model result	Fitting result	Model result	Fitting result	Model result	Fitting result	Model result	Fitting result
0	0.353	0.353	0.152	0.151	0.495	0.496	0.152	0.151
0.1	0.355	0.355	0.158	0.157	0.503	0.504	0.142	0.141
0.2	0.358	0.358	0.164	0.163	0.511	0.512	0.131	0.130
0.3	0.361	0.361	0.170	0.169	0.520	0.521	0.119	0.118
0.4	0.364	0.364	0.177	0.176	0.530	0.531	0.106	0.106
0.5	0.367	0.367	0.185	0.183	0.541	0.542	0.092	0.092
0.6	0.371	0.370	0.193	0.192	0.552	0.553	0.077	0.077
0.7	0.375	0.374	0.202	0.201	0.565	0.566	0.061	0.060
0.8	0.379	0.378	0.212	0.210	0.579	0.580	0.042	0.042

0.1 and 0.1 in the VIS waveband, respectively. We select 16 groups of its total LAI (TLAI) in various growth phases as: 0.1, 0.2, 0.5, 0.8, 1, 1.5, 2, 2.5, 3, 4, 5, 6, 7, 8, 9, 10. Then, we can obtain 16 corresponding groups of basic solutions of canopy reflectance (zero soil reflectance) by the model under diffuse radiation as: 0.009, 0.015, 0.028, 0.035, 0.038, 0.043, 0.045, 0.046, 0.047, 0.047, 0.047, 0.047, 0.047, 0.047, 0.047, 0.047; and 16 groups of canopy transmittance as: 0.917, 0.844, 0.672, 0.543, 0.473, 0.339, 0.245, 0.179, 0.132, 0.073, 0.040, 0.023, 0.013, 0.007, 0.004, 0.002. The fitting curves for the basic solutions (canopy reflectance and transmittance with zero soil reflectance) are α_0 and β_0, as in the following:

$$\alpha_0(\mathrm{TLAI}) = 0.04663\exp(0.0009856\times\mathrm{TLAI}) - 0.04452\exp(-1.696\times\mathrm{TLAI})\,, \quad (10a)$$

$$\alpha_0(\mathrm{TLAI}) = 0.2613\exp(-1.426\times\mathrm{TLAI}) + 0.729\exp(-0.5774\times\mathrm{TLAI})\,. \quad (10b)$$

See a and b in Fig. 5: the fitting results are in good agreement with the model results.

Coupling the fitting curves (10a) and (10b) with the analytical solutions (2)–(5) we can obtain the canopy reflectance, transmittance, absorptance, and the ground absorption for a given type of vegetation (with certain leaf optical properties), various growth phases (shown by the LAI), and various soil reflectance values.

For example, the basic solutions of canopy reflectance α_f and transmittance β_f with $\rho_s = 0$ are 0.047 and 0.055 by the fitting equations (10a) and (10b) for the above mentioned vegetation with LAI=4.5. Take the basic solutions into formulas (2)–(5), and we can get canopy reflectance, transmittance, absorptance, and ground absorption with various soil reflectance (see Table 6), and find that the results are in good agreement with those by the model, and that the maximum difference is 0.001.

Using the same method, we can deal with radiation for other wavebands, under diffuse radiation, for var-

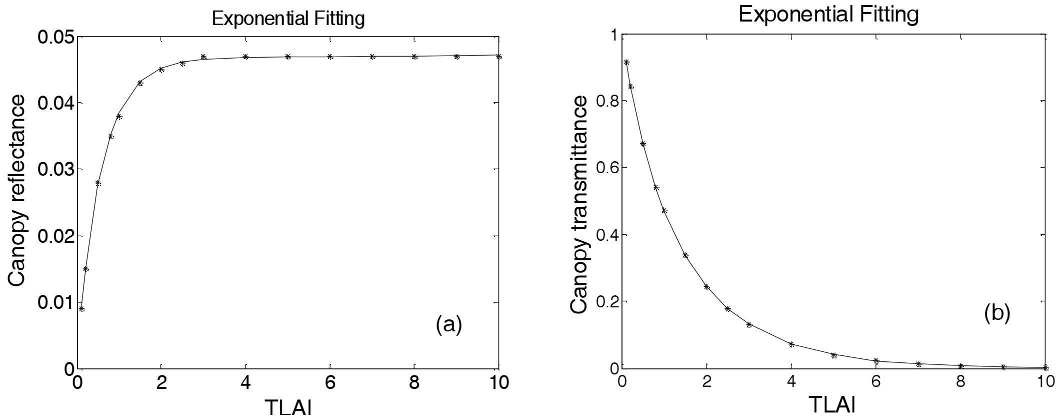

Fig. 5. Fitting curves for the basic solutions of (a) canopy reflectance and (b) transmittance in the VIS waveband. ($\rho_l = 0.1, \tau_l = 0.1$; spherical leaf angle distribution; $\rho_s = 0$.)

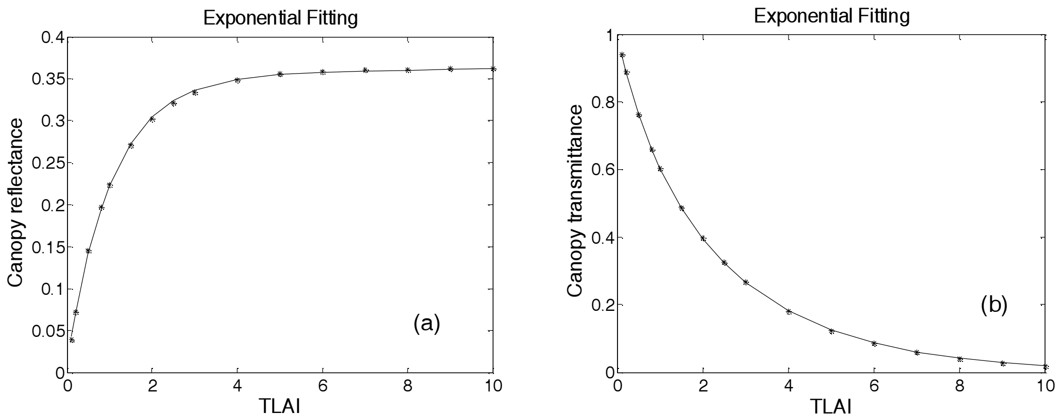

Fig. 6. Fitting curves for the basic solutions of (a) canopy reflectance and (b) transmittance in the VIS waveband. ($\rho_l = 0.5, \tau_l = 0.3$; spherical leaf angle distribution; $\rho_s = 0$.)

ious vegetations and give their specific fitting curves. For a case in the NIR waveband in diffuse radiation, the leaf reflectance ρ_l and transmittance τ_l are 0.5 and 0.3 respectively, with a spherical leaf angle distribution, we select its 16 groups of TLAIs in different growth phases as: 0.1, 0.2, 0.5, 0.8, 1, 1.5, 2, 2.5, 3, 4, 5, 6, 7, 8, 9, 10. The corresponding 16 groups of basic solutions with 16 groups of total LAIs of canopy reflectance (zero soil reflectance) as: α_0=0.039, 0.072, 0.146, 0.197, 0.224, 0.271, 0.302, 0.321, 0.334, 0.349, 0.356, 0.359, 0.361, 0.361, 0.362, 0.362; and 16 groups canopy transmittance (zero soil reflectance) as: β_0=0.941, 0.889, 0.762, 0.661, 0.604, 0.487, 0.397, 0.325, 0.267, 0.181, 0.123, 0.085, 0.059, 0.040, 0.028, 0.019. The fitting curves for the canopy reflectance and transmittance with zero soil reflectance are α_0 and β_0 as following:

$$\alpha_0 = 0.3534\exp(0.002533 \times \text{TLAI}) - 0.3412\exp(-0.9493 \times \text{TLAI})\,, \tag{11a}$$

$$\beta_0 = 0.205\exp(-1.207 \times \text{TLAI}) + 0.7846\exp(-0.3676 \times \text{TLAI})\,. \tag{11b}$$

See Fig. 6: the fitting results are in good agreement with the model results.

Coupling the fitting basic solutions with the analytical solutions for the same vegetation with LAI=4.5 in the VIS waveband, we can obtain the radiative transfer results. Comparison with the model results shows that the two results are in good agreement (Table 7). The maximum difference of the two results is 0.002.

For direct beam radiation, we still take the above vegetation, and select the incident inclination angle as 45°. The fitting canopy reflectance and transmittance with $\rho_s = 0$ are $\alpha_{0,b}$ and $\beta_{0,b}$ in Eq. (12) for the VIS waveband and (13) for the NIR waveband. Comparisons with the model results are shown in Tables 8 and 9. We can see that the maximum difference is 0.001 and 0.004 respectively, and that the two results are in

Table 8. Comparison of the radiative transfer results by the model and those by fitting coupling analytical solutions (VIS waveband, direct beam radiation).

	Canopy reflectance α_c		Canopy transmittance β_c		Canopy absorptance γ_c		Ground absorption γ_g	
ρ_s	Model result	Fitting result	Model result	Fitting result	Model result	Fitting result	Model result	Fitting result
0	0.044	0.044	0.053	0.053	0.904	0.903	0.053	0.053
0.1	0.044	0.044	0.053	0.053	0.909	0.908	0.048	0.048
0.2	0.044	0.045	0.053	0.054	0.913	0.913	0.042	0.043
0.3	0.045	0.045	0.053	0.054	0.918	0.918	0.037	0.038
0.4	0.045	0.045	0.054	0.054	0.923	0.922	0.032	0.032
0.5	0.045	0.045	0.054	0.054	0.928	0.927	0.027	0.027
0.6	0.045	0.046	0.054	0.055	0.933	0.932	0.022	0.022
0.7	0.046	0.046	0.054	0.055	0.938	0.937	0.016	0.016
0.8	0.046	0.046	0.055	0.055	0.943	0.943	0.011	0.011

Note: $\alpha_b = 0.047$, $\beta_b = 0.055$.

Table 9. Comparison of the radiative transfer results by the model and those by fitting coupling analytical solutions (NIR waveband, direct beam radiation).

	Canopy reflectance α_c		Canopy transmittance β_c		Canopy absorptance γ_c		Ground absorption γ_g	
ρ_s	Model result	Fitting result	Model result	Fitting result	Model result	Fitting result	Model result	Fitting result
0	0.340	0.341	0.155	0.156	0.506	0.503	0.155	0.156
0.1	0.343	0.343	0.161	0.162	0.512	0.511	0.145	0.146
0.2	0.345	0.346	0.167	0.168	0.521	0.520	0.134	0.134
0.3	0.348	0.349	0.174	0.175	0.530	0.529	0.122	0.122
0.4	0.351	0.352	0.180	0.182	0.541	0.539	0.108	0.109
0.5	0.354	0.355	0.188	0.189	0.552	0.550	0.094	0.095
0.6	0.358	0.359	0.196	0.198	0.564	0.562	0.078	0.079
0.7	0.361	0.363	0.204	0.207	0.577	0.575	0.061	0.062
0.8	0.365	0.367	0.213	0.217	0.592	0.589	0.043	0.044

Note: α_b=0.353, $\beta_b = 0.151$.

good agreement.

$$\alpha_{0,b} = 0.04426\exp(-0.0009679\times\text{TLAI}) - 0.04259\exp(-1.325\times\text{TLAI})\ , \tag{12a}$$

$$\beta_{0,b} = 1.003\exp(-0.6533\times\text{TLAI}) + 2.268\times10^{-8}\times\exp(0.9929\times\text{TLAI})\ , \tag{12b}$$

$$\alpha_{0,b} = 0.3492\exp(-0.0002673\times\text{TLAI}) - 0.345\exp(-0.8372\times\text{TLAI})\ , \tag{13a}$$

$$\beta_{0,b} = 0.1165\exp(-0.9586\times\text{TLAI}) + 0.8834\exp(-0.388\times\text{TLAI})\ . \tag{13b}$$

5. Conclusions

In this study, we simplified the complicated GRTM with nine radiation angle sectors and nine leaf angle classes to a version with three radiation angle sectors and three leaf angle classes (corresponding to a six-stream model). Many verifications proved that the simplified version possesses almost the same accuracy as the GRTM, and carries the added advantage of being able to save on computer resources for use in land surface models.

Using the derivation method in atmospheric radiation, we derived a set of simple analytical formulas from the series similar to the Taylor ones for radiative transfer in the canopy under separate diffuse and direct beam radiation, which can deal with arbitrary underlying soil reflectance. This set of formulas can easily and rapidly obtain the radiative transfer components, such as canopy reflectance, transmittance, ground absorption etc., by using two set of basic solutions: one set is the canopy reflectance and transmittance with zero soil reflectance for a downward radiation, and the other is that for an upward radiation. The analytical formulas of the radiative components for the arbitrary underlying ground surfaces we derived are general ones suitable for various conditions: for example, canopies composed of different vegetation types (different overstory and understory vegetation in the model) in the vertical layers; different abaxial and adaxial leaf optical properties; different plant shapes vertically; for non-uniform downward radiations; and so on. These analytical formulas can be widely used: in the GRTM, but also in other radiative models (e.g., the two-stream model) of a linear system. The key point is to develop one or two basic solutions used in these formulas.

Comparison of the results by the GRTM and the analytical formulas shows that the two results are the same for various conditions, such as different combinations of various LAIs, leaf angle distributions, leaf optical properties, whether under direct beam or diffuse sky radiations. These analytical formulas are accurate and time-saving, simplifying the computation and reducing the level of computer resources necessary.

The equation fittings of canopy reflectance and transmittance changing with the LAI under the condition of zero soil reflectance, are indicative of the growing conditions of the vegetation through the seasons. By coupling the equations with the analytical formulas derived above, many important issues for radiative transfer in various vegetation types changing with the seasons can be obtained. In this paper, we have fitted the basic solutions of one vegetation type versus LAI, and calculated the radiative transfer in the canopy by coupling this fitting solution to the analytical solutions. Further simplification work could also be extended to all of the 12 or more vegetation types in land surface processes in the future, which will improve the accuracy of radiative transfer estimation within the canopy, as well as continuing to save on computer resources.

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APPENDIX

Design of Experiment

The experiments n1–n12 indicate the status about various combinations of leaf area index (LAI), leaf reflection (ρ), transmission (τ), and soil reflection (ρ_s).

n1: LAI=1, $\rho = 0.1, \tau = 0.1, \rho_s = 0.8$;
n2: LAI=1, $\rho = 0.1, \tau = 0.1, \rho_s = 0.2$;
n3: LAI=5, $\rho = 0.1, \tau = 0.1, \rho_s = 0.8$;
n4: LAI=5, $\rho = 0.1, \tau = 0.1, \rho_s = 0.2$;
n5: LAI=8, $\rho = 0.1, \tau = 0.1, \rho_s = 0.8$;
n6: LAI=8, $\rho = 0.1, \tau = 0.1, \rho_s = 0.2$;
n7: LAI=1, $\rho = 0.5, \tau = 0.3, \rho_s = 0.8$;
n8: LAI=1, $\rho = 0.5, \tau = 0.3, \rho_s = 0.2$;
n9: LAI=5, $\rho = 0.5, \tau = 0.3, \rho_s = 0.8$;
n10: LAI=5, $\rho = 0.5, \tau = 0.3, \rho_s = 0.2$;
n11: LAI=8, $\rho = 0.5, \tau = 0.3, \rho_s = 0.8$;
n12: LAI=8, $\rho = 0.5, \tau = 0.3, \rho_s = 0.2$.

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